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THE APPLICATION OF A KINEMATIC WAVE MODEL TO URBAN SURFACES

by

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ABSTRACT

A mathematical rainfall-runoff model based on kinematic wave theory has been developed for application to urban subcatchments. The model has been tested against laboratory rainfall-runoff data from concrete surfaces installed at the Imperial College of Science & Technology, London. The model reproduced hydrographs very faithfully under these circumstances, but a brief study on field data was not so impressive.

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1. INTRODUCTION

The purposes of a model of the rainfall-runoff process is to gain a better understanding of the physical processes at work and to use this knowledge in engineering design. This report describes one such model and its application to observed rainfall-runoff data.

It is useful to identify two classes of mathematical model: on the one hand, there is the physically-based approach which attempts to simulate the individual processes (such as overland flow in this case). On the other hand, there is the lumped conceptual approach which, assuming the individual processes to be too complex for satisfactory mathematical description, pretends that the overall process is governed by a simpler mechanism which is itself more conducive to analytical manipulation. This latter approach requires the parameters of this simpler mechanism to be related to physical properties of the prototype catchment, generally by regression analysis involving as much rainfall-runoff data as is available.

The choice of approach may be seen as a trade-off between scientific rigour and engineering expedience. To date, the Institute of Hydrology's urban research programme has concentrated on the latter approach (Kidd & Lowing, 1979) due to the constraints placed upon it by the need to develop improved design. This present study develops a model based on the first approach. In this way, it should be possible to assess the effect of the decision to adopt the lumped approach in terms of predictive performance.

The specific objective has been to study the model under circumstances most conducive to predictive success. In this respect, the model has been applied only to catchments which lend themselves well to division into overland flow and gutter flow elements. The data used were collected during experiments on a laboratory catchment at the Imperial College of Science and Technology (Johnston et al., 1978), brief details of which have also been given by Makin & Kidd (1979). A rainfall simulator was used to generate artificial rainfall inputs (varieties of intensities, durations and profiles) over a variety of catchment sizes (between 17 and 36 m^2) and slopes. The catchments contain a single overland flow segment plus a single gutter flow segment down one edge.

Limited tests were also done with a catchment using observed data deriving from field experiments. Like the laboratory catchments, this catchment lent itself easily to the process of dividing into segments.

TABLE 1: The Laboratory Catchments

Catchment	Size (m^2)	Slopes (%)	Area (m^2)
101	9.75 * 3.66	½ * 1	35.67
102	"	"	32.56 reduced area
103	7.32 * 3.66	½ * 1	26.76
104	7.32 * 2.44	½ * 1	17.84
105	9.75 * 3.66	1 * 2.5	35.67
106/107	9.75 * 4.88	1 * 2.5	47.57

THE MODEL

Introduction

The St Venant momentum equation for long waves in open channels can be defined as:

$$\underbrace{g\frac{\partial h}{\partial x}}_{1} + \underbrace{\frac{\partial v}{\partial t}}_{2} + \underbrace{v\frac{\partial v}{\partial x}}_{3} = g\,(S_o - S_f) \qquad (1)$$

where
- v = velocity ($m\ s^{-1}$)
- g = acceleration due to gravity ($m\ s^{-2}$)
- S_o = channel slope (-)
- S_f = friction slope (-)
- x = distance in downstream direction (m)
- t = time (s)

term 1 can be called the diffusion term
" 2 " " " " acceleration term
" 3 " " " " dynamic term

Under certain circumstances (for instance, where lateral inflow predominates) these three terms can safely be ignored, so that the momentum equation becomes

$$g(S_o - S_f) = 0 \quad \text{or} \quad S_o = S_f$$

With empirical relationships for S_f which are available, it is possible to find a solution for the rate of discharge as a function of the cross-sectional area of flow in the form:

$$Q = \alpha A^n \qquad (2)$$

For instance, values for α and n can be found from the shape of the cross-section and Chezy's friction equation

$$\bar{v} = C\, S_o^{\frac{1}{2}}\, R^{\frac{1}{2}}$$

where
- $\bar{v}$ = mean velocity (mS^{-1})
- C = Chezy's roughness coefficient ($m^{\frac{1}{2}}\ s^{-1}$)
- R = hydraulic radius (m)

A continuity equation for open channel flow can be written as:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \qquad (3)$$

where
- A = cross-sectional area of flow (m^2)
- Q = rate of discharge ($m^3\ s^{-1}$)
- q = lateral inflow ($m^2\ s^{-1}$)

The model defined by equations (2) and (3) (and derived by ignoring the three terms on the left hand side of equation (1) is known as the Kinematic Wave model. This pair of equations has no analytical solution, and the extensive literature on the subject (e.g. Lighthill & Whitham, 1955; Ruh-Ming Li et al., 1975 Lyngfelt, 1978) is mostly concerned with determining numerical schemes for solving them. In this case, the solution is based upon the work of Singh (1975).

A numerical solution

Q can be eliminated from equation 3 by substituting equation 2, so that the continuity equation can be written as:

$$\frac{\partial A}{\partial t} = q - n\,\alpha\,A^{n-1}\,\frac{\partial A}{\partial x} \qquad (4)$$

A Taylor expansion gives:

$$A(x,t+\Delta t) = A(t) + \Delta t.\,\frac{\partial A}{\partial t} + \frac{(\Delta t)^2}{2}\,.\,\frac{\partial^2 A}{\partial t^2} + \text{HOT} \qquad (5)$$

where HOT stands for Higher Order Terms, which will be ignored from now on.

$\frac{\partial^2 A}{\partial t^2}$ can be found by differentiation of equation 4

$$\frac{\partial^2 A}{\partial t^2} = \frac{\partial q}{\partial t} - \frac{\partial}{\partial x}\left\{ n\,\alpha\,A^{n-1}\left(q - \alpha\,\frac{\partial A^n}{\partial x}\right)\right\} \qquad (6)$$

Substituting equations 4 and 6 in 5 gives:

$$A(x,t+\Delta t) = A(x,t) + \Delta t\left(q - n\,\alpha\,A^{n-1}\,\frac{\partial A}{\partial x}\right) +$$

$$\frac{(\Delta t)^2}{2}\left\{\frac{\partial q}{\partial t} - \frac{\partial}{\partial x}\left[n\,\alpha\,A^{n-1}\left(q - \alpha\,\frac{\partial A^n}{\partial x}\right)\right]\right\} \qquad (7)$$

There is no analytical solution for equation 7, which is solved by a finite difference scheme. A fixed grid scheme (see figure 1) has been adopted.

By replacing partial derivatives by differences, one obtains the following finite difference solution:

$$A_j^{i+1} = A_j^i + \Delta t\left(q^i - \alpha\,\frac{A_{j+1}^{i\,n} - A_{j-1}^{i\,n}}{2\Delta x}\right) +$$

$$\frac{(\Delta t)^2}{2}\left(\frac{q^{i+1} - q^i}{\Delta t} - \frac{n\alpha}{\Delta x}\left[\frac{A_{j+1}^{i\,n-1} + A_j^{i\,n-1}}{2}\left(q^i - \alpha\,\frac{A_{j+1}^{i\,n} - A_j^{i\,n}}{\Delta x}\right) - \frac{A_j^{i\,n-1} + A_{j-1}^{i\,n-1}}{2}\left(q^i - \alpha\,\frac{A_j^{i\,n} - A_{j-1}^{i\,n}}{\Delta x}\right)\right]\right) \qquad (8)$$

A_j^i is the cross-sectional area of flow at point j and at time i, and q^i

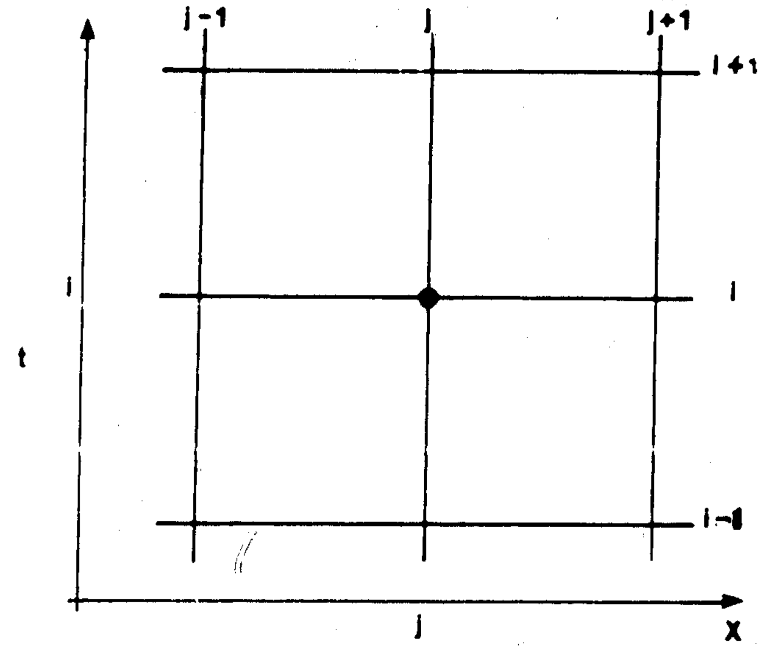


FIGURE 1 The fixed finite-difference grid

is the lateral inflow at time i. Now the cross-sectional area A_j^{i+1} can be calculated from A_{j-1}^i, A_j^i, A_{j+1}^i and q^i and q^{i+1}. So when boundary conditions for A are given for i=0 and for j=0, all the values of the array A can be computed, except for the point at the downstream end of the flow. For this point a first order approximation can be used:

$$A_j^{i+1} = A_j^i + \Delta t\left(q^i - \alpha\,\frac{A_j^{i\,n} - A_{j-1}^{i\,n}}{\Delta x}\right) \qquad (9)$$

The analytical solution

If the lateral inflow q from equation 3 is zero (e.g. overland flow after cessation of rainfall), an analytical solution for the differential equation can be obtained.

By substituting equation 2, equation 3 can then be written as:

$$\frac{\partial A}{\partial t} + \frac{\partial(\alpha A^n)}{\partial x} = 0 \qquad (10)$$

A solution can be found by the 'separation of variables' method. Assuming two independent functions T(t) and X(x) such that

$$A(x,t) = T(t).X(x) \qquad (11)$$

then $\frac{\partial A}{\partial t} = X \frac{\partial T}{\partial t}$ (12a) and $\frac{\partial A}{\partial x} = T \frac{\partial X}{\partial x}$ (12b)

After rearranging terms, the differential equation becomes:

$$\frac{1}{T^n} \cdot \frac{\partial T}{\partial t} = - n \alpha X^{n-2} \frac{\partial X}{\partial x} = - \mu^2 \qquad (13)$$

As the term on the left hand side is depending only on t and the term in the middle only on x, they can be equal to each other only if μ^2 is a constant.

