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THE ESTIMATION OF CATCHMENT AVERAGE POINT RAINFALL PROFILES

By

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ABSTRACT

Methods for estimating the average point rainfall profile for a catchment combining daily and autographic raingauge data are presented. The total rainfall depth is estimated by considering triangles of daily raingauges surrounding points on a mesh over the catchment. A method for detecting spatial variations in the daily rainfall totals at the raingauges which may be unacceptable for uniform input lumped rainfall-runoff models is included. The autographic profiles are aligned with reference to their most prominent features, and are scaled by the daily rainfalls and the average taken. The method is designed to maintain consistency in regions of sparse or dense daily raingauge coverage, and between events on the same catchment when different daily and autographic raingauges are operating.

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1 INTRODUCTION

Rainfall data for a particular catchment will usually consist of daily totals from storage gauges, together with hyetographs at shorter time intervals (frequently hourly) from autographic (recording) gauges. For many applications, especially for use as input to lumped catchment rainfall-runoff models, it is necessary to construct a catchment average hyetograph (or profile) combining the available data.

There are two ways in which a catchment average rainfall profile could be defined. Consider a hypothetical area with an infinitely dense network of autographic raingauges - an area totally covered with square collecting funnels for example. Over this area, the ordinates of the areal average profile may be determined exactly as the total volume collected in each time interval divided by the area. That is, the areal profile would simply be the average of all the individual gauge profiles. If there were differences in the shapes or timing of the individual profiles over the catchment, this areal average profile would not have the same shape as any of the individual ones; it would be attenuated, making the peak lower and the shape smoother.

The opposite extreme is the more abstract concept of the areal average point profile. In this, an attempt is made to find the average shape of the individual profiles, without reference to the timing, using a suitable alignment. It is then necessary to determine an average timing for use with this average shape. This form of average profile will not be so severely attenuated: the peak will usually be reduced much less, and any smoothing will tend to involve less important detail away from the peak.

For a real catchment, an estimate of an average profile must be made from available data. A simple estimate of the areal average profile was used in the U.K. Flood Studies (NERC, 1975, Vol IV pp.25-27) which was otained by means of a weighted average of all available profiles. However, this has been found to produce a profile whose smoothness is highly dependent on the number of gauges used. This bias is transmitted to the rainfall-runoff model and could affect the model parameters determined in some cases; for example, the smoother the rainfall profile used, the more peaked the unit hydrograph which is required to model a particular event. The problem here is basically one of the objective: it is not easy to extrapolate from the small and variable number of profiles observed for an event on a typical catchment to the areal average profile.

This problem of extrapolation does not arise if the objective is changed to the average point profile. In cases where only one gauge is available, it is more appropriate to regard its profile as the best estimate of the average point profile, and where there is more than one gauge, the profiles should be combined in such a way as to retain a representative shape. This would tend to remove the bias in catchment models due to different numbers of gauges operating for different events.

Therefore, the problem of deriving an areal average profile may be seen as consisting of two parts. Firstly it is necessary to develop a method for estimating the average point profile from the hourly and daily rainfall data and secondly it may be felt desirable that the areal average profile should be related to this average point profile as the former is perhaps closer to the ideal for input to a model. The first part is considered in this report, while the second is the subject of continuing research.

The problem of estimation of the average point profile may be further divided into two parts. Firstly, using all the available information from daily raingauges the average total depth in the storm is estimated and at the same time significant variations in the depths over the catchment may be detected. Secondly, the profiles observed at one or more autographic gauges are used to estimate the average profile shape. Applying the average depth to the average shape gives the average point profile.

Much of the material in this report is concerned with the development of a practical method for performing the calculations outlined above. When data are collected from a wide range of instrumented catchments for use in calibrating a lumped rainfall-runoff model such as the unit hydrograph model of the U.K. Flood Studies (NERC, 1975) the initial selection of suitable catchments and events is often made on the basis of the quality of the flow record which is available. This means that the rainfall data, although satisfying a minimum standard, may often be less than ideal, and the coverage of autographic and daily-read rainfall gauges will vary markedly between catchments and even between events on the same catchment. It is therefore desirable to develop a method which will provide an estimate of the catchment average rainfall for a given event in such a way as to eliminate as far as possible the effects of gauge density and distribution, and which will enable the input of consistent data to the rainfall-runoff model. The method proposed in this report uses a number of simple algorithms which are designed to work in all circumstances: more sophisticated techniques are available when large numbers of gauges can be used but when a large sample of catchments is selected it is inevitable that sparse distributions will be encountered.

Lumped rainfall-runoff models require the input of a catchment average rainfall profile which should be representative of the whole catchment if derived model parameters are to be compared between events. It is therefore necessary to check the raingauge data for spatial uniformity and storm movement. The method proposed includes a technique for detecting "unacceptable" spatial variations in daily rainfall totals (Section 4) but rapid storm movement cannot be observed reliably and analysed without a large number of autographic raingauges and so is beyond the scope of this report, although a method of analysis is provided by Marshall (1980).

2 ESTIMATION OF TOTAL RAINFALL DEPTH - THEORY

The estimation of areal average rainfall given the point falls at a number of irregularly distributed sites is a problem which has received much attention in the literature. However, it will be seen in the following review that no existing solution to this problem fully satisfied the stringent requirements of the present study. A new technique, the "Triangle Method" has been developed and is summarised in Section 2.2.

2.1 Review of possible techniques

The method used in the U.K. Flood Studies (NERC, 1975, Vol IV pp 26-27) for estimating the catchment average total storm rainfall depth is one of the simplest available. Daily raingauges were found within a quadrilateral surrounding the catchment. For each day of the storm, the rainfall at each gauge was expressed as a percentage of the gauge's average annual fall - determined for a standard period by the Meteorological Office - and these percentages were averaged over all the gauges. The percentage for each day thus obtained was converted back to a depth using the catchment annual average rainfall, and the individual days of the autographic profiles were then scaled by the corresponding days' depths. In this method it is assumed that a prior mapping of annual average rainfall is available from which to calculate the catchment value; this is the case for the U.K.

Although this method has some desirable features, such as the use of percentages of the annual average rainfall to produce more uniform readings, particularly in mountainous regions where the majority of gauges may be in the lower-lying areas, it does have two drawbacks which may become apparent for some events. The application of scaling to the days of a storm separately can distort the profile in an extreme manner when rain which is recorded at some daily gauges is not recorded at the autographic gauges. This can be avoided by considering the days of the storm together, but the other main drawback is more difficult to overcome. This is the fact that all the raingauges on the catchment have the same weighting when averages are taken, which is unsatisfactory because of the irregular distribution; the presence of a cluster of gauges in an area where the observed rain is atypical can distort the catchment average. Thus it is desirable to consider methods which provide weights for the gauges for use in the averaging process.

A set of raingauge weights should ideally have the following properties for use in the context envisaged in this study:

(i) the algorithm used for the determination of weights should be well-suited to computer implementation;

(ii) the weights for all the gauges within a close cluster should be roughly equal, so that gauges on the outside do not have much higher weightings than those on the inside of the cluster;

(iii) the total of the weights for all the gauges within a cluster should be roughly the same as the weight for a single gauge at the same site;

(iv) the method used should be applicable when the number of gauges available is small, which may be because the catchment is small or because it is located in a region where there are few gauges.

Properties (ii) and (iii) when taken together should, as far as possible, ensure consistency in weighting between different events on the same catchment with different sets of operational gauges.

The most widely used technique for weighting is the Thiessen polygon (or Voronoi polygon) method (Thiessen, 1911). In this method, the catchment is divided into polygons by the perpendicular bisectors of lines joining raingauges; in this way the polygon surrounding each gauge is that part of the catchment which is nearer to that gauge than to any other. The weighting for each gauge is the area of its surrounding polygon divided by the total catchment area. Although this is intuitively attractive, it is a method ill-suited to computer application, and will give low weights to gauges in the centre of a cluster, contrary to requirement (ii) above. Computational algorithms have been developed, for example geometrical methods as described by Forrest (1980). A Monte Carlo method was described by Diskin (1969); it was developed into a method using gridded trial points (Diskin, 1970), and this later method will be used in a modified form for the simpler problem of weighting autographic raingauges to be described in Section 5.2.

A number of other geometrical constructions lead to definitions of weights. The methods of Akin (1971) and of Goel and Aldabagh (1979) derive weights from dissections of the catchment based on triangles with raingauges at their vertices. Although these do produce satisfactory results, the dissections are regarded as a matter for hand computation, which would be impractical when considering large numbers of events and catchments. The method described by Bethlamy (1976) calculates weights based on the angles each gauge subtends at the major and minor axes of the catchment. Although it is an elegant method, it could lead to the total weight for a cluster becoming large by comparison with a single gauge elsewhere on the catchment. The method of Pande and Al-Mashidani (1978) is similar to the Thiessen method and can suffer from the same disadvantage although it is computationally simpler.

Other methods for estimating catchment average rainfall involve the representation of the catchment by a mesh of points, which is usually rectangular. The rainfall depths at the mesh points are then calculated by some method, and these are averaged to give the areal value.

Suppose there are M mesh points $G_1, G_2, \ldots G_M$ and there are N raingauges $R_1, R_2, \ldots R_N$ whose annual average rainfalls are $\sigma_1, \sigma_2, \ldots \sigma_N$ and which have readings $r_1, r_2, \ldots r_N$. Then if the estimated rainfall g_i at grid point G_i (expressed as a proportion of the annual average) is a weighted average of the observed falls at some or all of the raingauges, g_i may be written as

$$g_i = \sum_{j=1}^{N} w_{ij} \; r_j/\sigma_j \qquad (2.1)$$

where $\sum_{j=1}^{N} w_{ij} = 1$ for all i

and $w_{ij} \geq 0$ for all i, j

Now the catchment average rainfall P_A will be an average of the mesh point estimates. In the most general case, this average will allow a weight Z_i for each mesh point G_i, although in practice these weights may be equal (taking the value $^1/M$). Thus,

$$P_A = \left(\sum_{i=1}^{M} Z_i \; g_i \right) \sigma_A \qquad (2.2)$$

where $\sum_{i=1}^{M} Z_i = 1$

with $Z_i \geq 0$ for all i

and σ_A is the catchment annual average rainfall.

Hence, combining (2.1) and (2.2),

$$P_A = \left(\sum_{j=1}^{N} u_j \; r_j/\sigma_j \right) \sigma_A \qquad (2.3)$$

where $u_j = \sum_{i=1}^{M} Z_i w_{ij}$ (2.4)

with $\sum_{j=1}^{N} u_j = \sum_{i=1}^{M} Z_i \left(\sum_{j=1}^{N} w_{ij} \right) = \sum_{i=1}^{M} Z_i = 1$

and $u_j \geq 0$ for all j

So it can be seen that the catchment average rainfall is a weighted average of the raingauge readings, the weights being $\{u_j\}$ as given by (2.4). Thus the problem of determining point depths at mesh points is equivalent to that of determining raingauge weights, provided we assign weights to the mesh points in advance.