Now we have two simpler differential equations:

$$\frac{1}{T^n} \cdot \frac{\partial T}{\partial t} = - \mu^2 \quad (14a) \quad \text{and} \quad n \alpha X^{n-2} \frac{\partial X}{\partial x} = \mu^2 \qquad (14b)$$

$$\text{or} \quad \frac{\partial T}{\partial t} = - \mu^2 T^n \quad (15a) \quad \text{and} \quad \frac{\partial X}{\partial x} = \frac{\mu^2}{n\alpha} X^{2-n} \qquad (15b)$$

From which it follows that:

$$T = \{(1-n)(c_1 - t\mu^2)\}^{\frac{1}{1-n}} \quad (16a) \quad \text{and} \quad X = \{(n-1)(c_2 + \frac{\mu^2}{n\alpha} x)\}^{\frac{1}{n-1}} \quad (16b)$$

so that

$$A(x,t) = \left(\frac{c_2 + \frac{\mu^2}{n\alpha} x}{\mu^2 t - c_1} \right)^{\frac{1}{n-1}} \qquad (17)$$

Now the constants c_1, c_2 and μ must be eliminated. This can be done by using the boundary conditions:

$$A(0,t) = 0 \qquad (18a)$$

$$A(L,t_o) = A_{L,t_o} \qquad (18b)$$

where L is the length of the flow region (m)

and t_o is the time from which the lateral inflow is zero (s)

From equations 17 and 18a, it can be seen that $c_2 = 0$. From equations 17 and 18b, it follows that:

$$c_1 = \mu^2 t_o - \frac{\mu^2}{n\alpha} L A_{L,t_o}^{1-n} \qquad (19)$$

In equation 17, μ^2 is eliminated automatically. When c_1 and c_2 are substituted in equation 17 the solution for A is found:

$$A(x,t) = A_{L,t_o} \left(\frac{x}{L + (t-t_o)\alpha n A_{Lt_o}^{n-1}} \right)^{\frac{1}{n-1}} \qquad (20)$$

This solution is valid for $t > t_o$ and for $A(o,t) = 0$ (without upstream inflow). By generalizing equation 18a it is possible to find a solution for cases with upstream inflow also.

Assume that:

$$A(o,t) = A_{o,t} \qquad (21)$$

Then c_1 and c_2 can be found in the same way:

$$c_2 = (\mu^2 t - c_1) * A_{o,t}^{n-1} \qquad (22a)$$

$$c_1 = \mu^2 t_o - \frac{\frac{\mu^2}{n\alpha} L}{A_{L,t_o}^{n-1} - A_{o,t_o}^{n-1}} \qquad (22b)$$

The solution for A is then

$$A(x,t) = \left(A_{o,t}^{n-1} + \frac{x}{n\alpha(t-t_o) + \frac{L}{A_{L,t_o}^{n-1} - A_{o,t_o}^{n-1}}} \right)^{\frac{1}{n-1}} \qquad (23)$$

Parameters α and n

As mentioned earlier the precise form of equation 2 (ie values of α and n) can be obtained by reference to Chezy's friction equation

$$\bar{v} = C S_o^{\frac{1}{2}} R^{\frac{1}{2}} \qquad (24a)$$

On the other hand, Manning's formula is:

$$\bar{v} = k_m S_o^{\frac{1}{2}} R^{2/3} \tag{24b}$$

There is some conflict between the exponents of R in these 2 equations, so that a general equation can be written in the form:

$$\bar{v} = C S_o^{\frac{1}{2}} R^{m-1} \tag{24c}$$

As $R = \frac{A}{WP}$ (WP = wetted perimeter) and $Q = \bar{v}A$, it follows that

$$Q = C S_o \frac{A^m}{WP^{m-1}} \tag{25}$$

For overland flow, where depth of flow is small in comparison to the width, it can be seen that $WP \sim b$. For a unit width, $WP \approx 1$ and $A \approx h$. So for overland flow

$$Q = C S_o^{\frac{1}{2}} A^m \tag{26}$$

so that $\alpha = C S_o^{\frac{1}{2}}$ and $n = m$.

For flow through a gutter (with triangular cross-section) α and n depend on the shape of the cross-section as well. It holds that $A = a_1 h^2$ and $WP = a_2 h$ (h depth of flow, a_1 and a_2 are constants, depending on the sideslopes). Then:

$$WP = \frac{a_2}{a_1^{\frac{1}{2}}} A^{\frac{1}{2}} \tag{27}$$

When this is applied to equation 25, we obtain

$$Q = C S_o^{\frac{1}{2}} a_1^{\frac{1}{2}(m-1)} a_2^{(1-m)} A^{\frac{1}{2}(m+1)} \tag{28}$$

so that for gutter flow $\alpha = C S_o^{\frac{1}{2}} a_1^{\frac{1}{2}(m-1)} a_2^{(1-m)}$

and $n = \frac{1}{2}(m+1)$

Table 2 gives a survey of the values for α and n obtained depending on which form of the friction equation is used.

a_1 and a_2 are easy to solve. For instance when the side slopes of the gutter are θ_1 and θ_2,

Table 2. Values for α and n from the equation $Q = A^n$ for three cases

	Overland flow			Triangular gutter flow		
	Chezy's eq $m = \frac{3}{2}$	Manning's eq $m = \frac{5}{3}$	exponent m free	Chezy's eq. $m = \frac{3}{2}$	Manning's eq $m = \frac{5}{3}$	exponent m free
α	$CS_o^{\frac{1}{2}}$	$k_m S_o^{\frac{1}{2}}$	$CS_o^{\frac{1}{2}}$	$CS_o^{\frac{1}{2}} a_1^{\frac{1}{4}} a_2^{-\frac{1}{2}}$	$k_m S_o^{\frac{1}{2}} a_1^{\frac{1}{3}} a_2^{\frac{2}{3}}$	$CS_o^{\frac{1}{2}} a_1^{\frac{1}{2}(m-1)} a_2^{(1-m)}$
n	$\frac{3}{2}$	$\frac{5}{3}$	m	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{1}{2}(m+1)$

$$a_1 = \frac{1}{2} * \left(\frac{1}{\text{tg } \theta_1} + \frac{1}{\text{tg } \theta_2}\right) \text{ and } a_2 = \frac{1}{\sin \theta_1} + \frac{1}{\sin \theta_2}$$

At this stage, no prior decision was made as to which friction relationship was most appropriate. The general equation 24c was used which effectively encompasses the other two.

Application of the model

The model as described makes it possible to simulate

(1) flow over rectangular planes (overland flow) with length L_{ov} and longitudinal slope S_{ov} and for time varying rainfall RF(t) and top end inflow $TE_{ov}(t)$.

(2) flow through a gutter with triangular cross-section, side slopes θ_1 and θ_2, length L_{gu}, longitudinal slope S_{gu} for time-varying lateral inflow q(t) and top end inflow $TE_{gu}(t)$.

For certain situations in an urban environment, it is possible to distinguish rectangular planes and gutters, forming a group of segments over which a 'cascade' of flow takes place. The variables L_{ov} and S_{ov} or θ_1, θ_2, L_{gu} and S_{gv} can then be obtained from an examination of catchment topography.

In fact, the overland flow will not take place in the direction of the longitudinal slope S_{ov} (i.e. rectangular to the gutter section) due to a longitudinal cross-slope associated with the gutter itself (see Figure 2). This causes the lateral inflow to the gutter to depend on space as well as time. As the model does not allow this, a simplification is necessary, so in the model as conceived it is assumed that the water flows in the direction of the longitudinal slope (dotted arrows). It is considered that this simplification will not be to the significant detriment of the performance of the model.

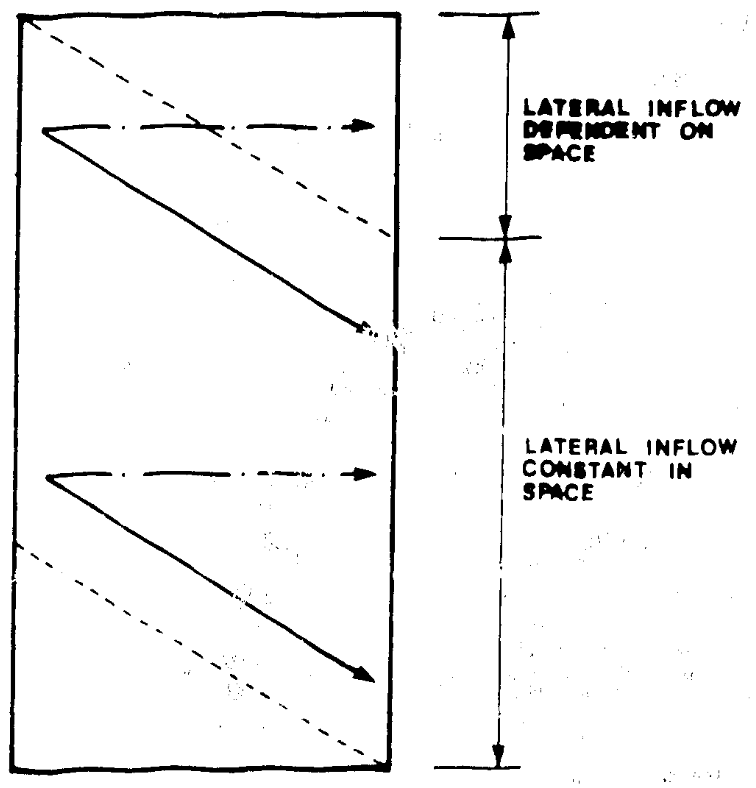


FIGURE 2 Simplification of flow direction for overland flow

When the gutter is formed by overland sections as in Figure 3a a second problem arises. Here the flow over the rectangular plane section ends at the water level in the gutter (Point B in Figure 3b) and the lateral inflow into the gutter from the plane section should be computed for point B. Point B however varies with the rate of flow in the gutter, and thus also in time and space. For simplicity, the rate of flow from the plane-section at Point A is taken as the lateral inflow into the gutter. For that reason, the rainfall that falls on the water surface in the gutter is not determined because it has already been taken into account between the points B and A and C and A as lateral inflow on the rectangular plane sections. This assumption greatly simplifies the application of the model, and is not considered to be significantly detrimental to its performance.

As the lateral inflow from the rectangular plane sections never becomes zero, the analytical solution is not applicable for gutter sections with lateral inflow. Furthermore, from equation 26 it can be seen that the analytical solutions is not usable for $A_{o,t_o} = A_{L,t_o} = 0$.