Some methods have been proposed (e.g. English, 1973) which involve the fitting of surfaces of varying degrees of complexity to nearby raingauge readings by a least-squares technique to give an estimate of the mesh point values, but these methods do not adapt well to very sparse distributions of gauges. The method of Chidley and Keys (1970) is similar, but unlike the nonlinear methods, it can be interpreted in terms of raingauge weightings as described above.

An appropriate and popular method for obtaining weights is that of reciprocal distances, whereby w_{ij} is given by

$$w_{ij} = \left(\frac{1}{D_{ij}{}^b} \right) \Big/ \left(\sum_{j=1}^{N} \frac{1}{D_{ij}{}^b} \right) \qquad (2.5)$$

where D_{ij} is the distance between gauge R_j and mesh point G_i and b is a parameter.

This method is most commonly used with b=2; Simanton and Osborn (1980) found that for an experimental network in south-west U.S.A., there were no significant differences in exponents in the range $1 \leq b \leq 3$. Salter (1972) used only the 6 nearest gauges to each mesh point and found that b=2 was preferable to b=1.

The method has two disadvantages. Firstly, it is intuitively unsatisfactory to have the rainfall at a point determined by all the gauges over an area; it would be preferable for it to be determined by nearby gauges only. The second problem is more fundamental: if there is an uneven distribution of gauges with a large cluster, together with others more widely spread, those in the cluster will have approximately equal weightings, and the total of the weightings for the cluster would be very much larger than the weighting of each of the scattered gauges, which could lead to biassed average rainfalls, contrary to requirement (iii) for raingauge weights proposed earlier in this section.

The first disadvantage is overcome and a potential means for avoiding the second is provided by the method of Dean and Snyder (1977). In this, the nearest four or five raingauges such that the lines joining them enclose the given mesh point are taken, and a reciprocal-distance weighting is applied to these surrounding gauges. Again, an inverse-square weighting was found satisfactory. From the nature of this construction, it is to be expected that only one gauge from a cluster would be used for each mesh point.

For the purposes of developing the simplest suitable method, the technique of Dean and Snyder (1977) was adapted, using a triangle to surround each mesh point. The modification to use a triangle rather than a quadrilateral was made in order that the method should be effective for more sparse distributions of gauges.

2.2 The triangle method

The chosen method may be summarised as follows.

(i) A mesh is constructed to represent the catchment.

(ii) For each mesh point, a search is made to find a triangle of raingauges which surround it. The search for gauges is limited to those which are within a given distance, D_o say, of the point. If no such triangle of gauges can be found for a particular mesh point, the three closest gauges are used.

(iii) The weightings w_{ij} for the raingauges in equation (2.1) are now calculated - an inverse-square distance method is used. Suppose the three gauges to be used for mesh point G_i are R_a, R_b, R_c at distances d_a, d_b, d_c respectively from G_i. Then,

$$w_{ij} = \begin{cases} \dfrac{\frac{1}{d_j^2}}{\frac{1}{d_a^2} + \frac{1}{d_b^2} + \frac{1}{d_c^2}} & j = a, b, c \\ 0 & \text{otherwise} \end{cases} \qquad (2.6)$$

The methods for constructing the mesh, and determining the weights $\{Z_i\}$ for the mesh points, for finding gauges within a catchment, for finding the triangles for the mesh points, and for determining the maximum distance D_o will be discussed in Section 3.

2.3 Scaling of autographic gauge profiles

The methods outlined so far may be used to obtain a catchment average total storm rainfall, which could be used to scale the observed hyetographs at each autographic raingauge on the catchment. However, when there is more than one autographic gauge on a catchment, a more satisfactory result is obtained with a slight modification: a separate total for each autographic gauge is derived using the weightings from the triangles as derived in the previous section, but using only the contributions to the weightings taken from those mesh points for which it is the nearest autographic gauge. In addition to providing a more accurate "corrected" hyetograph, this enables the total fall in each observed hyetograph to be compared with a total representative of nearby daily gauges.

Suppose there are ν autographic gauges $H_1, H_2, \ldots H_\nu$, each with a hyetograph T intervals long which is such that the fall at gauge k in interval t is ρ_{kt}, $1 \leq k \leq \nu$, $1 \leq t \leq T$. Define δ_{ik} as

$$\delta_{ik} = \begin{cases} 1 & \text{if } H_k \text{ is the nearest autographic gauge to grid point } G_i \\ 0 & \text{otherwise} \end{cases} \qquad (2.7)$$

Using this, an expression may be obtained for the scaled profile ρ'_{kt} at gauge H_k:

$$\rho'_{kt} = \sigma_A \, \rho_{kt} \left[\frac{\sum_{i=1}^{M} \delta_{ik} z_i g_i}{\left(\sum_{t=1}^{T} \rho_{kt}\right)\left(\sum_{i=1}^{M} \delta_{ik} z_i\right)} \right] \qquad (2.8)$$

where g_i is determined using equation (2.1).

3 ESTIMATION OF TOTAL RAINFALL DEPTH - APPLICATION

3.1 Construction of mesh

The techniques described in this report are designed for computer-based application and it is therefore necessary to define the catchment area in some way which is suitable for numerical calculations. Although in some cases catchment boundary coordinates may be available in digitised form, this is not always the case, and a much simpler representation is therefore used in the present scheme which may be supplemented by the detailed information, if it is available, for some of the algorithms.

On a map of the catchment, a quadrilateral is drawn which represents the catchment boundary as closely as possible, fitting by eye. This defines the inner box which is used in place of the catchment boundary for subsequent analysis; its suitability is checked by comparing its area with the known catchment area: if it is too large or too small it should be redrawn. An appropriate tolerance for this check would be 20%.

For numerical calculation, it is convenient to use a mesh of points to represent the catchment, as discussed in Section 2.1. Such a mesh is now constructed as follows: each side of the inner box is divided into m equal parts, and $M = m^2$ sub-boxes are constructed by joining the corresponding division points on opposite sides of the inner box. Each sub-box yields a mesh point at the intersection of the lines joining the midpoints of opposite sides. Thus there is a set of mesh points $G_1, G_2, \ldots G_M$, with associated areas $A_1, A_2, \ldots A_M$ which are the areas of the corresponding sub-boxes. This construction is illustrated in Fig. 1.

Each mesh point G_i may now be assigned a weight Z_i where

$$Z_i = A_i/\hat{A} \qquad (3.1)$$

and $\hat{A}$ is the area of the inner box.

A method for determining a suitable value for the number of points in the mesh M has been determined experimentally; this is described in Section 3.3.

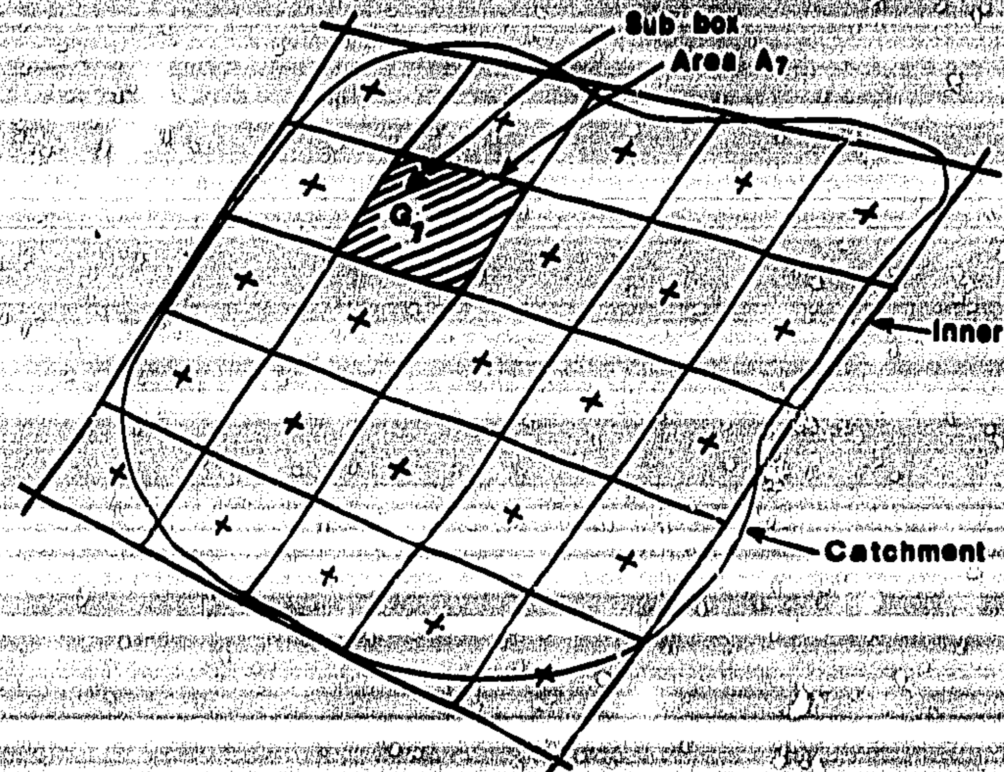

FIGURE 1 Example of grid representation of a catchment showing a grid point, sub-box and associated area

3.2 The triangle method algorithm

Following the construction of the mesh, the next step in the application of the procedure is to locate all the daily raingauges which are available for the catchment and storm concerned. The search for gauges should not be confined to the catchment itself; gauges which are outside but nearby may also help to estimate the rainfall in parts of the catchments, particularly near its boundary. The inverse-square distance weighting which is included in the algorithm will ensure that even if very distant gauges are used, their influence on the result will be small.

The search area for gauges is therefore not restricted to the inner box; an outer box is constructed, with sides parallel to those of the inner box, but at a distance E times further away from the point of intersection of the lines joining midpoints of opposite sides (see Fig 2). The expansion factor E is necessarily arbitrary, but a value E = 1.5 will usually lead to the inclusion of all relevant gauges; it may be necessary to use larger factors when the raingauge distribution is very sparse or if the catchment has an unusual shape which requires a long, narrow inner box to be drawn.

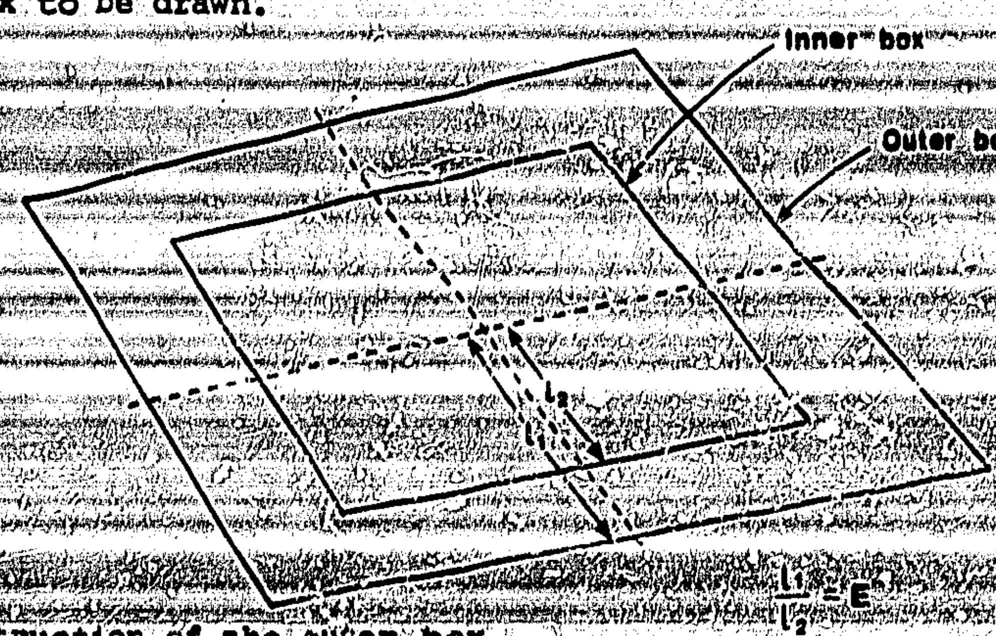

FIGURE 2 Construction of the outer box

The search algorithm, which is used for both daily and autographic gauges, is specific to the U.K. raingauge network. Raingauge data are held in a three-level database system of random-access computer files, as follows.

First level : Ordered index of raingauge reference numbers, with pointers to the location of each gauge's entry at the second level.

Second level : Unordered index containing

(i) essential parameters for each gauge such as National Grid reference, annual average rainfall for a standard period, and gauge altitude,

and (ii) pointers for each gauge's entries at the third level.

Third level : Data files of different structures for daily and autographic data.

(i) Daily data is stored in a highly packed form with one file per year covering the period 1961-1980 inclusive. New years of data will be added as validation checks are completed.

(ii) Autographic data are stored on a single file: only data corresponding to runoff events on a similar database of storm hydrographs are held.

The raingauges operating for a given storm on a particular catchment are found by searching the three levels of the database in turn. Raingauges in the U.K. are numbered in such a way that the numbers within Hydrometric Areas form mutually exclusive sequences. The Hydrometric Area in which a catchment lies will always be known; however the way in which the outer box is constructed will mean that it may over-lap with neighbouring Hydrometric Areas. Thus the search proceeds as follows (see also Fig 3):

(i) Search the first-level index to find gauges within the Hydrometric Area containing the catchment and also within all adjacent Hydrometric Areas.

(ii) For each of these gauges take the National Grid reference from the second-level index and find if it lies inside a rectangle drawn along National Grid lines to contain the outer box. Then find if the gauge lies within the outer box itself: the sum of the angles (taking account of sign) subtended by the four sides of the box at the gauge will be 2π if it is inside or zero if it is outside.

(iii) For each gauge within the outer box, examine the third-level data files to find if it is operating during the storm, and if so, the rainfall data are extracted.

Once the gauges have been found, and the mesh has been set up, calculation of the weights for the daily raingauges may proceed. The maximum distance allowed from a mesh point to any of the gauges in its triangle, D_O, is calculated as

$$D_O = 2\sqrt{\bar{A}/N} \qquad (3.2)$$

where $\bar{A}$ is the area of the outer box

and N is the number of daily raingauges found for the event.

For each mesh point the triangle search algorithm considers each combination of 3 gauges which are within D_O of the point and checks whether the mesh point is within the triangle they form. The sequence in which gauges are used is illus-

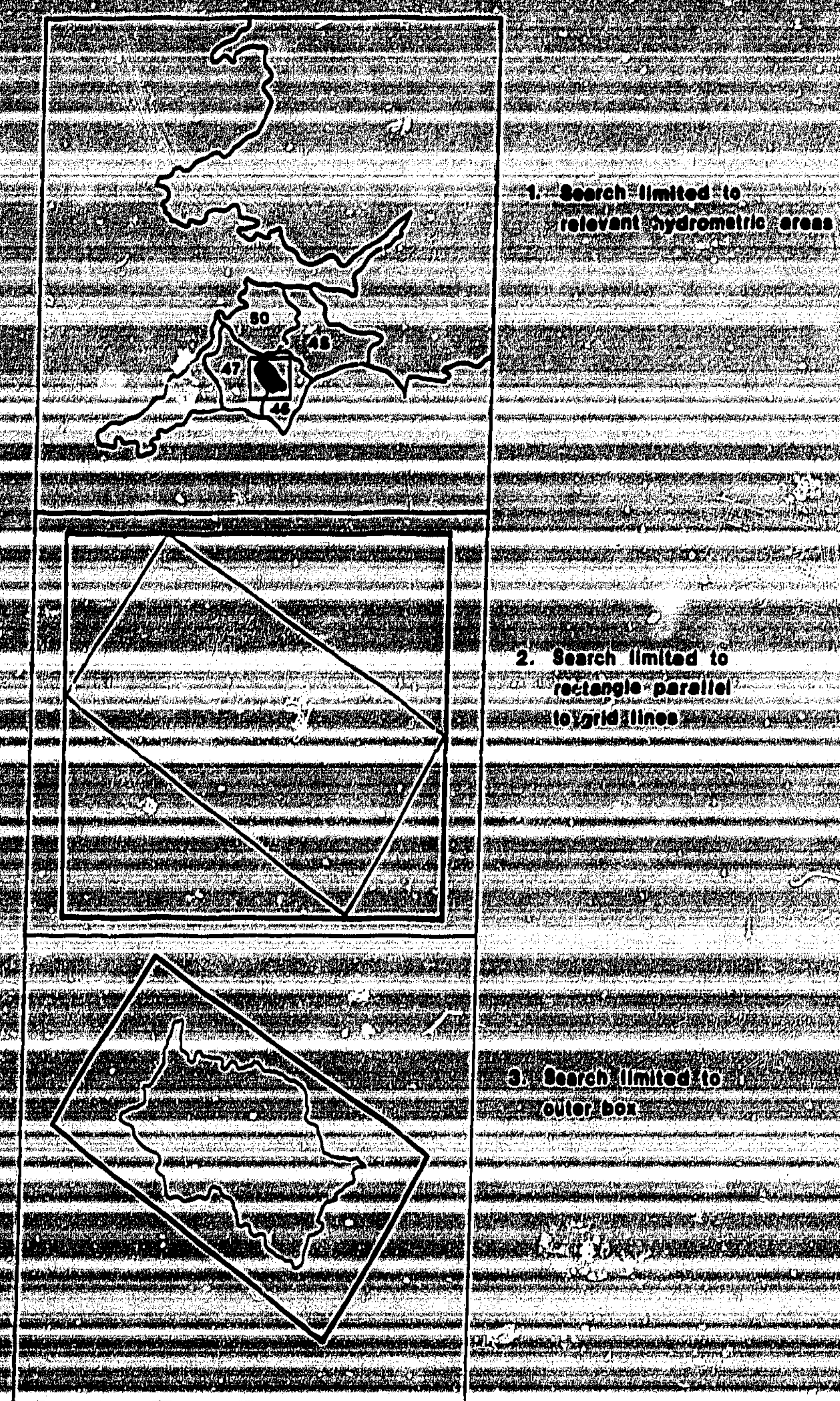

FIGURE 3 Stages in the search for raingauges

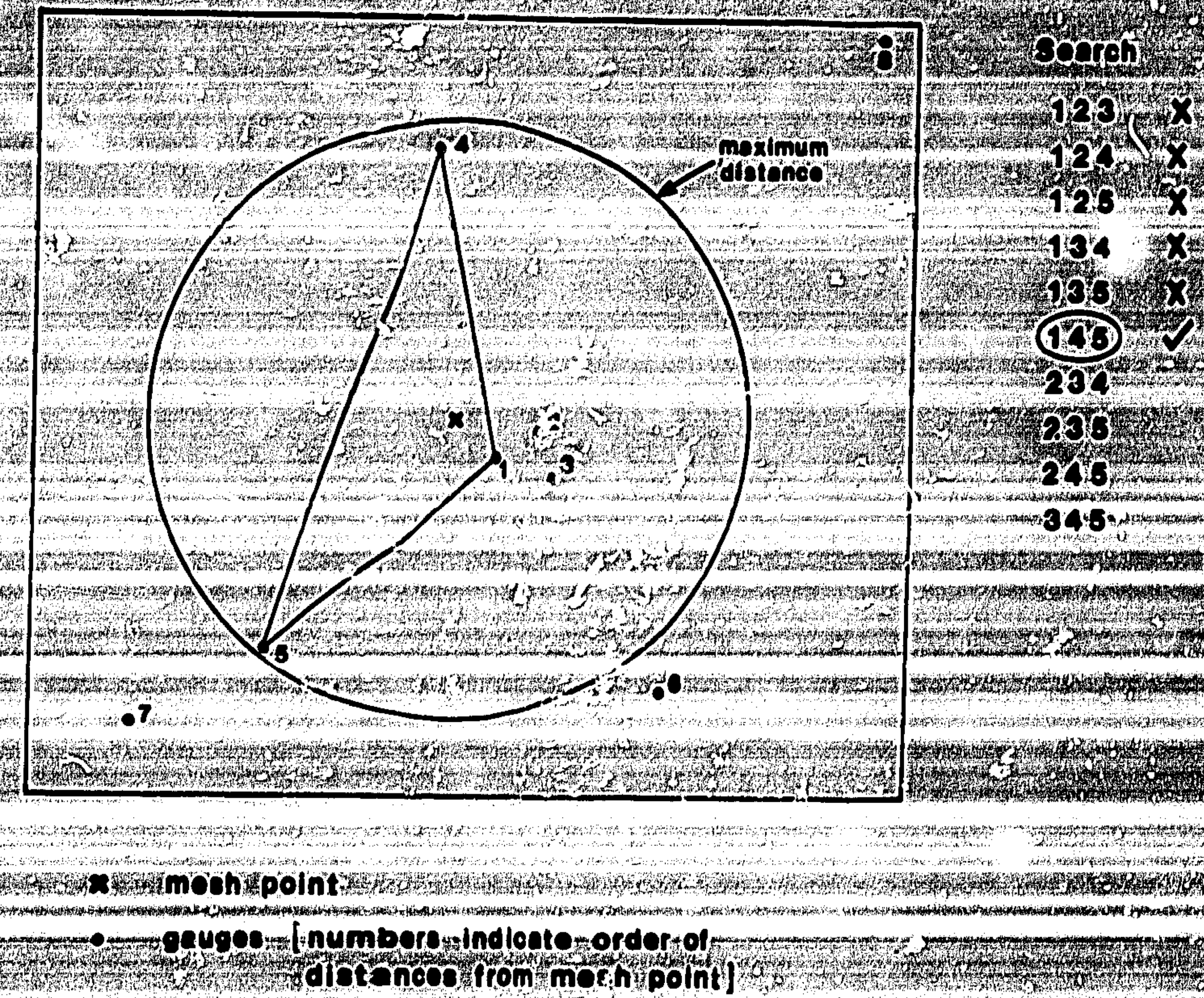

FIGURE 4 Example of sequence of checking triangles

trated in Fig 4. The emphasis is on attempting to have at least one of the vertices of the triangle as close as possible to the mesh point. Once the triangle has been found, the calculations proceed as described in Section 2.2.

3.3 Choice of mesh spacing

The construction of an M-point mesh to represent the catchment has been described in Section 3.2. The way in which the value of M was determined by numerical experiment and was related to the number of gauges available is described in this section.