So for gutter segments with no lateral inflow the rising limb of the hydrograph must be calculated numerically as well.

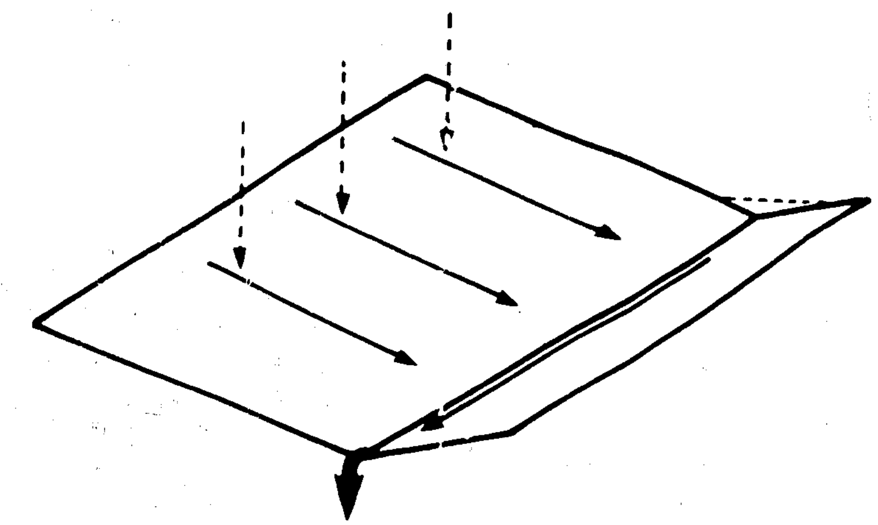


FIGURE 3a Gutter segment formed by two overland segments

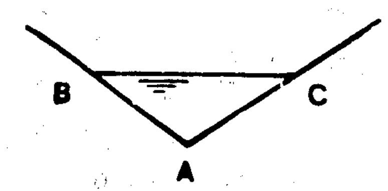
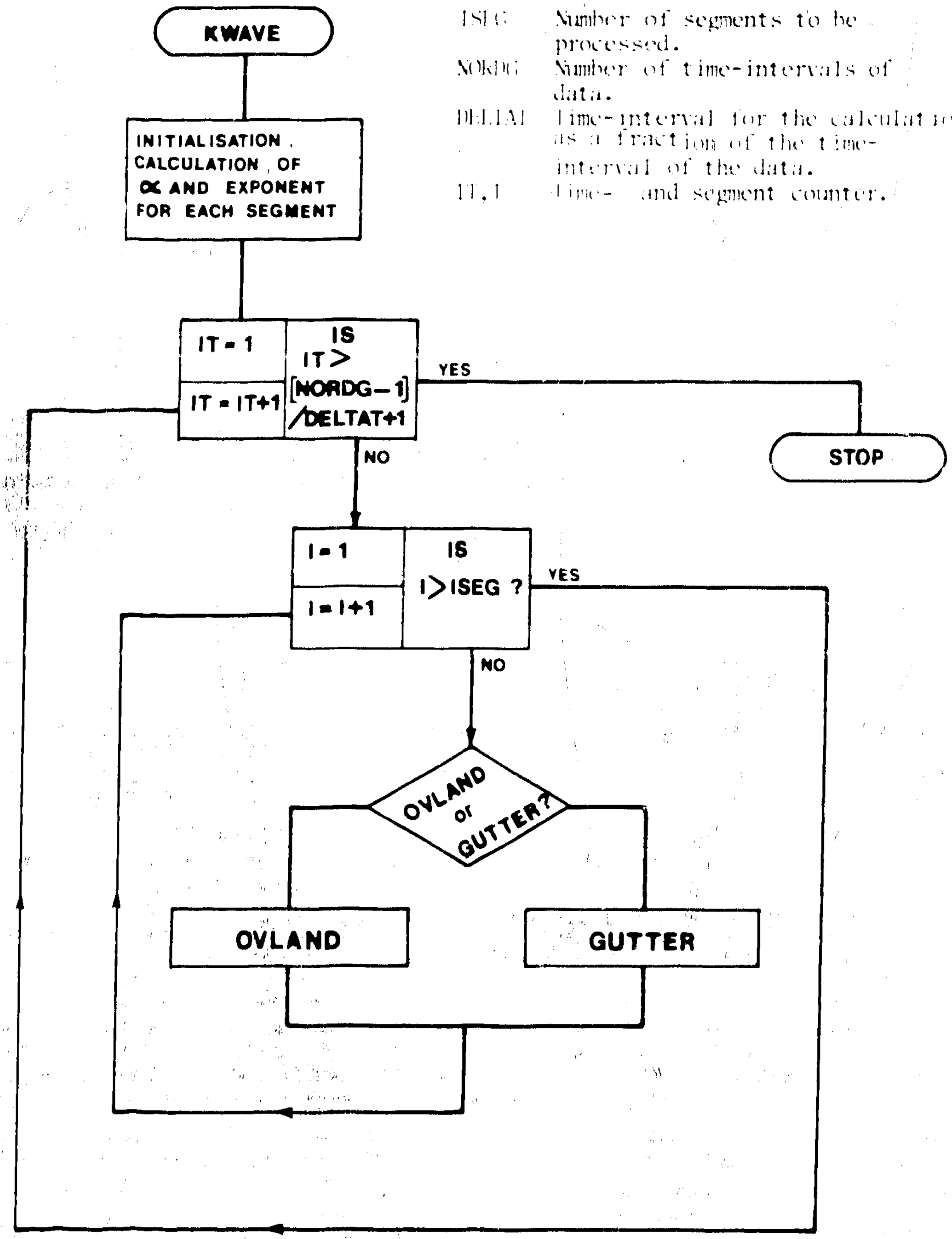


FIGURE 3b Simplification of boundary between overland and gutter segment

For the application of the model, the catchment must be divided into segments, which can either be for overland flow or for gutter flow. A brief summary of the algorithm is shown in Figure 4 overleaf. The complete source program is shown in Appendix A.

Analysis on grid size

The numerical solution given is an explicit finite difference scheme. As such the efficiency of the solution is dependent upon the grid size used. It is possible that the calculation becomes unstable when either the time or the space-step chosen is too large. As the time and/or space-step decreases, the simulation converges towards solution but the required computational effort also increases.



ISEG Number of segments to be processed.
NORDG Number of time-intervals of data.
DELTAT Time-interval for the calculation as a fraction of the time-interval of the data.
IT, I Time- and segment counter.

FIGURE 4 Summary of the algorithm of the kinematic wave model

To investigate the effect of decreasing the time-step a simulation of a hypothetical storm was carried out for different time steps (C, n and a number of space-steps fixed on 30.0, 1.71 and 5 respectively) on catchment 105. The rainfall was kept at a constant 200 mm/hr for a period that was long enough to reach the equilibrium runoff. At first sight, one might expect the accuracy to increase with decreasing time-step. Figure 5 demonstrates that, with a 5s time-step, the runoff goes more or less straight to the equilibrium flow and stays there until the rainfall stops. Simulation with smaller time-steps have progressive tendency to oscillate around the equilibrium, before becoming stable. A simulation with Δt = 10s gave stability problems.

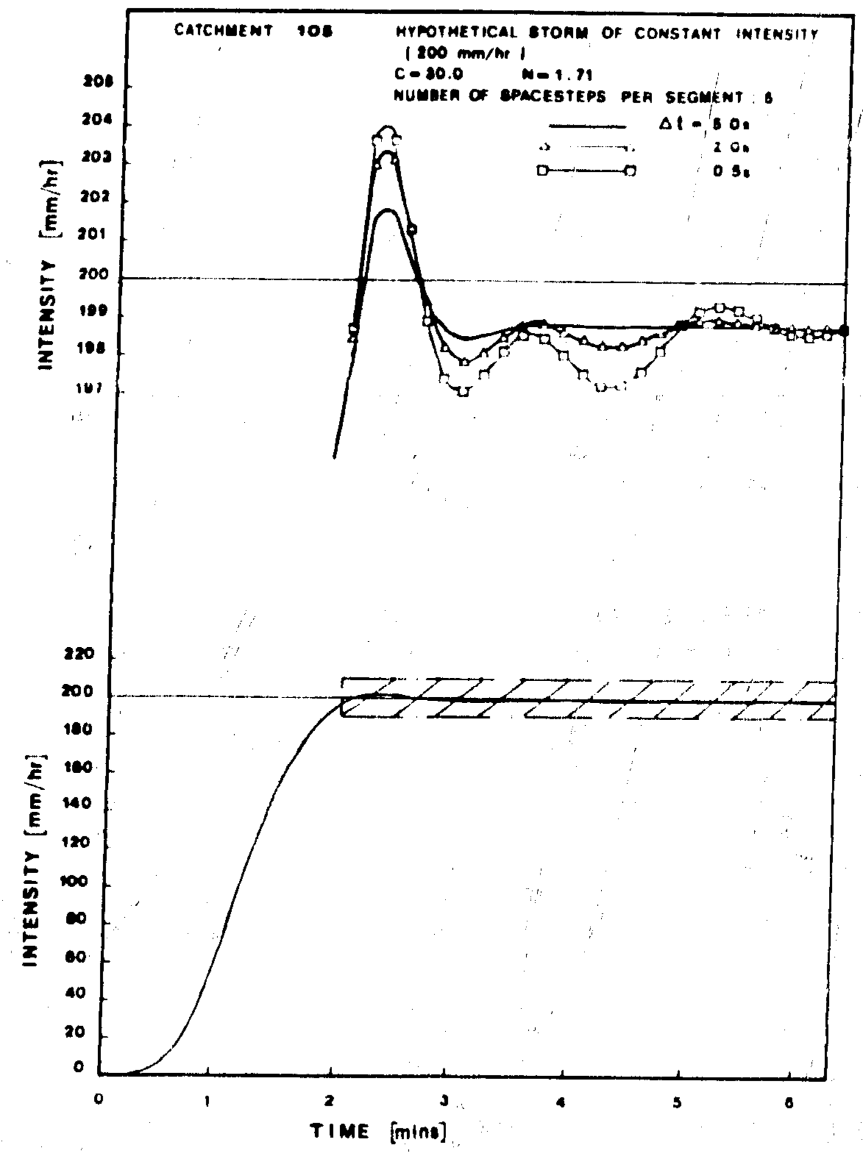


FIGURE 5 Simulations with decreasing time-steps. The top-curves are an enlargement of the hatched section

From this it may be concluded that, for a chosen space-step, the largest time-step that does not cause stability problems gives the best simulation. How large this time-step will be must be found by trial and error. The maximum time-step that does not give instability depends upon the chosen C, n and rainfall intensity. It can be seen that once a satisfactory pair of Δx-Δt values has been found, decreasing the spacestep only will lead to stability problems.