The raingauge weights obtained by the triangle method are dependent on the number of points in the mesh. It is therefore desirable to choose the number of mesh points to ensure "convergence" of the weightings, i.e. to ensure that the use of a finer mesh would not lead to a significantly different set of weights. For the purposes of determining this convergence, all the daily raingauges available for the arbitrary date of 1st April 1976 were located for Hydrometric Area 0392 (Upper Thames area, see Fig 5). 166 such gauges were found. The triangle method is scale-independent, so that any size of area could have been used for this study; this region was chosen to allow easy searching for gauges and to ensure that there was a large number of gauges available. Random subsets of various sizes were

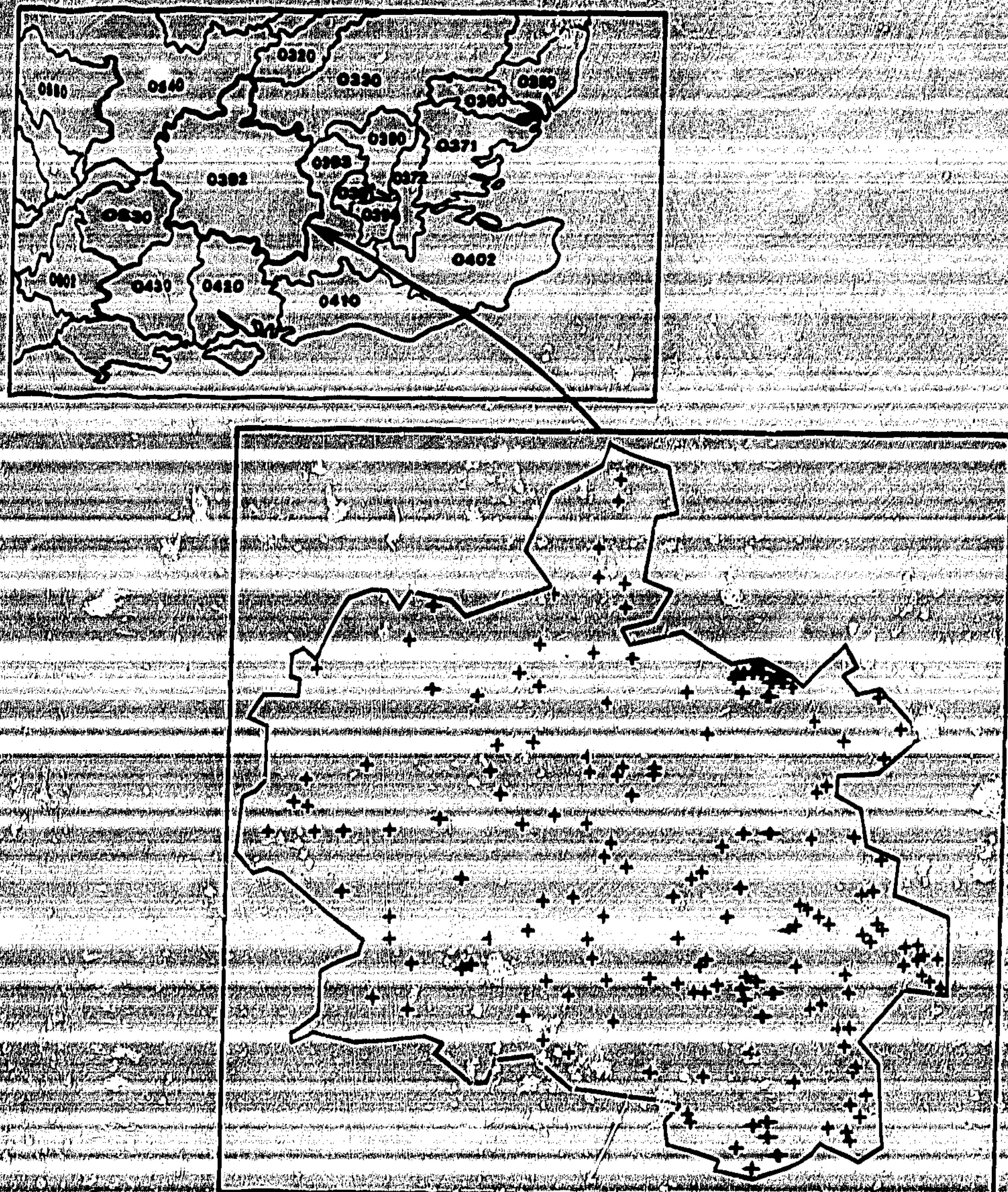

FIGURE 5 R... gauges in Hydrometric Area 0392

selected from among the gauges and their weights computed by the triangle method for meshes of size from 5 x 5 up to 20 x 20 (M = 25 to 400).

Convergence was defined to have occurred when increasing the value of m by one (where $M = m^2$) changed the triangle method weights for the N gauges in the subset being used from the old values $\{u_j^{(1)} : 1 \leq j \leq N\}$ to the new values $\{u_j^{(2)} : 1 \leq j \leq N\}$ such that

$$RMS = \left\{ \frac{1}{N} \sum_{j=1}^{N} \left(u_j^{(1)} - u_j^{(2)} \right)^2 \right\}^{\frac{1}{2}} \tag{3.3}$$

was less than 10% of the mean gauge weight, i.e. so that

$$RMS \leq \frac{1}{10\,N} \tag{3.4}$$

The study was split into two phases:

(i) Several trials were made with each of N = 5, 10, 20, 35, 50, 70 to determine whether the value of m at which convergence occurs is consistent between runs with the same value of N. For each value of N, these values of m were within ± 2 of the mean.

(ii) In addition, trials were made with random values of N to obtain a spread of samples in the range $5 \leq N \leq 70$.

The results from (i) and (ii) were pooled (Fig 6) and a regression analysis was carried out to determine the dependence of the convergence value of m on N. A regression of log m on log N led to the equation:

$$m = 2.296\, N^{0.485} \tag{3.5}$$

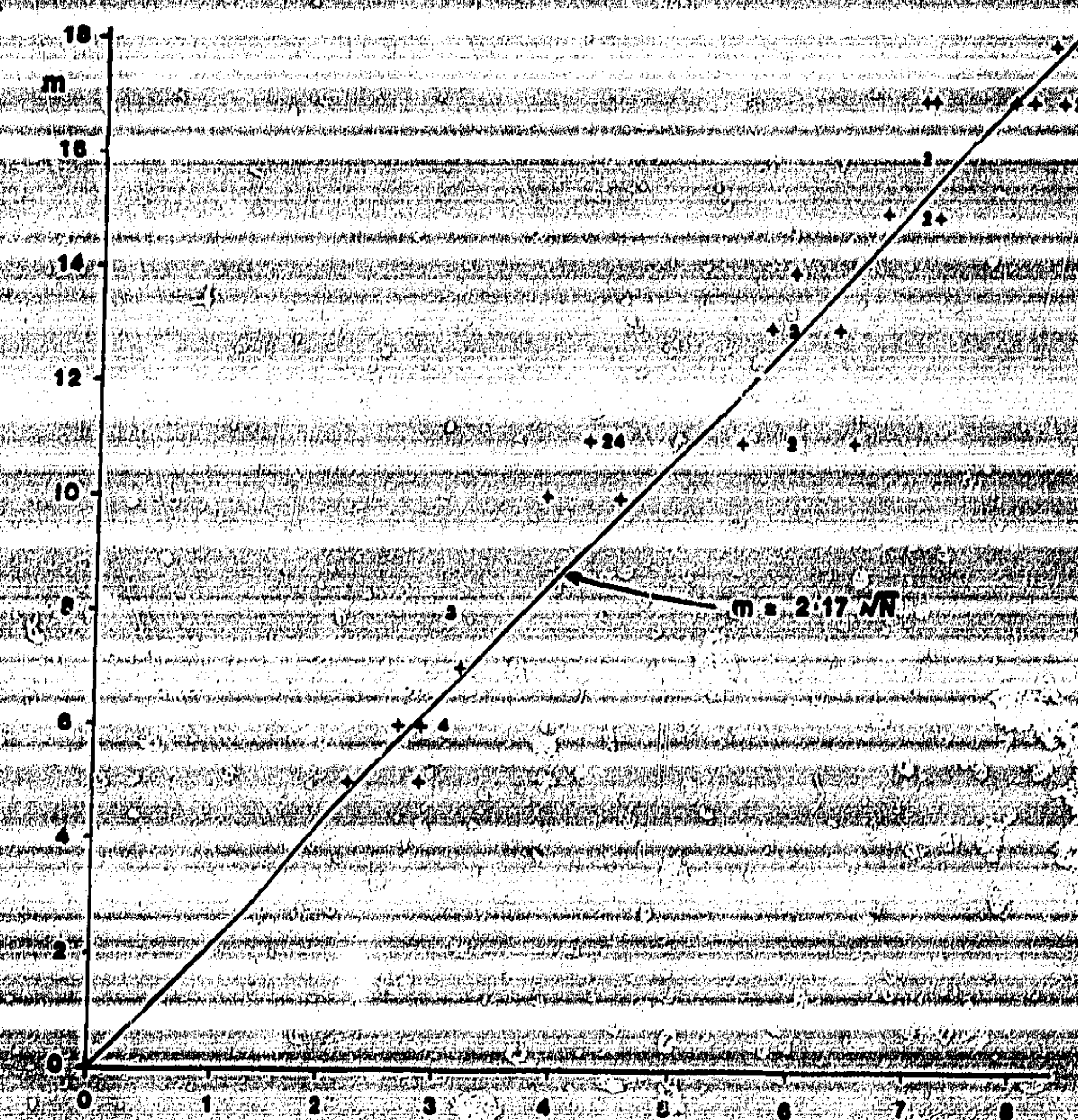

FIGURE 6 Results of mesh size analysis

This confirms an intuitive expectation that m would be proportional to $\sqrt{N}$ so that the number of mesh points required is proportional to the number of gauges. Therefore, the following simplified equation was obtained:

$$m = 2.17\sqrt{N} \tag{3.6}$$

which has an R^2 value of 92.4%.

The value of m obtained from equation (3.6) is included in the mesh generation algorithm. It should be noted that, in theory, N could vary between events on the same catchment. It is therefore recommended that the value of N used in equation (3.6) should be an estimate of the maximum number which could be used, e.g. the total number of gauges found within the outer box in stage (ii) of the search described in section 3.2.

3.4 Comparison of methods

The reasoning behind the development of the triangle method which is presented in this report is of a largely theoretical nature, based on the four requirements for raingauge weights proposed in section 2.1. These, in turn, are largely based on the aim of consistency in the gauge weightings between different events on the same catchment. This section describes the results from trials which were designed to show how some of the methods for obtaining sets of raingauge weights which are described in section 2.1 perform.

The trials were conducted using the 166 daily gauges operational on 1st April 1976 in Hydrometric Area 0392 (see section 3.3 and Fig. 5). Random samples of various sizes were taken from among these gauges; the aim of the trials was to assess the effect on the weights assigned to the gauges in a sample if some of them were "missing". This was achieved by calculating the weights before and after random subsets were dropped from each sample.

Raingauge weights were calculated by the following five methods:

(i) Triangle — The method described in this report.

(ii) Thiessen — The numerical version of the Thiessen method described by Diskin (1970) using gridded trial points (a 200 x 200 grid was used).

(iii) Inverse square — A method using the same mesh as the triangle method but with inverse-square distance weightings for all gauges at each mesh point.

(iv) Bethlamy — Bethlamy's (1976) method based on calculating the angles subtended by each gauge at the major and minor axes of the catchment.

(v) Nearest Three — A variant of the triangle method using the nearest three gauges at each mesh point instead of the surrounding triangle.

Several criteria were used to measure the performance of the methods. Suppose the original sample consisted of the N + n gauges $R_1, R_2, \ldots, R_{N+n}$ from which the n gauges $R_{N+1}, R_{N+2}, \ldots, R_{N+n}$ were then dropped to examine the effect on the remaining N gauges.

Now suppose these gauges had weights $\{u_j^{(1)} : 1 \leq j \leq N+n\}$ with the full sample and $\{u_j^{(2)} : 1 \leq j \leq N\}$ with the sample after dropping n gauges, and that the gauges have annual average rainfalls $\{\sigma_j : 1 \leq j \leq N+n\}$ and altitudes $\{A_j : 1 \leq j \leq N+n\}$.

The following five measures (C1 - C5) were used for comparing the methods:

C1: RMS change : The root-mean-square change in weight for the remaining N gauges when the n gauges are dropped

$$C1 = \left\{ \frac{1}{N} \sum_{j=1}^{N} (u_j^{(1)} - u_j^{(2)})^2 \right\}^{\frac{1}{2}} \tag{3.7}$$

C2: Maximum change: The maximum absolute change in weight for any of the N remaining gauges.

$$C2 = \max_{1 \leq j \leq N} | u_j^{(1)} - u_j^{(2)} | \tag{3.8}$$

C3: Radius : A measure of how far away the effect of dropping a gauge will be significant. Suppose that for $1 \leq j \leq N$, d_j is the distance from remaining gauge j to the nearest dropped gauge, then the measure is

$$C3 = \frac{\sum_{j=1}^{N} | u_j^{(1)} - u_j^{(2)} | \, d_j}{\sum_{j=1}^{N} | u_j^{(1)} - u_j^{(2)} |} \tag{3.9}$$

C4: AAR change : The absolute difference in the areal average value of annual average rainfall calculated before and after dropping the n gauges.

$$C4 = \left| \sum_{j=1}^{N+n} u_j^{(1)} \sigma_j - \sum_{j=1}^{N} u_j^{(2)} \sigma_j \right| \tag{3.10}$$

C5: Altitude change : The absolute difference in the areal average value of altitude calculated before and after dropping the n gauges

$$C5 = \left| \sum_{j=1}^{N+n} u_j^{(1)} A_j - \sum_{j=1}^{N} u_j^{(2)} A_j \right| \tag{3.11}$$

A total of 12 different combinations of total number of gauges and numbers of gauges dropped were used. Within each of these 12 runs, ten different random selections of dropped gauges were used. The means and standard deviations for each of criteria C1 - C5 for each run are shown in Tables 1 - 5.

The results from all runs were pooled to determine significant differences between methods. The resulting three-way unbalanced table (with factors: method, number of gauges at start, number of gauges dropped) was analysed using

TABLE 1 : MEANS AND STANDARD DEVIATIONS OF COMPARISON CRITERION C1 : RMS CHANGE

RUN NO.	GAUGES AT START	GAUGES DROPPED	TRIANGLE		THIESSEN		INVERSE SQUARE		BETHLAMY		NEAREST THREE	
			mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
1	20	3	0.0125	0.0041	0.0169	0.0076	0.0089	0.0015	0.0092	0.0016	0.0089	0.0018
2	15	2	0.0158	0.0034	0.0211	0.0065	0.0122	0.0019	0.0115	0.0015	0.0128	0.0022
3	20	5	0.0216	0.0043	0.0272	0.0058	0.0237	0.0097	0.0165	0.0010	0.0241	0.0119
4	15	4	0.0323	0.0054	0.0396	0.0057	0.0253	0.0046	0.0255	0.0036	0.0261	0.0055
5	30	5	0.0100	0.0024	0.0141	0.0029	0.0068	0.0007	0.0065	0.0007	0.0069	0.0013
6	8	1	0.0209	0.0170	0.0314	0.0306	0.0279	0.0275	0.0171	0.0301	0.0281	0.0304
7	20	1	0.0059	0.0028	0.0086	0.0050	0.0029	0.0012	0.0026	0.0004	0.0024	0.0013
8	15	1	0.0161	0.0116	0.0314	0.0324	0.0041	0.0015	0.0057	0.0013	0.0034	0.0018
9	8	2	0.0649	0.0154	0.0869	0.0441	0.0670	0.0260	0.0502	0.0068	0.0690	0.0316
10	30	2	0.0052	0.0024	0.0076	0.0033	0.0046	0.0034	0.0025	0.0029	0.0045	0.0044
11	15	3	0.0225	0.0110	0.0288	0.0176	0.0177	0.0047	0.0169	0.0025	0.0183	0.0060
12	20	2	0.0087	0.0033	0.0098	0.0045	0.0108	0.0073	0.0056	0.0010	0.0116	0.0091

TABLE 2 : MEANS AND STANDARD DEVIATIONS OF COMPARISON CRITERION C2 : MAXIMUM CHANGE

RUN NO.	GAUGES AT START	GAUGES DROPPED	TRIANGLE		THIESSEN		INVERSE SQUARE		BETHLAMY		NEAREST THREE	
			mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
1	20	3	0.0318	0.0115	0.0539	0.0333	0.0163	0.0031	0.0147	0.0029	0.0169	0.0045
2	15	2	0.0344	0.0077	0.0545	0.0202	0.0176	0.0029	0.0157	0.0019	0.0213	0.0039
3	20	5	0.0474	0.0143	0.0693	0.0186	0.0752	0.0268	0.0242	0.0013	0.0772	0.0347
4	15	4	0.0681	0.0146	0.0864	0.0229	0.0467	0.0107	0.0374	0.0052	0.0518	0.0129
5	30	5	0.0284	0.0102	0.0517	0.0109	0.0121	0.0016	0.0094	0.0012	0.0136	0.0031
6	8	1	0.0414	0.0336	0.0771	0.0714	0.0479	0.0382	0.0210	0.0042	0.0468	0.0437
7	20	1	0.0193	0.0102	0.0330	0.0216	0.0122	0.0050	0.0035	0.0006	0.0100	0.0056
8	15	1	0.0481	0.0379	0.1136	0.1230	0.0151	0.0054	0.0083	0.0020	0.0125	0.0068
9	8	2	0.1065	0.0328	0.1948	0.1180	0.1136	0.0240	0.0708	0.0097	0.1111	0.0327
10	30	2	0.0174	0.0090	0.0331	0.0173	0.0185	0.0051	0.0038	0.0005	0.0177	0.0078
11	15	3	0.0500	0.0223	0.0768	0.0448	0.0359	0.0116	0.0245	0.0023	0.0397	0.0163
12	20	2	0.0249	0.0093	0.0338	0.0176	0.0294	0.0112	0.0081	0.0014	0.0319	0.0176

TABLE 3 : MEANS AND STANDARD DEVIATIONS OF COMPARISON CRITERION C3 : RADIUS

RUN NO.	GAUGES AT START	GAUGES DROPPED	TRIANGLE		THIESSEN		INVERSE SQUARE		BETHLAMY		NEAREST THREE	
			mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
1	20	3	164.05	59.6	148.10	72.3	242.34	58.4	248.53	63.0	227.42	54.3
2	15	2	179.25	28.2	158.04	37.2	301.50	58.7	306.70	53.5	273.84	45.1
3	20	5	170.52	23.3	142.66	24.7	262.55	62.3	244.22	46.0	260.47	61.9
4	15	4	183.36	45.1	144.33	47.5	255.58	52.9	258.49	45.5	249.74	52.6
5	30	5	145.20	14.5	120.40	22.2	220.59	25.8	222.92	30.3	209.91	23.1
6	8	1	362.98	88.2	181.09	126.0	520.11	160.0	542.84	79.0	522.98	153.0
7	20	1	197.32	78.6	149.25	96.9	497.22	104.0	469.01	78.8	457.82	102.0
8	15	1	231.32	94.0	159.45	104.0	518.23	188.0	415.77	118.0	488.37	200.0
9	8	2	243.19	44.0	182.52	59.0	273.28	54.7	281.88	40.8	273.07	49.7
10	30	2	159.88	28.5	125.92	31.0	319.19	94.2	330.42	47.3	298.59	92.3
11	15	3	191.21	84.1	175.07	109.0	216.57	24.7	249.43	30.3	204.17	33.6
12	20	2	143.52	18.0	119.92	29.4	287.43	92.5	288.79	77.2	274.21	84.3

TABLE 4 : MEANS AND STANDARD DEVIATIONS OF COMPARISON CRITERION C4 : AAR CHANGE

RUN NO.	GAUGES AT START	GAUGES DROPPED	TRIANGLE mean	TRIANGLE s.d.	THIESSEN mean	THIESSEN s.d.	INVERSE SQUARE mean	INVERSE SQUARE s.d.	BETHLAMY mean	BETHLAMY s.d.	NEAREST THREE mean	NEAREST THREE s.d.
1	20	3	4.906	4.83	5.341	6.27	2.954	2.49	3.525	2.05	3.145	3.02
2	15	2	2.864	2.20	2.860	2.39	4.727	3.21	4.993	3.00	4.543	3.40
3	20	5	5.349	3.29	4.844	3.80	4.650	5.74	4.343	3.52	4.802	5.97
4	15	4	7.333	4.18	6.211	5.64	9.402	6.88	10.246	7.20	9.645	6.33
5	30	5	3.962	3.09	4.062	2.75	3.955	3.50	3.478	2.80	3.871	3.98
6	8	1	6.536	7.33	10.017	14.1	8.245	9.91	4.885	5.41	8.363	10.3
7	20	1	1.668	1.89	1.217	1.82	0.909	0.835	2.131	1.02	0.793	0.823
8	15	1	2.128	2.32	2.504	1.47	2.189	0.769	2.331	1.74	1.748	0.783
9	8	2	22.96	12.7	18.68	25.0	23.64	15.3	20.20	9.61	25.00	16.7
10	30	2	1.855	1.02	2.057	1.89	4.514	11.0	2.529	1.92	4.864	12.6
11	15	3	6.779	5.16	6.099	4.86	4.972	3.38	6.849	4.84	4.846	3.49
12	20	2	3.571	2.20	2.680	1.48	6.024	5.44	3.728	1.83	6.372	6.08

TABLE 5 : MEANS AND STANDARD DEVIATIONS OF COMPA

RUN NO.	GAUGES AT START	GAUGES DROPPED	TRIANGLE mean	TRIANGLE s.d.	TH mea
[illegible]	20	3	1.853	1.54	5.34
2	15	2	2.739	3.07	4.07
3	20	5	3.608	2.43	3.14
4	15	4	4.467	3.72	5.63
5	30	5	2.230	2.28	2.88
6	8	1	3.622	3.01	3.35
7	20	1	0.812	0.727	0.78
8	15	1	1.850	1.65	3.00
9	8	2	16.50	9.87	16.10
10	30	2	1.972	1.38	1.955
11	15	3	6.396	9.65	5.908
12	20	2	2.925	2.07	2.354