Using the above scheme for choosing the time-step, it is possible to investigate the effect of decreasing the space-step (still keeping C and n constant). As the space-step decreases the simulations tend to converge towards some absolute solution. Satisfactory results can be obtained with approximately 5 space-steps for any given segment. Adopting this criterion time-steps between 2.5 and 5.0s are found to be appropriate for the type of data under investigation.

2. ANALYSIS OF LABORATORY CATCHMENT DATA

Introduction

The model described in the previous section uses variables, for which values can be found from the properties of the catchment (catchment configuration, length and slope of segments), and model parameters (roughness C and exponent n). This section is concerned with the performance of the model for a variety of these variables. (The dimension of C is dependent on n: meter^{2-n} second^{-1}. For this reason the dimension is omitted; but the units may be assumed to be metric).

For convenience the Kinematic Wave routines have been made compatible with the Institute of Hydrology Modelling package. This computer-based package is described in detail elsewhere (Kidd, 1978), and has the following capabilities:

1. easy and efficient data access,
2. a variety of loss-models,
3. a variety of surface routing models (to which the Kinematic wave model has now been appended),
4. Optimization of up to 10 model parameters (using a Rosenbrock algorithm) on individual or combined events,
5. error surface mapping for designated pairs of model parameters,
6. a variety of objective functions for optimization. For the purpose of this investigation, optimization has been performed using the Integral Square Error, which is

a normalised version of the last squares estimate, and given by

$$ISE = \frac{\Sigma (Q_o - Q_m)^2}{\Sigma Q_o} * 100\%$$

where Q_o = observed runoff
Q_m = modelled runoff
ISE = integral square error

For this analysis the data from the experiments on laboratory catchments with concrete surfaces at Imperial College (Johnston *et al.*, 1978) have been used. As the catchment surfaces have been wetted immediately before the actual experiments, it is assumed that no noticeable depression storage takes place nor any other losses by percolation or evaporation. The rainfall volumes have been proportionally corrected to match the observed runoff volumes (volumes have effectively been forced). This correction is small due to the controlled nature of the experiments. The number of space-steps has been taken as 5 for all segments and the time-step as 2.5 seconds.

For each catchment (- see Table I) a group of events has been selected that has been used to perform the optimization. The characteristics of these events are shown in Table 3.

TABLE 3. Characteristics of events used for the analysis

Catch No.	Storm no.	Duration (s)	Intensity (mm/hr)	Shape (SW = square wave, T = triangular wave)
101	100018	60	250	SW
	100019	30	250	SW
	100005	60	200	SW
	100010	60	150	SW
	100022	60	126	SW
	100025	300	208 (max)	T
103	100055	90	146	SW
	100058	90	198	SW
	100060	150	250	SW
	100061	90	250	SW
	100063	300	204 (max)	T
	100064	300	153 (max)	T
104	100069	60	250	SW
	100070	45	250	SW
	100073	90	198	SW

	100076	90	76	SW
	100078	300	307 (max)	T
	100079	300	253 (max)	T
105	100085	60	251	SW
	100086	45	251	SW
	100089	60	199	SW
	100093	60	148	SW
	100097	60	128	SW
	100102	300	307 (max)	T
	100103	120	307 (max)	T
106	100116	60	256	SW
	100117	120	256	SW
	100113	60	196	SW
	100111	60	146	SW
	100137	60	200 (max)	T
	100130	180	196	SW

Estimation of optimum parameters for individual catchments

For each catchment, the two model parameters were optimised. For this purpose, a global optimum was obtained by doing the analysis on all events combined. It immediately became clear that the error function surface contained a very clearly-defined steep-sided valley caused by a strong interdependency between the two parameters. An example of this phenomenon is shown in Figure 6 for catchment 105. The line of the bottom of the valley is very flat, so that satisfactory simulations can be obtained from an infinite variety of pairs of C and n values.

The Rosenbrock algorithm does not work very efficiently under these circumstances, so that a search for the true optimum was performed by studying cross-sections of the valley at different points.

Table 4 gives the global optima found for each catchment. In Figure 7, the optima have been plotted to a common scale. Through each optimum has been drawn a line representing the bottom of the valley (as plotted on Figure 6 for catchment 105). From this it can be seen that, while there is some variation in the position of the optimum for each catchment, the relative locations of the valley bottoms are almost coincident. The apparent disparity in the position of the optimum for catchment 106 may be associated with the fact that these data were processed somewhat differently than those for the other catchments.

TABLE 4. Global optimal C and n for each catchment (found by optimisation on events of Table 3 combined).

Catchment No.	n_{opt}	C_{opt}
101	1.70	65.383
102	1.73	68.826
104	1.72	70.850
105	1.71	59.914
106	1.40	12.589

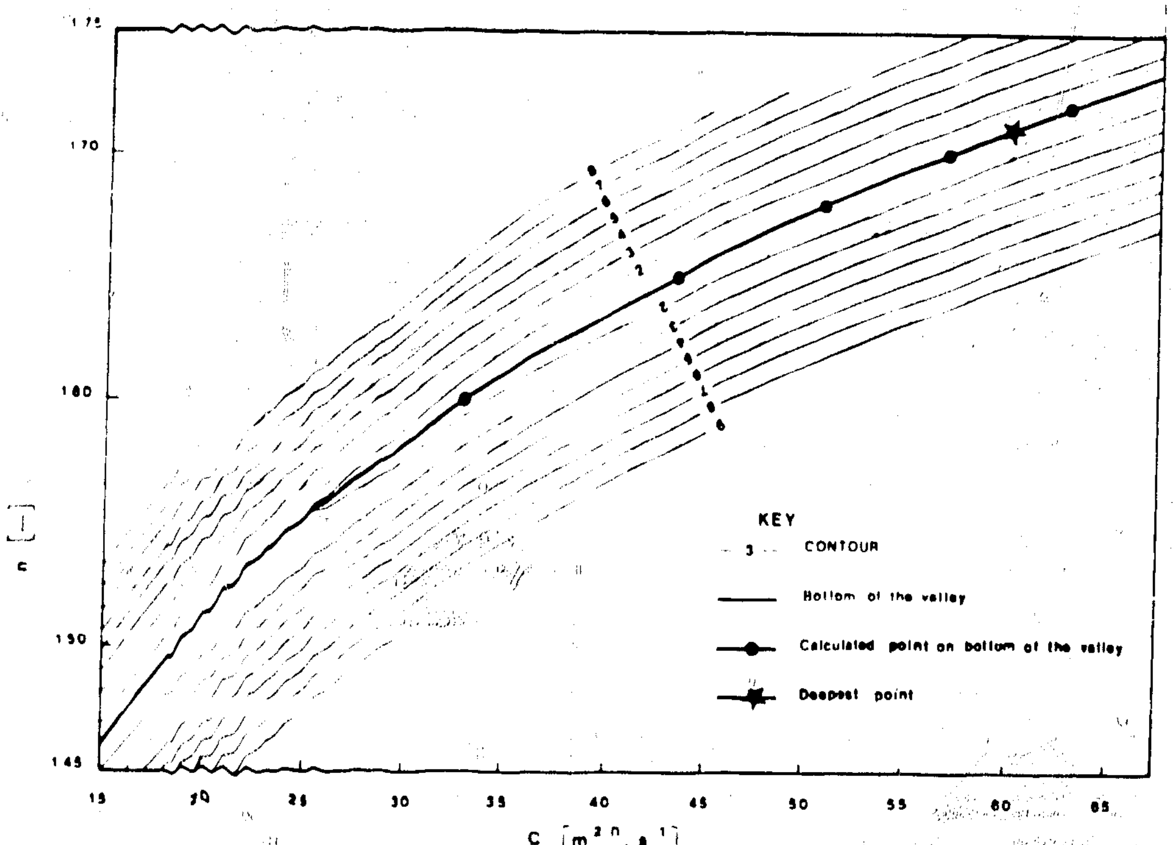


FIGURE 6 Error surface mapping for catchment 105

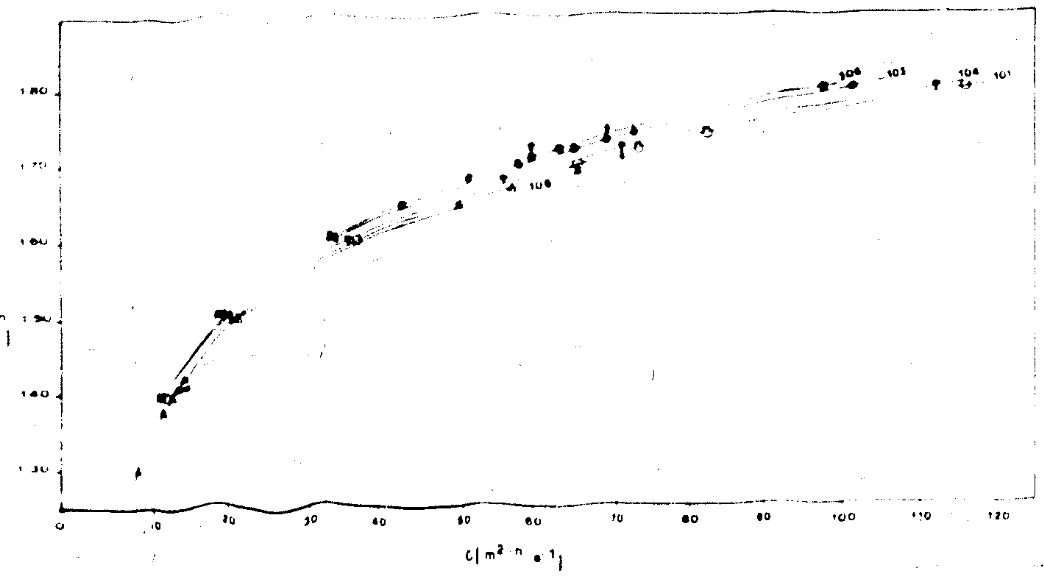


FIGURE 7 Bottoms of the valleys for the five laboratory catchments

Estimation of overall parameter values

As Manning's value of $\frac{5}{3}$ (see equations 24) for the exponent n is close to the optima of four catchments, the effect of taking $n = \frac{5}{3}$ for each catchment instead of the optimal value has been investigated. Table 5 shows the values of the objective function for n optimal and for $n = \frac{5}{3}$ for each catchment and their relative differences. As can be expected, the difference in the values of the objective function is only significant for catchment 106. Even in this case the difference is not large. Overall the choice of $n = \frac{5}{3}$ for all catchments is considered justifiable.