RISON CRITERION/CS : ALTITUDE CHANGE

ESSEN	INVERSE SQUARE		BETHLAMY		NEAREST THREE	
s.d.	mean	s.d.	mean	s.d.	mean	s.d.
6.27	2.954	2.49	3.525	2.05	3.145	3.02
4.50	3.181	2.34	3.684	2.22	2.857	2.06
2.29	3.467	2.58	3.562	3.03	3.728	2.76
3.85	4.423	4.07	6.226	4.90	4.456	3.35
1.61	2.959	1.83	2.888	2.07	2.753	1.90
5.16	7.974	9.29	5.471	1.92	7.726	9.81
0.554	0.632	0.582	1.254	0.940	0.496	0.498
2.88	0.919	0.445	1.121	0.576	0.699	0.355
20.4	11.03	7.06	11.26	7.27	12.20	7.50
1.79	1.293	1.21	1.882	1.28	1.428	1.55
9.73	5.000	3.80	7.611	5.62	4.820	3.73
1.87	2.980	1.74	3.393	2.39	2.915	1.60

an extension of the method described by Cox and Snell (1981). For each measure C1 – C5, the estimated standard error in the mean value was estimated from the residual mean square variation within runs, and this was used to determine differences between mean values for the five methods which are significant at the 1% and 5% levels. The results are shown in Table 6, in which it should be noted that the mean value of a measure for each method is adjusted to allow for interaction effects with the other two factors. Significant differences between the triangle method and the other four methods are highlighted in this table.

TABLE 6 : COMPARISON OF MEAN VALUES OF MEASURES C1 – C5

NOTE ++ indicates method better than Triangle method at 1% level
+ " " " " " at 5% level
-- " " worse " " " at 1% level

MEASURE	RESIDUAL MEAN SQUARE	ESTIMATED STANDARD ERROR OF MEANS	5% SIGNIFICANT DIFFERENCE IN MEANS	1% SIGNIFICANT DIFFERENCE IN MEANS	ADJUSTED MEANS TRIANGLE	THIESSEN	INV. SQUARE	BETHLAMY	NEAREST THREE
C1 RMS CHANGE	0.000156	0.00114	0.00224	0.00296	0.0232	0.0305 --	0.0212	0.0177 ++	0.0215
C2 MAXIMUM CHANGE	0.000889	0.00272	0.00536	0.00707	0.0480	0.0780 --	0.0415 +	0.0249 ++	0.0423 +
C3 RADIUS	6073.55	7.1143	14.008	18.483	190.212	143.123 ++	318.778 --	314.145 --	304.277 --
C4 AAR CHANGE	47.178	0.6270	1.2346	1.6290	7.1368	6.8592	7.6598	7.0810	7.8109
C5 ALTITUDE CHANGE	22.889	0.4367	0.8599	1.1346	4.7551	4.9510	4.4774	4.8588	4.5017

Residual mean square has 540 degrees of freedom

t-distribution: $t_{5\%,540} = 1.969$
$t_{1\%,540} = 2.598$

The following conclusions may be drawn from the results in Table 6.

(i) On average, there is no significant difference between the five methods in the effects on the calculated values of AAR and altitude when gauges are dropped. However, the differences may be significant in particular cases, as for example in the runs starting with 8 gauges in Tables 4 and 5.

(ii) The three criteria C1 – C3 illustrate the way in which the methods may be classified as "local" or "global". The Thiessen method is the most local of the five; it has relatively large changes in weights confined to a small radius around a dropped gauge. The opposite extreme is the Bethlamy method, which has smaller changes in weights spread out over a larger radius.

(iii) It is apparent that no one criterion can be used to judge the [illegible] of a particular method; this judgment is essentially subjective, and [illegible] on the nature of the application envisaged in a particular case, in which the results shown here may give some indication of the performance to be expected.

(iv) The Triangle method represents a compromise between local and global methods.

4 SPATIAL VARIATIONS OF DAILY RAINFALL

The procedure described in this report is designed to produce a single hyetograph representative of rainfall on a catchment during a storm event, which may then be used as an input to a lumped rainfall-runoff model. It would not be appropriate to use the procedure when there are marked variations in the rainfall across the catchment, either in total depth or in timing of the rainfall. In this section, a method is presented which can be used to quantify the variations in total storm depth across the catchment and hence enable a decision to be made as to whether the depths are sufficiently consistent to allow use with a lumped model. The question of differences in timing observed at autographic gauges is discussed in Section 5.3.

The falls recorded by daily raingauges sited in different parts of a catchment for the days of a storm cannot be expected to be identical. There will be differences due to:

(i) The topography of the catchment - the influence of mountains will cause variations in the rainfall depth. This source of variation can be removed to a large extent by considering the daily falls as a percentage of the annual average at each gauge.

(ii) Local random spatial variations within the storm, or more systematic variations which are not typical of the catchment, and are therefore not covered by case (i). This is not systematic movement across a catchment, which is unlikely to occur on such a timescale that it would be observable in daily data. This type of variation is undesirable in lumped rainfall-runoff modelling, since it will affect each event in a different way. Thus it is necessary to detect such variations when processing the rainfall data.

In the U.K. Flood Studies (NERC, 1975), a simple rule of thumb was applied whereby if the highest daily percentage observed for a particular day of an event was more than twice the lowest for that day, then the event was "unacceptable". However, when there was a good distribution of gauges on a catchment, the check was allowed to become more subjective, and one or two gauges outside the range could be discounted. It is, however, more desirable that the method should be entirely objective, and also that it should take account of raingauge weightings; the following procedure which has a simple statistical basis achieves this.

To obtain a criterion based on statistical properties, a distribution must be assumed for the daily percentage falls for a particular storm. As it is intended that any fluctuation allowed should be random, so that there is no systematic movement, the analysis is based on the supposition that the ratios of the observed falls to the corresponding annual average have a Normal $N(\mu, \sigma^2)$ distribution.

The most readily expressed type of allowed region for variations in the falls is $(1-\lambda)\mu$ to $(1+\lambda)\mu$ where λ is a constant to be determined. The criterion for acceptance may then be expressed as: the variations in daily falls are "acceptable" if and only if their ratios to the annual averages come from a distribution where the probability of observing a fall outside the permitted range is less than or equal to $^1/N_o$ where N_o is a constant number.

$$\text{i.e.}\quad P\{(1-\lambda)\mu \leq N(\mu,\sigma^2) \leq (1+\lambda)\mu\} > 1-{}^1/N_o \tag{4.1}$$

with μ and σ^2 estimated from the observed falls $r_1, r_2, \ldots r_N$ by the unbiassed estimators

$$\hat{\mu} = \sum_{j=1}^{N} u_j\, r_j/\sigma_j \tag{4.2}$$

$$\text{and}\quad \hat{\sigma}^2 = \frac{\sum_{j=1}^{N} u_j\,(r_j/\sigma_j - \hat{\mu})^2}{1 - \sum_{j=1}^{N} u_j^{\,2}} \tag{4.3}$$

where $\{u_j\}$ are the raingauge weightings defined in Section 2.

Equation (4.1) may be written as:

$$\int_{-\lambda\hat{\mu}/\hat{\sigma}}^{\lambda\hat{\mu}/\hat{\sigma}} \left(\frac{1}{\sqrt{2\pi}}\, e^{-y^2/2}\right) dy \geq \frac{N_o - 1}{N_o} \tag{4.4}$$

which leads to a criterion of the form

$$\frac{\hat{\mu}}{\hat{\sigma}} \geq C(\lambda, N_o) \tag{4.5}$$

where the function C is tabulated in Table 7.

TABLE 7 : VALUES OF THE VARIATION $C(\lambda, N_o)$ ALLOWED FOR DIFFERENT VALUES OF λ AND N_o

	$\lambda = 1/4$	$\lambda = 1/3$	$\lambda = 1/2$
$N_o = 6$	5.52	4.14	2.76
$N_o = 7$	5.88	4.41	2.94
$N_o = 8$	6.12	4.59	3.06
$N_o = 9$	6.36	4.77	3.18
$N_o = 10$	6.58	4.93	3.29

This analysis is quite general, but the choice of particular values of λ and N_o is at present purely subjective.

The choice $\lambda = {}^1/_3$ gives the region as ${}^2/_3\ \mu$ to ${}^4/_3\ \mu$, and corresponds to the "twofold" criterion mentioned above. The value $N_O = 8$ corresponds roughly to the subjective judgments which were made – one gauge in 8 was allowed to be outside the range and could be discounted. For $\lambda = {}^1/_3$ and $N_O = 8$, $C = 4.59$.

If we define

$$S = 4.59\hat{\sigma}/\hat{\mu} \tag{4.6}$$

then, if S is greater than unity, it is recommended that the storm should not be used for the calibration of lumped rainfall-runoff models. The criterion is found to correspond well with subjective decisions made before this analysis was available, although the inclusion of the weightings can make considerable differences in some cases.

5 ESTIMATION OF THE PROFILE SHAPE

The preceding Sections have described how the observations of total storm depths at daily raingauges may be used to scale each autographic gauge's observed hyetograph. There remains the problem of deriving the catchment average point profile from the individual ones in such a way as to minimise the inconsistencies which appear when different events on the same catchment have different subsets of the gauges operational.

5.1 Review of possible techniques

The method to be used to estimate the catchment average point profile must basically consist of two parts, namely

(i) a possible shifting in time of the individual profiles to take account of slight variations in timing between observations;

(ii) an averaging (with weights) of all the shifted profiles for each timestep.

The need for time-shifting of profiles has been pointed out by Aron et al. (1979). If hyetographs which essentially represent the same storm, but which have timing differences, are simply averaged interval by interval, the resultant hyetograph could be considerably attenuated compared to any of the individual profiles. Su (1981) pointed out in a discussion contribution that the method used by Aron et al. (1979) whereby the profile shape is taken from the gauge nearest the centre of the catchment does not make full use of the available data, and a weighted average of all shifted profiles is preferable. The method recommended by Su (1981) consists of calculating the time centroid of each hourly profile and then aligning them with all these centroids at the weighted average time. This is a satisfactory method, though some modification is necessary if sensible results are to be obtained from long, relatively low intensity profiles, and from cases where the profiles from different gauges on a catchment do not agree well in terms of shape.

When selecting a method for deriving weights for the autographic gauges, it must be remembered that there will be very few such gauges on a catchment; for this reason the triangle method would not be suitable. In the UK Flood Studies (NERC, 1975 Vol. IV pp 25-27), weights were based on the inverse distances from the centroid of the catchment, but this has the serious disadvantage that a gauge at or very near the centroid will have a very large weighting when compared to other gauges which may be on the catchment and would intuitively be considered representative of a large proportion of it. In this case, the Thiessen method (Thiessen, 1911) does not have the disadvantages described in Section 2.1 because the very low density of gauges will mean that clusters are unlikely to occur.