For this value, it has been investigated how the C value found by optimization on combined events corresponds to the C value found by optimization on individual events. Table 6 shows the C values for the events of 104 and 105 which have been found by individual optimizations. The value for C found by optimization on combined events differs in both cases less from the arithmetic mean than its standard deviation. While the differences in objective functions seem quite large, the absolute values are still small (< 5%).

Having fixed a value for n, it remains to ascertain whether a single value of C will suffice, as it should do if the model is justified, for all catchments. In Figure 8 the error function is plotted against the C value for $n = \frac{5}{3}$ for the five catchments showing the five minima.

TABLE 5. Changes in the objective function when $n = \frac{5}{3}$ is used instead of n_{opt}.

Catchment No.	n_{opt} (-)	C_{opt}	I.S.E. (n_{opt}) (%)	$C(n=\frac{5}{3})$	I.S.E. ($n=\frac{5}{3}$) (%)	rel. diff.* (%)
101	1.70	65.3	1.95	55.2	1.95	0.2
103	1.73	63.8	1.48	49.5	1.49	0.6
104	1.72	70.8	2.34	52.3	2.36	0.8
105	1.71	59.9	1.34	47.4	1.38	3.8
106	1.40	12.6	2.94	57.6	3.72	26.5

*) relative difference in values of objective functions:

$$\frac{\text{Obj}\ (n_{opt}) - \text{Obj}\ (n = \frac{5}{3})}{\text{Obj}\ (n_{opt})} * 100\%$$

TABLE 6. Optimal C's for $n = \frac{5}{3}$ for individual events of catchments 104 and 105.

Catchment No.	Event No.	C_{opt} $m^{\frac{1}{3}}\ s^{-1}$	I.S.E. (C_{opt}) (%)	I.S.E. (C=52.4) (%)	rel. diff. (%)
104	100069	53.8	3.10	3.17	2.1
	100070	55.0	3.82	4.04	5.6
	100073	51.1	2.33	2.78	19.2
	100076	49.9	2.56	2.75	7.4
	100078	51.0	0.91	0.93	1.9
	100079	49.6	0.85	0.91	7.52
arithmetic mean		51.7			
standard deviation		2.2			
global mean		52.3			
105	100085	47.7	.99	2.73	175.7
	100086	47.9	1.63	3.43	110.4
	100089	48.0	1.18	2.71	130.6
	100093	45.5	1.80	4.50	150.0
	100097	47.3	1.99	3.59	80.4
	100102	47.1	.54	.78	44.4
	100103	47.3	1.04	1.94	86.5
arithmetic mean		47.3			
standard deviation		0.8			
global mean		47.4			

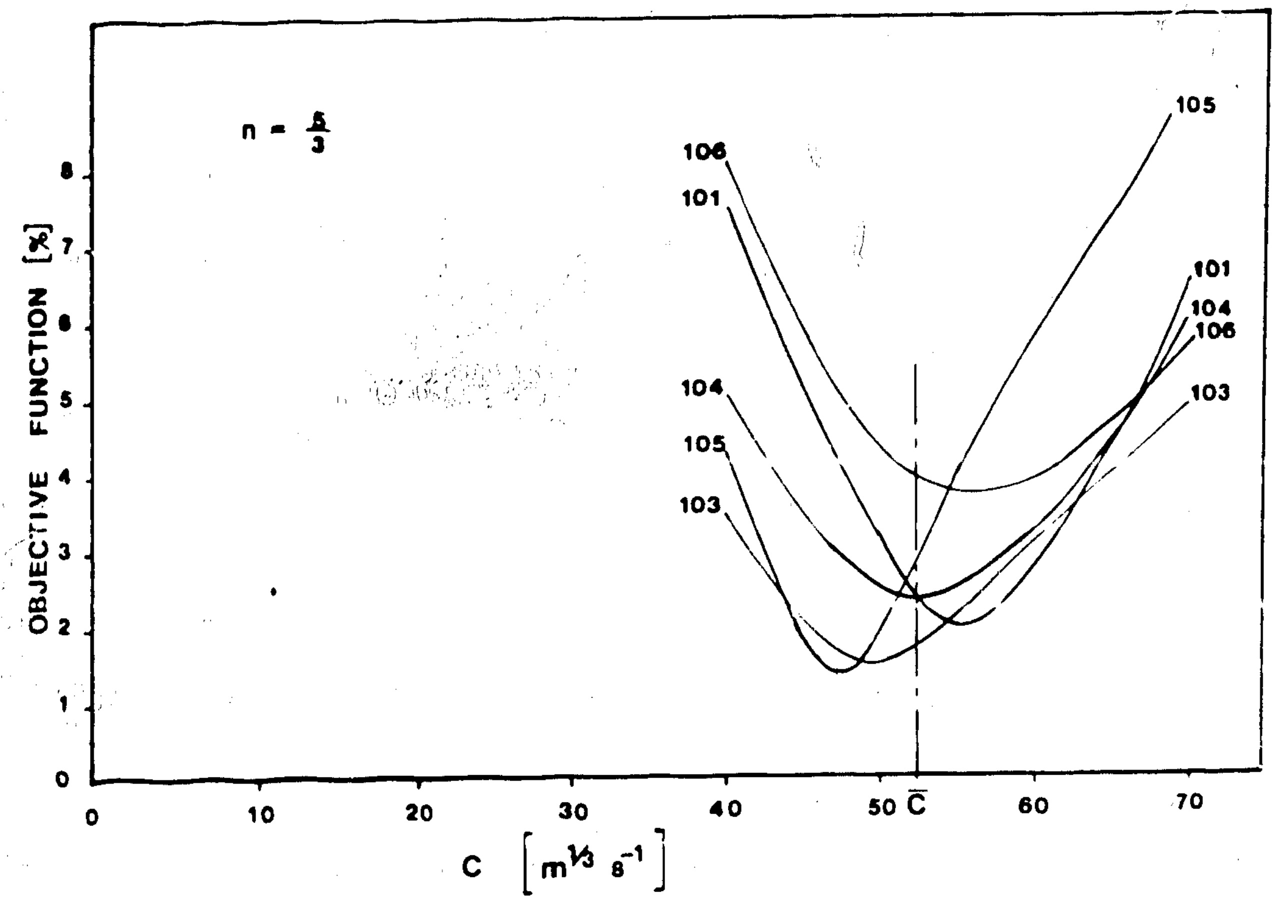


FIGURE 8 Relationship between objective function and C for five laboratory catchments

The arithmetic mean of the C values for which these minima occur is $\bar{C}$ = 52.4 (standard deviation 4.1). The values are close enough to suggest that the adoption of such a mean figure is justified. This hypothesis is tested by Table 7, which shows the values for the error function for C optimal per catchment and for $C = \bar{C} = 52.4$ and their relative differences. From this table and from Figure 8 it can be expected that the goodness of fit of the simulations of catchment 104 will be affected least and of catchment 105 most when $\bar{C}$ is used instead of C_{opt}. Simulations with $n = \frac{5}{3}$ $C = C_{opt}$ and $C = \bar{C}$ for events 100078 and 100085 are shown in Figures 9a and 9b. These two represent the best and the worst results that could be expected from the adoption of global overall parameter values.

TABLE 7. Changes in the objective function when $C = \bar{C} = 52.4$ is used instead of $C = C_{opt}$ for each catchment $(n = \frac{5}{3})$

Catchment	C_{opt} $m^{1/3}s^{-1}$	I.S.E. (C_{opt}) (%)	Obj (52.4) (%)	Relative difference*) (%)
101	55.2	1.95	2.26	15.9
103	49.5	1.49	1.69	13.4
104	52.3	2.36	2.36	0.0
105	47.4	1.38	2.89	109.4
106	57.6	3.72	3.92	5.3

*) Relative difference = $\frac{Obj\ (C_{mean}) - Obj\ (C_{opt})}{Obj\ (C_{opt})} * 100\%$

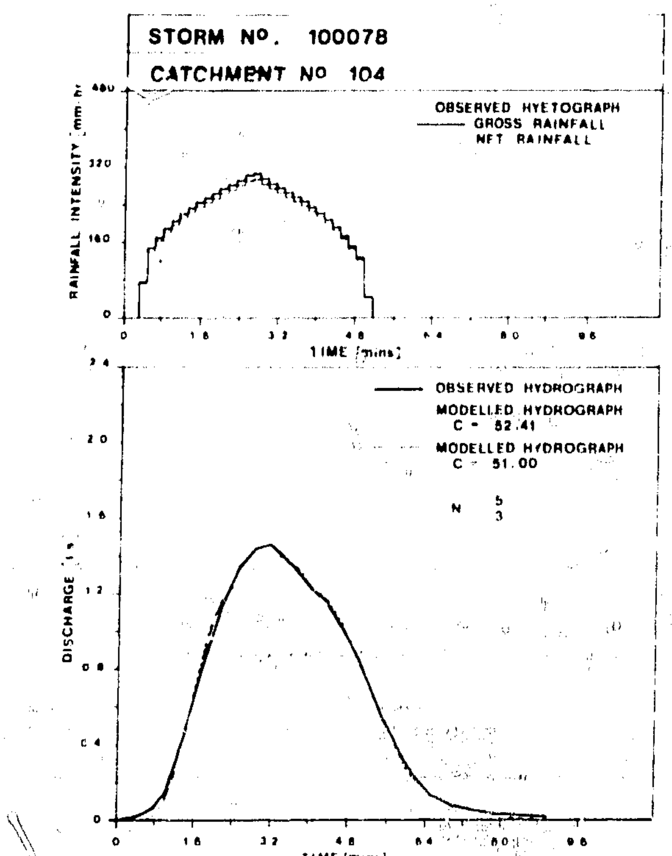


FIGURE 9a Simulations for C optimal for the catchment and for the mean value of C for all catchments

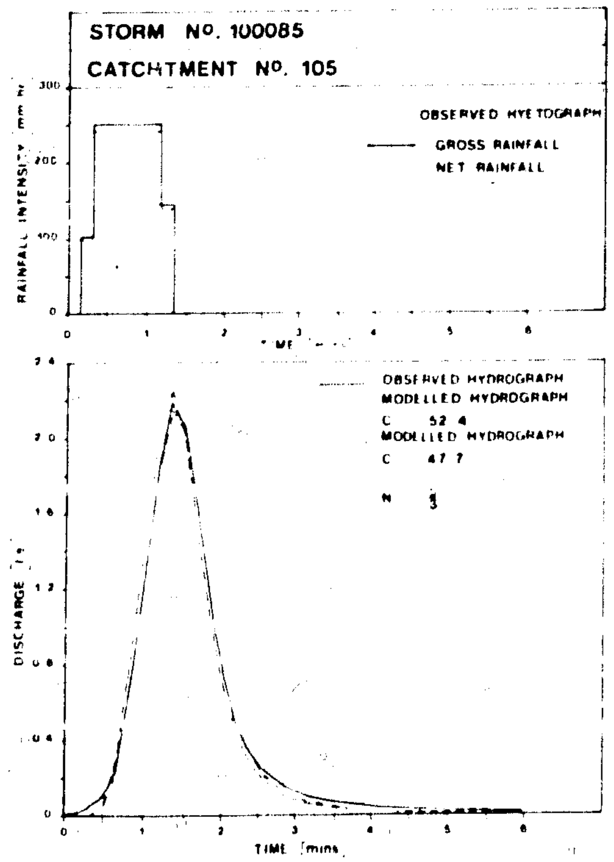


FIGURE 9b Simulations for C optimal for the catchment and for the mean value of C for all catchments

The effect of rainfall impact on roughness

It has been suggested in the literature (e.g. Ruh-Ming Li *et al*, 1975) that rainfall impact increases roughness. To investigate the importance of this phenomenon, the subroutine that calculates the objective functions was changed in such a way that it became possible to do the calculation for a reduced part of the event. Then it was possible to carry out optimizations for the rising limb, the recession limb or for the whole event. For long-lasting events with a square-wave rainfall profile, the time before the fall-off of the hydrograph can be assumed to represent the catchment under rainfall and the period after this point to represent the catchment under no rainfall.