5.2 Definition of autographic gauge weightings

The method for calculating autographic gauge weightings designed for use in the procedure described in this report is developed from the Thiessen polygon technique as adapted by Diskin (1970), using the mesh set up for the daily rainfall analysis in Section 3.1. For each autographic gauge, the weight is the sum of the subbox areas corresponding to those mesh points for which the gauge is the nearest one, divided by the total area of the inner box. Using the notation defined in equation 2.7 and Section 3.1, the autographic gauge weights are $V_1, V_2, \ldots V_\nu$, where

$$V_k = \frac{1}{\hat{A}} \sum_{i=1}^{M} \delta_{ik} A_i \tag{5.1}$$

$$\text{or} \quad V_k = \sum_{i=1}^{M} \delta_{ik} z_i \tag{5.2}$$

where $\{z_i\}$ are the weights for the mesh points, $z_i = A_i/\hat{A}$.

If no time-shifting of the profiles were applied, the scaled profiles from equation (2.8) could be used to estimate the catchment average profile:

$$\bar{\rho}_t = \sum_{k=1}^{\nu} V_k \rho'_{kt} \tag{5.3}$$

for $1 \leq t \leq T$.

The catchment average total rainfall will then be

$$\sum_{t=1}^{T} \bar{\rho}_t = \sum_{k=1}^{\nu} \sum_{t=1}^{T} V_k \rho'_{kt} \tag{5.4}$$

and substituting from equations (5.2) and (2.8) for V_k and ρ'_{kt} respectively, leads to the result

$$\sum_{t=1}^{T} \bar{\rho}_t = \sigma_A \sum_{i=1}^{M} z_i g_i \tag{5.5}$$

which is the same as the total rainfall P_A calculated in equation (2.2) using daily gauges alone. This shows that the choice of weighting for the autographic gauges represented by equation (5.1) is compatible with the scaled profiles in equation (2.8).

5.3 Definition of rainfall blocks

In order that the method should be capable of analysing any observed rainfall profiles, it is necessary to define and isolate their most significant features with the aim of using them to calculate the required alignment in time. The technique devised puts a numerical framework on an intuitive approach to the problem. The rainfall profiles are divided into blocks, which are defined by considering all the ordinates which are above a given low threshold, and requiring that all the blocks should then be separated by at least three time intervals in which the rain is below the threshold (see Figure 7). In practice, the threshold may be dependent on the accuracy to which rainfall intensities are measured and digitised, and some subjective judgment may be required; however it is suggested that a value of 0.25 mm/hr is adequate for the majority of cases.

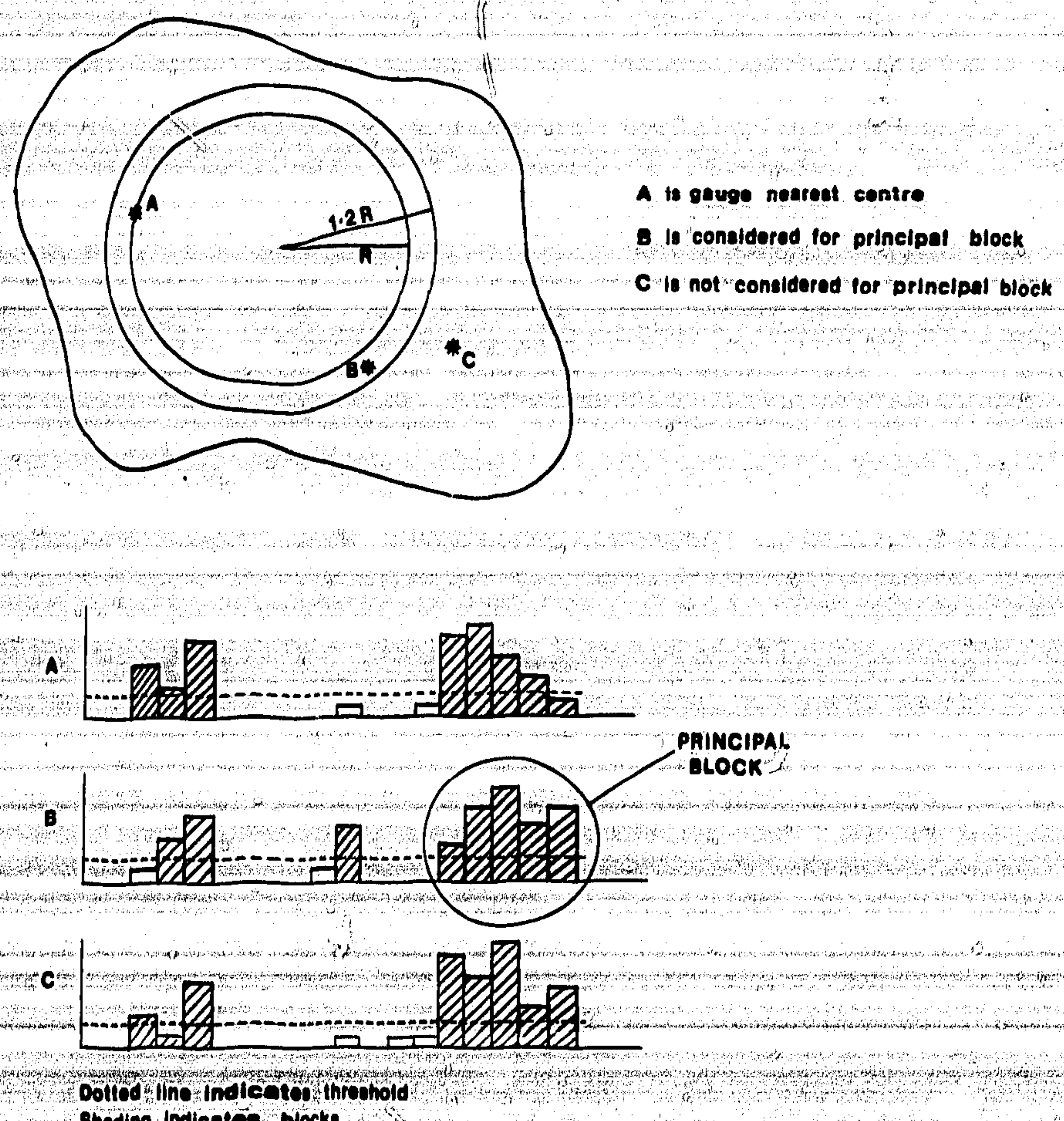

FIGURE 7 Example of definition of blocks

The principal block for a particular storm on a given catchment is now defined as the largest block, by total rainfall depth, amongst the gauges which are within a radius 20% greater than the distance of the nearest gauge from the centre of the catchment (Fig 7).

For all the other gauges, the corresponding block must be found. Although this will usually be unambiguous, there will be cases where the profiles do not agree well. The corresponding block for each gauge is defined as the largest block which has no more than four time intervals between its centroid and that of the principal block (see Fig 8). If no block satisfies this rule, the profiles should be considered too dissimilar for use in producing a catchment average point profile for lumped catchment modelling, although it would be possible to continue the analysis using the largest block.

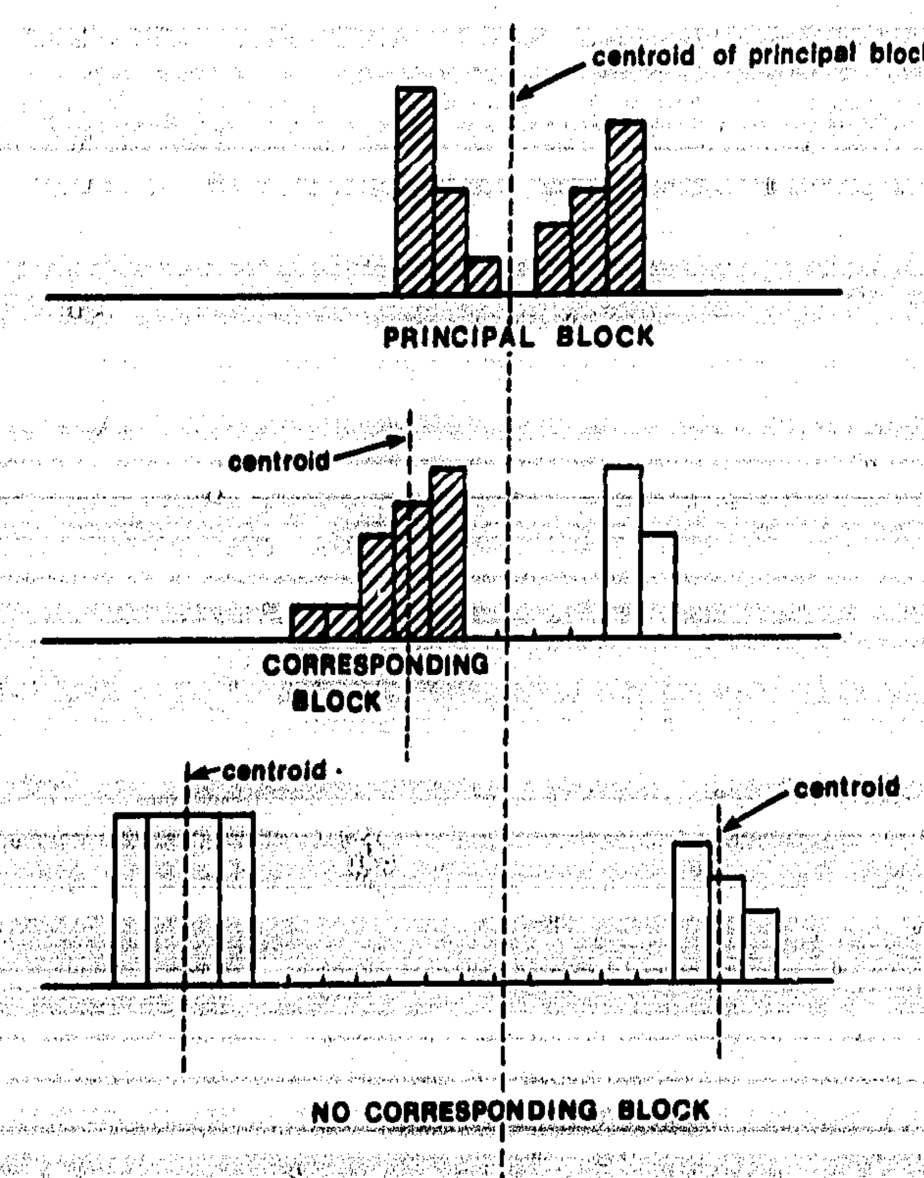

FIGURE 8 Examples of definition of corresponding blocks

To complete the analysis of the rainfall profiles, the individual scaled hyetographs should now be aligned so that the time centroids of the principal block and the corresponding blocks are moved to their weighted mean time, aligning to the nearest time interval. The catchment average point profile may now be estimated by taking the weighted mean of the shifted profile for each interval. The weights to be used for this process are taken from equation (5.1).

Suppose the principal block or its corresponding block for hourly gauge H_k lies in the time interval $S_k \leq t \leq F_k$. Then the centroid of this block is at time C_k given by

$$C_k = \left(\sum_{t=S_k}^{F_k} t\rho'_{kt} \right) \Big/ \left(\sum_{t=S_k}^{F_k} \rho'_{kt} \right) \tag{5.6}$$

and the time shift to be applied to the profile at gauge H_k is

$$\tau_k = \mathrm{Int}\left\{ \sum_{p=1}^{\nu} v_p C_p - C_k \right\} \tag{5.7}$$

where Int represents the nearest integer to the expression in brackets. Hence the catchment average point rainfall profile $\hat{\rho}_t$ is given by

$$\hat{\rho}_t = \sum_{k=1}^{\nu} v_k \rho'_{k,t-\tau_k} \tag{5.8}$$

6 EXAMPLE

To provide an illustration of the operation of the techniques presented in this report, an event has been taken from the catchment of the Dart at Austins Bridge, draining part of southern Dartmoor in south-west England. An outline of the catchment, together with the distribution of raingauges and the inner and outer boxes used (with an expansion factor E=1.5) and the observed rainfall profiles are shown in Figure 9.

The results from the daily and hourly rainfall analyses are shown in Tables 8 and 9 respectively. From these, the following points may be noted:

(i) The initial search found 68 daily raingauges which were within the outer box; however only 21 of these were operating on the required date. The mesh size was calculated using equation (3.6), with N=68, and the result was an 18 x 18 mesh.

(ii) The inner box has an area 15% greater than the catchment area, which is within the 20% tolerance suggested in Section 3.1.

(iii) The annual average rainfall for the catchment calculated using the raingauges available for this event is 6% smaller than the standard Meteorological Office value, which, although it is not entirely unsatisfactory, does indicate that the daily raingauges may not be completely representative of the catchment. The same impression can be gained from examining the locations of the gauges in Figure 9.

(iv) The value of the variation statistic S=1.82, calculated using equation (4.6), indicates that there are large spatial variations in the daily rainfall over the catchment, and so the event should be regarded as unsuited for use in lumped catchment modelling.

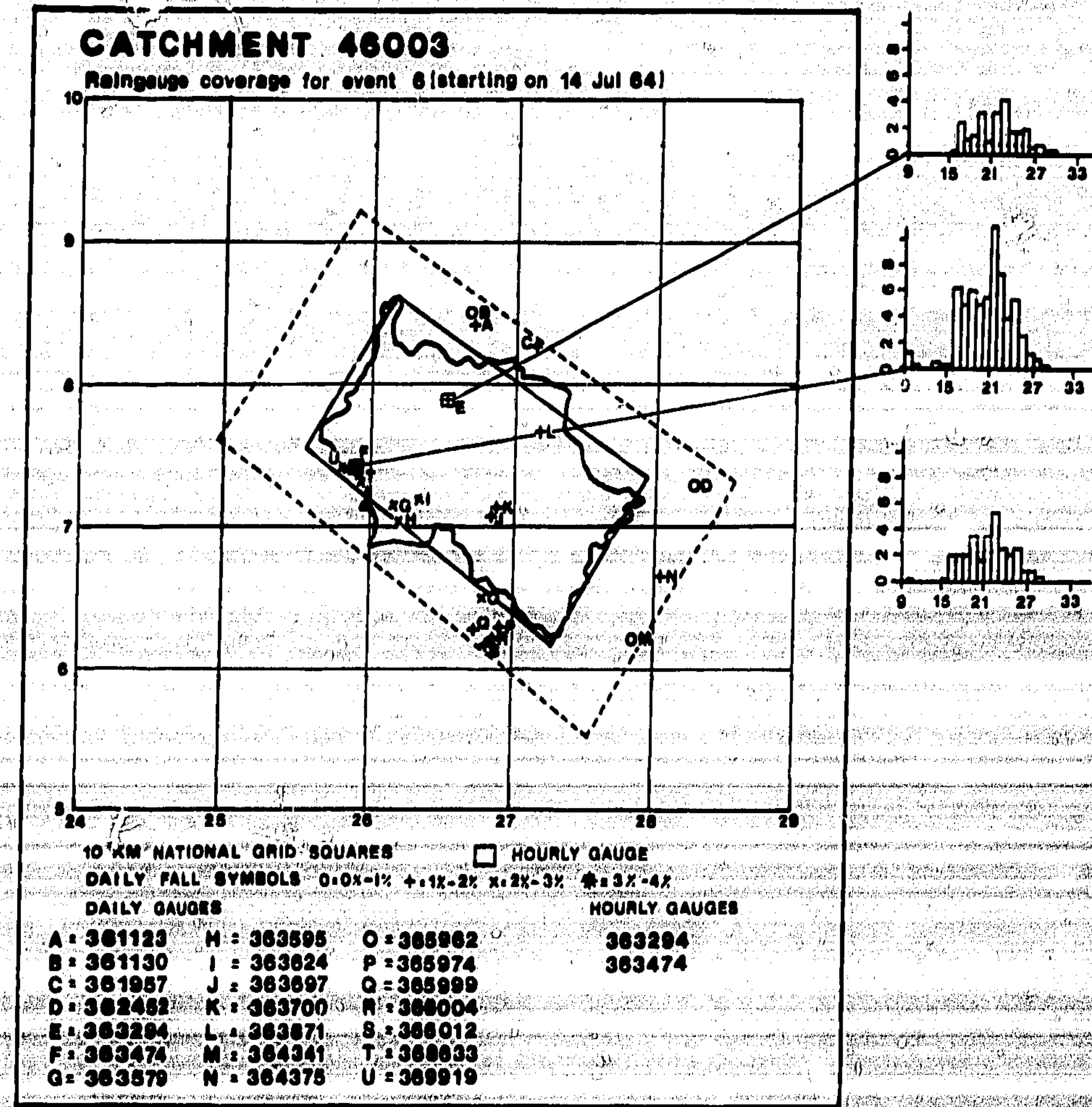

FIGURE 9 Example of event on Dart at Austins Bridge catchment

(v) Further evidence for non-uniformity of the rainfall emerges from the difference between the total rainfall observed at hourly gauge 363474 and the total estimated from nearby daily gauges.

TABLE 8 DART AT AUSTINS BRIDGE

Daily Rainfall data on 14 July 1964

Catchment Area = 248.0 km^2
Area of Inner Box = 285.3 km^2

Catchment Annual Average Rainfall = 1821 mm
Catchment AAR estimated from gauges = 1711 mm

	Gauge	Annual Average (mm)	Weight	Daily Fall (mm)	% of AAR
A	361123	1916	0.0400	25.4	1.33
B	361130	1783	0.0289	0.0	0.00
C	361957	1584	0.0164	17.3	1.09
D	362452	1006	0.0287	9.4	0.93
E	363294	1724	0.1619	29.2	1.69
F	363474	2136	0.0623	67.6	3.16
G	363579	1823	0.0232	40.1	2.20
H	363595	1877	0.0153	44.5	2.37
I	363624	1918	0.0691	41.9	2.18
J	363697	1773	0.0726	31.8	1.79
K	363700	1664	0.1231	26.9	1.62
L	363871	1437	0.1440	14.5	1.01
M	364314	1195	0.0140	11.7	0.98
N	364375	1081	0.0387	10.9	1.01
O	365962	2005	0.0648	43.9	2.19
P	365974	1837	0.0299	35.1	1.91
Q	365999	2068	0.0004	39.9	1.93
R	366004	1798	0.0012	19.8	1.10
S	366012	1874	0.0022	33.3	1.78
T	368633	2000	0.0251	51.1	2.55
U	369919	1862	0.0382	52.4	2.81

Variation statistic S = 1.89

Mean daily percentage = 1.71%

Catchment Average Total Rainfall = 31.16 mm

Triangles found for 250 out of 324 mesh points.

TABLE 9 DART AT AUSTINS BRIDGE

Hourly Rainfall data on 14 July 1964

Gauge	Weight	Actual Total (mm)	Total from Daily Gauges (mm)	Check Gauge Total (mm)
363294	0.7724	24.88	25.25	29.20
363474	0.2276	63.63	48.73	67.60

Threshold 0.25 mm
Principal block is at gauge 363294

Time (hr)	363294		363474		Average
9	0		1.40		0
10	0		0.38		0.24
11	0		0.13		0.07
12	0		0.13		0.02
13	0		0.51		0.02
14	0		0.25		0.09
15	0.25		0.25		0.24
16	2.54		6.35	BLOCK	2.03
17	1.27		4.95		2.10
18	1.52		6.10		2.05
19	3.30	BLOCK	4.95		3.65
20	1.02		5.59		1.66
21	3.30		11.05		3.56
22	4.32		7.37		5.31
23	1.78		3.94		2.68
24	1.78		5.46		2.08
25	2.03		2.54		2.54
26	0.51		1.14		0.[illegible]
27	0.76		0.76		0[illegible]
28	0.25		0.25		0.[illegible]
29	0.25		0.13		0.[illegible]
30	0		0		0.02
31	0		0		0
32	0		0		0

Block Centroid	Block Centroid	Alignment Centre
at 21.44	at 20.09	at 21.22
Shift 0	Shift +1	

7 DISCUSSION AND CONCLUSIONS

The concept of a catchment average point rainfall profile is an abstract one; such profiles are not observable in practice and so there is no simple way in which an estimate [illegible] compared with "reality" to assess its performance. In order to allow such a comparison, it would be necessary to obtain the areal average profile, which could [illegible] compared with radar-derived rainfall data, however the rainfall data which is available for a typical case is better suited to the estimation of the average point profile.

It is therefore preferable to assess the method by considering the intuitive suitability of each component, and the capability to produce acceptable results from poor quality or non-uniform data. The methods described in this report have been developed with this as the principal aim because they are intended for computer application to large numbers of storms on a variety of catchments with a minimum of user intervention required. It is acknowledged that other methods may be more appropriate in specific cases.

One area which is consistently neglected in the development of methods for deriving catchment average rainfall profiles is that of variations in raingauge availability between events on the same catchment. For lumped catchment modelling to be meaningful, it is essential that these variations should, as far as possible, be eliminated from the rainfall input so that the addition of raingauges simply improves the accuracy of the estimate without affecting the fundamental shape of the profile. It is felt that the methods described here are a significant step towards achieving this, through the inclusion of daily raingauge weightings and timeshifts for the hourly profiles which are specifically designed for this purpose.

The average point profiles obtained using the methods described in this report may be used for input to lumped catchment models. It should be borne in mind that the difference between the average point profiles and the areal average profiles will be included in the model parameters thus obtained, but it is hoped that these differences will be more consistent between events for a catchment than the errors which would arise if the same raingauge data were used to estimate the areal profile. Further research is required to explore the nature of the relationship between average point profiles and areal profiles.

The principal components of the method described in this report may be summarised as follows:

1) Derivation of daily raingauge weights from an irregular mesh representation of a catchment, using an algorithm based on triangles of raingauges surrounding each mesh point. (Sections 2 and 3).

2) Detection of unacceptable variations in daily rainfall totals (Section 4).

3) Scaling of individual hourly profiles according to representative daily gauges (Section 2.3).

4) Alignment of hourly profiles by an objective means (Section 5).

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