For eight events on three different catchments optimizations were performed for C for the rising limb, the falling limb and the whole event, resulting in roughness estimates C_{ri}, C_{fa} and C_{wh} respectively. (Values for n have been taken from Table 4 for each catchment). Table 8 gives a summary of the results of this analysis. The roughness coefficient is lower for the falling limb (i.e. a higher roughness) in

TABLE 8. Results of optimization of C for the rising limb, the falling limb and the whole event (C_{ri}, C_{fa}, C_{wh})

Catchment No.	Event No.	Duration (s)	Intensity (mm/hr)	n*)	C_{ri}	C_{fa}	C_{wh}
101	100007	240	200	1.70	64.7	62.0	64.0
	100008	150	150	1.70	65.0	57.5	63.1
	100015	120	250	1.70	65.0	59.4	64.3
105	100088	150	199	1.71	58.7	56.5	58.2
	100092	150	148	1.71	50.7	[illegible]	[illegible]
	100096	150	128	1.71	58.2	52.8	56.9
106	100112	120	196	1.40	13.7	13.1	13.5
	100117	120	256	1.40	11.8	11.6	11.7

*n values taken from table 4

all cases, although the difference is not great. This is the opposite effect to that expected if rainfall impact does increase roughness. A possible explanation of this phenomenon is that, as the recession has a greater proportion of low flows than the rising limb, the roughness will increase with decreasing depth, so the two phenomena may be effectively cancelling each other out. The conclusion from this analysis is that the effect of rainfall impact may safely be ignored.

Conclusion

The analyses described above suggest that the Kinematic Wave model simulates the data very well. However, these analyses have been performed with data from highly controlled conditions. The implication is that with roughness C = 52.4 and exponent $n = \frac{5}{3}$, the model reproduces events occurring over well defined urban subcatchments of surface texture the same as the concrete used in the laboratory. The effect of applying the model to less perfect circumstances is investigated in the next section.

3. ANALYSIS OF FIELD DATA

Introduction

The analyses described in this section represent a first attempt to evaluate the model under circumstances less amenable to accurate modelling than the laboratory data used in the previous section. A limited amount of analysis has been [illegible] on catchments which lend themselves well to the type of division [illegible] segments necessary for the application of the model.

School Close, Stevenage (catchment 206/2) is a subcatchment instrumented as part of the Institute of Hydrology's data collection programme, and details can be found elsewhere (Makin and Kidd, 1979). Figure 10 is a representation of the catchment as programmed. Four events have been used in the analyses.

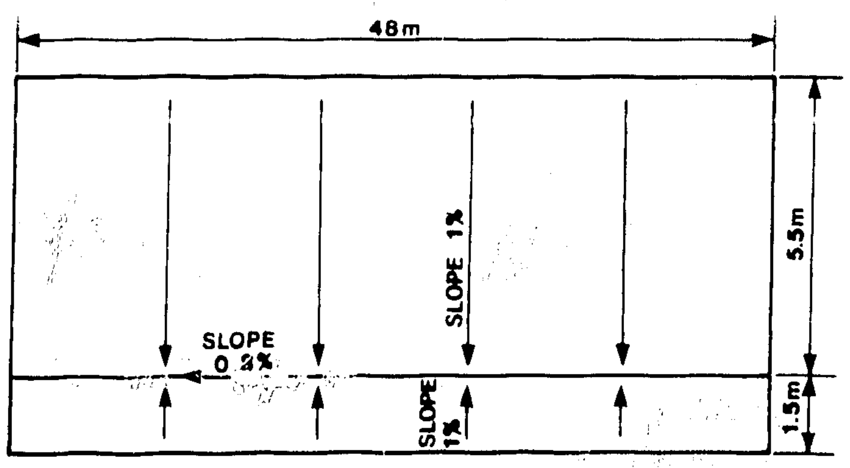


FIGURE 10 Schematic representation of catchment 206/2

Analysis

Catchment data have been obtained from a 1:100 map (Makin & Kidd, 1979). The catchment has been divided in two rectangular overland flow segments converging into one gutter segment. These segments had to be idealised to rectangular planes in order to obtain a longitudinal slope and a length for each segment. As the catchment was dry at the start of each event a depression storage had to be applied. In an earlier study (Kidd & Lowing, 1979) an average depression storage of 0.53 mm had been found for this catchment. After an initial allowance for depression storage, the volume of rainfall has been corrected to the measured runoff volume with the constant proportional loss model (see Kidd, 1978; Kidd & Lowing, 1979).

This proportional loss is given by:

$$CP = 1 - \frac{RO}{(RF - DEPSTOG)*AREA} \tag{29}$$

where

CP	constant proportional loss (-)
RF	gross rainfall volume (mm)
DEPSTOG	depression storage (mm)
AREA	total area of the segments (m^2)
RO	measured runoff volume (ltr)

For event 206019 the kinematic wave model parameters have been optimised, giving a similar valley as found for the laboratory catchments, with an optimum at n = 1.71 and C = 73.0. Figure 11 shows the simulation for the optimal values of the model parameters. Figure 12 shows the line of the bottom of the valley for event 206019 together with that for catchment 105. For the value of $n = \frac{5}{3}$ obtained in the previous section, an optimum C = 55.6 is obtained as against 52.4 for the concrete surfaces.

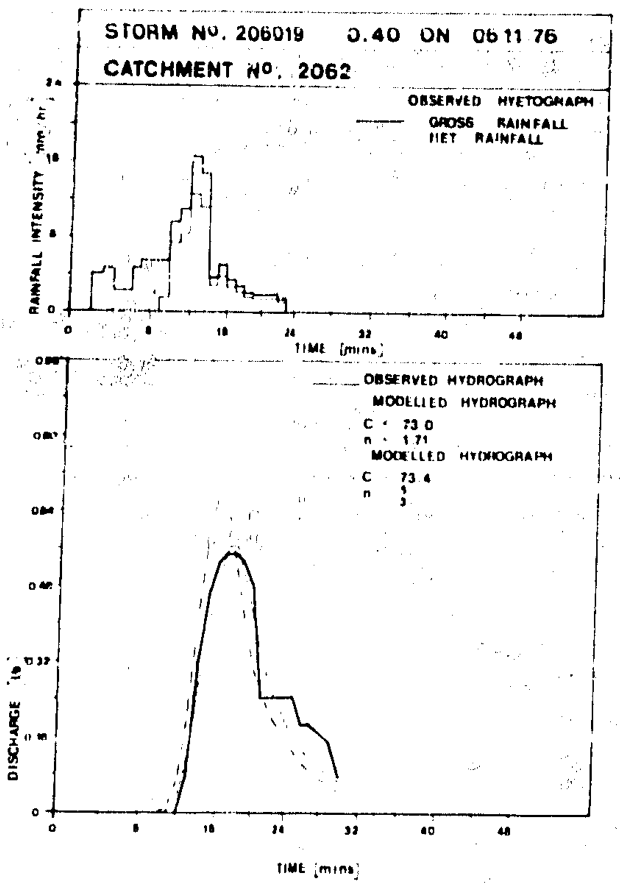


FIGURE 11 Simulation with C and n optimal for the event and with $n = \frac{5}{3}$ and the mean value of C from four events

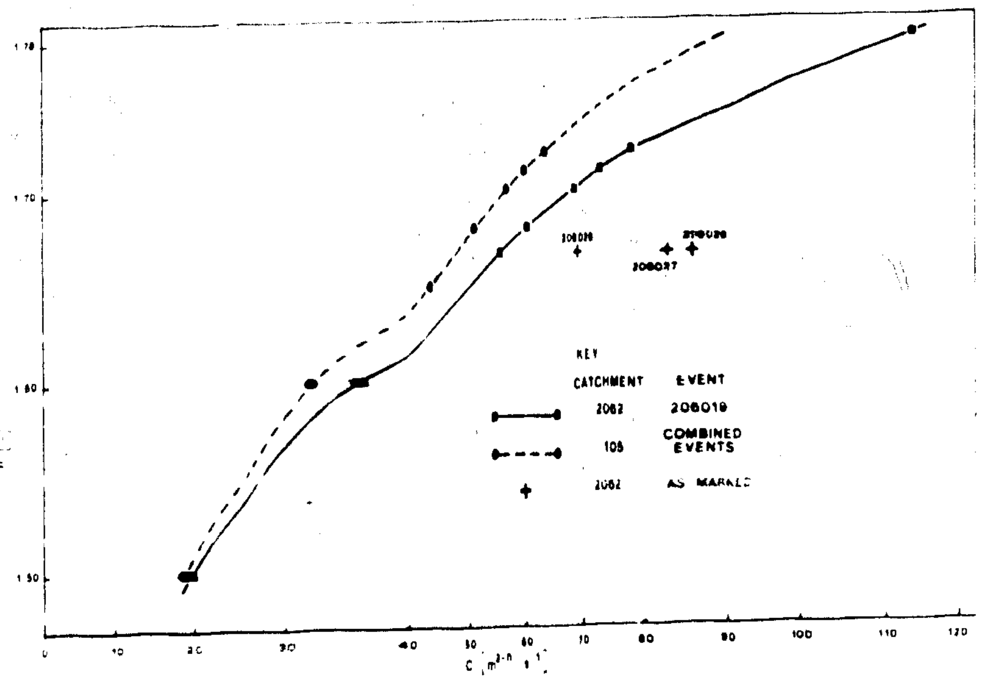


FIGURE 12 Bottom of the valley for event 206019 and catchment 105 and optimal C values for three other events of catchment 2062 for $n = \frac{5}{3}$

For $n = \frac{5}{3}$, C has been optimised for three other events on the same catchment. The resulting optimum C values, as marked on Figure 12, show a much greater variability than was found for the laboratory catchment events. Event 206019 has been rerun with C = 73.7 and $n = \frac{5}{3}$, and the resulting simulation is shown in Figure 11.

For conditions less ideal than those of the laboratory catchments, the variability in optimum roughness is much greater. However Figure 11 shows that a reasonable simulation can be found with the kinematic wave model under these conditions.

4. SUMMARY AND CONCLUSIONS

A mathematical model based on kinematic wave theory has been developed for application to urban subcatchments. Within a spectrum of mathematical models of varying complexity, this model represents the practicable limit as far as sophistication is concerned. The model uses an explicit solution to the kinematic wave equations, and uses a fixed-grid finite difference formulation. The model has been programmed in Ascii Fortran, and the routines are in Appendix A.

Performance testing against laboratory data has demonstrated that the model does indeed simulate the overland processes very well. A strong interdependence of the two parameters was observed but one pair of parameter values (roughness C = 52.4, exponent $n = \frac{5}{3}$) is capable of satisfactorily reproducing observed rainfall-runoff data for all five catchments in the laboratory study. A brief attempt to apply the model to the less ideal circumstances of field data was less rewarding in terms of reproduction of observed runoff hydrographs.

In practical application, the advantages of scientific exactitude tend to be masked by such elements as (a) data error, (b) uncertainties in modelling the rainfall loss in space and time, (c) shortcomings in the assumption of uniform sheet overland flow, and (d) idealisation of catchment topography. This study has shown that under ideal circumstances (where the effect of these elements are kept to a minimum) the model is excellent. The manner in which this performance falls off as the above elements (and, most importantly, items (b) and (d)) become important should be the subject of further study. It is in this respect that this model can then be compared to its more simplistic lumped counterpart.

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APPENDIX A

Source programs in FORTRAN

The common blocks make the routines compatible with the Institute of Hydrology's urban hydrology modelling package. Input and output is done by this package.

DOE-URBAN*REINOUT(1).KWAVE

```
C       THIS ROUTINE DOES A KINAMATIC WAVE SIMULATION USING THE
C       THEORY WHICH IS EXPLAINED IN  CHAPTER ONE OF
C       "THE APPLICATION OF A KINEMATIC WAVE MODEL TO URBAN SURFACES"
C       BY RJMG AND CHRK.
C       PROGRAMMED BY RJMG ; MAY '79
C       SUBROUTINES THAT ARE CALLED DIRECTLY BY THIS ROUTINE ARE
C       GUTTER, OVLAND AND QLINT.
C
C       EXPLANATION OF VARIABLES (THE VALUES OF THE VARIABLES
C       THAT ARE MARKED WITH A "*" ARE AFFECTED BY THIS ROUTINE.):
C
C     * ALFA(I)          ALFA FROM TABLE 1 FOR SEGMENT I (M**(3-2*EXP(I))/S).
C     * EXP(I)           EXPONENT N FROM TABLE 1 FOR SEGMENT I (-).
C     * QLOLD(I)         LATERAL INFLOW DURING PREVIOUS INTERVAL FOR SEGMENT
C                        I (M**2/S).
C     * QLNEW            LATERAL INFLOW DURING NEXT INTERVAL (M**2/S).
C     * AOTO(I)          A*(TZERO) FROM EQ. 22 FOR SEGMENT I WHERE TZERO IS TIME
C                        FROM WHICH CALCULATION WILL BE DONE ANALYTICALY.
C     * ALTO(I)          AZERO FROM EQ. 18B FOR SEGMENT I;IDEM; (M**2)
C     * A(I,J,K)         CROSS-SECTIONAL AREA OF FLOW FOR SEGMENT I,ON GRIDPOINT
C                        J AND AFTER TIME INTERVAL J-1; (M**2).
C     * QOUT(I)          FLOW FROM THE BOTTOM END OF SEGMENT I (M**3/(ITINT*S)
C     * IT               TIMEINTERVAL COUNTER (-).
C     * ITHYE            NUMBER OF TIMEINTERVALS AFTER WHICH RAINFALL STOPS (-).
C     * DX(I)            DISTANCE BETWEEN TWO GRIDPOINTS FOR SEGMENT I (M).
C     * DELTAT           TIMESTEP  FOR THE CALCULATION AS A FRACTION OF ITINT.
C     * NK(I)            NUMBER OF GRIDPOINTS FOR SEGMENT I (-)
C       ISEG             NUMBER OF GRIDPOINTS THAT HAS TO BE PROCESSED (-)
C                        0 < ISEG < NSEG.
C       ITYPE(I)         SEGMENT TYPE OF SEGMENT I;
C                        ITYPE(I)=1 FOR OVERLANDFLOW SEGMENTS;
C                        ITYPE(I)=2 FOR GUTTERFLOW SEGMENTS.
C       AL(I)            LENGTH OF SEGMENT I IN FLOWDIRECTION (M).
C       SLOPE(I)         SLOPE OF SEGMENT I IN FLOWDIRECTION AS A FRACTION:
C                        SO ABS(DH/DX) NOT IN %,DEGREES,GRADES,RADIANS ETC.
C                        SLOPE(I) > 0,I=1,ISEG.
C       WID(I)           NOT USED.
C       ZLT(I) & ZRT(I) LEFT AND RIGHT SIDESLOPE FOR GUTTERSEGMENT I
C                        AS FRACTIONS.
C     * INCREM(I)        NUMBER OF GRIDPOINTS - 1 FOR SEGMENT I (-)
C                        IF P(NEXT+2) > 2 THE VALUES OF INCREM ARE MADE
C                        EQUAL TO P(NEXT+2);
C                        IF INCREM(I) > NX-1 INCREM IS MADE NX-1.
C       OVSEG1(I) & OVSEG2(I) SEGMENTNUMBER FROM WHICH LATERAL INFLOW
C                        TAKES PLACE IN SEGMENT I; IF EQUAL TO 0, NO LATERAL INFLOW
C                        IS ASSUMED.
C                        OVSEG1(I) < I AND OVSEG2(I) < I.
C       UPSEG(I)         NUMBER OF THE SEGMENT FROM WHICH WATER FLOWS
C                        INTO SEGMENT I AT THE TOP END;
C                        IF UPSEG(I) = 0 NO TOPEND INFLOW IS ASSUMED.
C                        USEG(I) < I.
C       NETRF(I) (REAL!) NETTO RAINFALL DURING TIMEINTERVAL I (MM/HR).
C     * RUNOFF(I)        RUNOFF FROM THE (CASCADE OF) SEGMENT(S) AFTER TIME-
C                        INTERVAL I, SO AFTER I*ITINT SECONDS (LTR/S)
C       P(I)             MODEL PARAMETERS AND VARIABLES; P(NEXT+I),I=0,3
C                        ARE USED BY THIS ROUTINE.
```

```
C      NORDG         NUMBER OF RAINFALL/RUNOFF DATA; THE ROUTINE DOES THE
C                    SIMULATION FOR NORDG*ITINT SECONDS.
C    * NEXT          NUMBER OF ELEMENT OF ARRAY P WHICH MUST BE USED FIRST.
C                    AFTER USING 4 ELEMENTS THE VALUE OF NEXT IS INCREASED BY 4.
C      ITINT         TIMEINTERVAL OF THE DATA (S) AND UNIT OF TIME : FOR THE
C                    CALCULATION.
C    * CHEZY         ROUGHNESS COEFFICIENT (M**.5/S IF EXPN=1.)
C    * EXPN          EXPONENT IN EQ 2 (-).
C    * TOUL & TOUR   SIDESLOPES OF A GUTTER (RADIANS).
C    * A1 & A2       A1 AND A2 FROM EQ. 27.
C    * RF            NETTO RAINFALL (M/S).
C     THE PARAMETERS  NSEG,NX,NT AND NT2 CORRESPOND WITH ARRAY DIMENSIONS
C     AND ARE USED TO MAKE  THE CHANGING OF THEM AS SIMPLE AS POSSIBLE.
C
C     ******************************************************************
C
      SUBROUTINE KWAVE
      COMMON /D/NETRF(200),RUNOFF(200),P(10),PMIN(10),PMAX(10),
     1NORDG,GF(4),IP(10)
      COMMON /E/VCA(200),NEXT,AREA
      COMMON /ACC1/ KATCH,CNAME(9),TAREA,PAREA,NEVENT,ITINT
      COMMON /ACC4/ISEG,ITYPE(10),AL(10),SLOPE(10),WID(10),ZLT(10)
     1,ZRT(10),INCREM(10),OVSEG1(10),OVSEG2(10),UPSEG(10)

      COMMON /KWA/ ALFA(10),EXP(10),QLOLD(10),QLNEW,AOTO(10),ALTO(10),
     1A(10,20,2),QOUT(10),IT,ITHYB,DX(10),DELTAT,NK(10)
      REAL NETRF
      PARAMETER NSEG=10
      PARAMETER NX=20
      PARAMETER NT=200
      PARAMETER NT2=20

      CHEZY=P(NEXT)*ITINT
      EXPN=P(NEXT+1)
      JXSTEP=P(NEXT+2)+.01
      DELTAT=1./AINT(P(NEXT+3)+.1)
      NEXT=NEXT+4

C     *** PER SEGMENT NUMBER OF SPACESTEPS IS ESTABLISHED     ***
C     *** AND ALFA AND EXP ARE CALCULATED                     ***

      IF (ISEG.LE.0.OR.ISEG.GT.NSEG) WRITE (6,800) ISEG,NSEG
  800 FORMAT (' ***WARNING*** NUMBER OF SEGMENTS : ',I10/
     1' THIS MUST BE POSITIVE AND LESS THAN ',I10)
      DO 10 I=1,ISEG
            IF (JXSTEP.GT.2.AND.JXSTEP.LT.NX) INCREM(I)=JXSTEP
            IF (INCREM(I).GT.NX-1) INCREM(I)=NX-1
            DX(I)=AL(I)/INCREM(I)
            NK(I)=INCREM(I)+1
            DO 11 JX=1,NK(I)
                  A(I,JX,1)=0.0
   11       CONTINUE


            *** FOR OVLAND SECTIONS ***

            IF (ITYPE(I).EQ.2) GO TO 20
                  ALFA(I)=CHEZY*SQRT(SLOPE(I))
                  EXP(I)=EXPN
                  GO TO 10

C                 *** FOR GUTTER SECTIONS ***

   20             A1=0.5*(1./ZLT(I)+1./ZRT(I))
                  TOUL=ATAN(ZLT(I))
                  TOUR=ATAN(ZRT(I))
                  A2=1./SIN(TOUL)+1./SIN(TOUR)
                  ALFA(I)=CHEZY*SQRT(SLOPE(I))*
     1            A1**((EXPN-1.)/2.)*A2**(1.-EXPN)
                  EXP(I)=(EXPN+1.)/2.

   10 CONTINUE


C     *** IT IS CALCULATED AFTER HOWMANY TIME-INTERVALS     ***
C     *** THE RAINFALL DEFINATELY STOPS                     ***

      DO 101 ITHYB=NORDG,1,-1
            IF (NETRF(ITHYB).GT.0.0) GO TO 102
  101 CONTINUE

  102 ITHYB=ITHYB/DELTAT+3

C     *** THE LATERAL INFLOW IS MADE EQUAL TO THE NETTO     ***
C     *** RAINFALL FOR TYPE 1 SEGMENTS                      ***

      IT=1
      RF=QLINT(NT,NETRF,DELTAT,IT,ITINT/3.6E6)
      DO 110 I=1,ISEG
            QLOLD(I)=RF
            IF (ITYPE(I).EQ.2) QLOLD(I)=0.0
  110 CONTINUE


C     *** RUNOFF FROM THE DIFFERENT SEGMENTS IS CALCULATED
C     *** FOR TIME= (IT+1)*DELTAT                            ***


C     *** ACTUAL CALCULATION OF THE RUNOFF. ***
      DO 130 IT=1,IFIX((NORDG-1.)/DELTAT+1.)
            DO 120 I=1,ISEG
                  IF (ITYPE(I).EQ.1) CALL OVLAND(I)
                  IF (ITYPE(I).EQ.2) CALL GUTTER(I)
  120       CONTINUE


C           *** THE RUNOFF IS BROUGHT IN THE ARRAY "RUNOFF"     ***
C           *** FOR ITINT SECONDS TIMEINTERVALS                 ***
C           *** AFTER BEING TRANSLATED IN LITERS/SECOND.        ***

            RUNOFF(IFIX((IT-1)*DELTAT+2.1))=QOUT(ISEG)/ITINT*1000.
  130 CONTINUE

      RETURN
      END
```

DOE-URBAN*REINOUT(1).OVLAND

```
C     THIS ROUTINE CALCULATES THE DISCHARGE FROM AN OVERLANDFLOW-SEGMENT J.
C     FOR WHICH THE EQUATION Q=ALFA(J)*A**EXP(J) IS VALID.
C     FOR ONE TIMEINTERVAL.
C     IT USES EQUATIONS 8 OF 9 IF IT+1 <= ITHYB (I.E. IN CASE
C     OF RAINFALL) AND EQUATION 24 IF IT+1 > ITHYB (I.E. IF RAIN-
C     FALL HAS DEFINATELY STOPPED FOR THE EVENT).
C     IT SEARCHES FOR THE POSSIBLE TOPEND INFLOW FROM SEGMENT UPSEG(J).
C     RAINFALL IS USED AS LATERAL INFLOW.
C     THE CROSS-SECTIONAL AREAS OF FLOW FOR THE GRIDPOINTS 1 UNTIL NX(J)
C     ARE SUPPOSED TO BE IN THE ELEMENTS A(J,JX,1),JX=1,NX(J)
C     THE CROSS-SECTIONAL AREAS AT THE END OF THE TIME INTERVAL
C     ARE PLACED IN THESE SAME ELEMENTS AGAIN FOR REUSE IN
C     THE NEXT CALL.
C     IT CALLS THE EXTERNAL FUNCTIONS WLAX25 AND QLINT.
C     THE INTERNAL FUNCTION WLAX25 DOES THE SOLUTION OF EQ. 24.
C     EXPLANATION OF VARIABLES:
C   * I               ABBREVIATION OF UPSEG(J).
C   * JX              GRIDPOINT COUNTER.
C     ALL OTHER VARIABLES HAVE THE SAME SENSE AS EXPLAINED
C     IN THE ROUTINE KWAVE.
C     THE VALUES OF THE FOLLOWING VARIABLES ARE POSSIBLY CHANGED
C     BY THIS ROUTINE:
C     A(J,-,-),AOTO(J),ALTO(J),QLOLD(J),QLNEW.

C     PROGRAMMED BY RJMG MAY '79.
C
C     ***********************************************************
C
      SUBROUTINE OVLAND(J)


      COMMON /D/NETRF(200),RUNOFF(200),P(10),PMIN(10),PMAX(10),
     1NORDG,GF(4),IP(10)
      COMMON /ACC1/ KATCH,CNAME(9),TAREA,PAREA,NEVENT,ITINT
      COMMON /ACC4/ISEG,ITYPE(10),AL(10),SLOPE(10),WID(10),ZLT(10)
     1,ZRT(10),INCREM(10),OVSEG1(10),OVSEG2(10),UPSEG(10)
      COMMON /KWA/ ALFA(10),EXP(10),QLOLD(10),QLNEW,AOTO(10),ALTO(10),
     1A(10,20,2),QOUT(10),IT,ITHYB,DX(10),DELTAT,NK(10)
      REAL NETRF,N
      INTEGER OVSEG1,OVSEG2,UPSEG
      PARAMETER NSEG=10
      PARAMETER NX=20
      PARAMETER NT=200
      PARAMETER NT2=2
      DEFINE          WLAX25(ALTO,AOTO,AOT,TZERO,ALFA,N,L,X,T
     1=(AOT**(N-1)+X/(N*ALFA*(T-TZERO)+L/
     2(ALTO**(N-1)-AOTO**(N-1))))**(1./(N-1))

C     *** UPSTREAM INFLOW IS CALCULATED. ***
      I=UPSEG(J)
      A(J,1,2)=0.0
      IF (I.NE.0) A(J,1,2)=(QOUT(I)/ALFA(J))**(1./EXP(J))

C     *** RAINFALL DURING THE NEXT INTERVAL IS CALCULATED.       ***
      QLNEW=QLINT(NT,NETRF,DELTAT,IT+1,ITINT/3.6E6)
      IF (IT+1.GT.ITHYB) GO TO 100
```

DOE-URBAN*REINOUT(1).GUTTER

```
C     THIS ROUTINE CALCULATES THE DISCHARGE FROM A GUTTERSEGMENT J
C     FOR WHICH THE EQUATION Q=ALFA(J)*A**EXP(J) IS VALID.
C     FOR ONE TIMEINTERVAL.
C     IT COLLECTS THE POSSIBLE TOPEND INFLOW OF LATERAL INFLOW
C     FROM PREVIOUSLY PROCESSED SEGMENTS.
C     IT CALLS THE FUNCTION WLAX25.
C     THE CROSS-SECTIONAL AREAS OF FLOW FOR THE GRIDPOINTS 1 UNTIL NX(J)
C     ARE SUPPOSED TO BE IN THE ELEMENTS A(J,JX,1),JX=1,NX(J)
C     THE CROSS-SECTIONAL AREAS AT THE END OF THE TIME INTERVAL
C     ARE PLACED IN THESE SAME ELEMENTS AGAIN FOR REUSE IN
C     THE NEXT CALL.
C     EXPLANATION OF VARIABLES:
C   * I               ABBREVIATION OF UPSEG(J).
C   * JX              GRIDPOINT COUNTER.
C     ALL OTHER VARIABLES HAVE THE SAME SENSE AS EXPLAINED
C     IN THE ROUTINE KWAVE.
C
C     THE VALUES OF THE FOLLOWING VARIABLES ARE POSSIBLY
C     CHANGED  BY THIS ROUTINE:
C     A,QLOLD,QLNEW,QOUTAOTO(J),ALTO(J).
C
C     PROGRAMMED BY RJMG MAY '79.
C
C     CHANGED AUGUST '79 BY RJMG
C     IF NO LATERAL INFLOW TAKES PLACE AND IT+1 > ITHYB THE  ANALYTICAL
C     SOLUTION IS USED.
C     THIS IS DONE BY THE INERNAL ROUTINE WLAX25.

C     ***********************************************************
C
      SUBROUTINE GUTTER(J)


      COMMON /D/NETRF(200),RUNOFF(200),P(10),PMIN(10),PMAX(10),
     1NORDG,GF(4),IP(10)
      COMMON /ACC1/ KATCH,CNAME(9),TAREA,PAREA,NEVENT,ITINT
      COMMON /ACC4/ISEG,ITYPE(10),AL(10),SLOPE(10),WID(10),ZLT(10)
     1,ZRT(10),INCREM(10),OVSEG1(10),OVSEG2(10),UPSEG(10)
      COMMON /KWA/ ALFA(10),EXP(10),QLOLD(10),QLNEW,AOTO(10),ALTO(10),
     1A(10,20,2),QOUT(10),IT,ITHYB,DX(10),DELTAT,NK(10)
      INTEGER OVSEG1,OVSEG2,UPSEG
      REAL NETRF,N
      DEFINE          WLAX25(ALTO,AOTO,AOT,TZERO,ALFA,N,L,X,T)
     1=(AOT**(N-1)+X/(N*ALFA*(T-TZERO)+L/
     2(ALTO**(N-1)-AOTO**(N-1))))**(1./(N-1))
      PARAMETER NSEG=10
      PARAMETER NX=20
      PARAMETER NT=200
      PARAMETER NT2=2

      *** UPSTREAM INFLOW IS CALCULATED ***
      I=UPSEG(J)
      A(J,1,2)=0.0
      IF (I.NE.0) A(J,1,2)=(QOUT(I)/ALFA(J))**(1./EXP(J))

C     *** LATERAL INFLOW IS CALCULATED. ***
```

```
C       ***CSA'S ARE CALCULATED FOR GRIDPOINTS USING EQ.'S 8 & 9         ***
        DO 10 JX=2,NK(J)
                A(J,JX,2)=WLAX28(J,JX,1,NSEG,NX,NT2,A,QLOLD(J),
     1          QLNEW,ALFA(J),EXP(J),DX(J),DELTAT,NK(J))
 10     CONTINUE

        QLOLD(J)=QLNEW
        IF (IT+1.NE.ITHYB) GO TO 900

C       *** IF RAINFALL DEFINATELY STOPS PARAMETERS FOR EQ. 24 ARE       ***
C       *** CALCULATED.                                                  ***
        AOTO(J)=A(J,1,2)
        ALTO(J)=A(J,NK(J),2)
        GO TO 900

C       *** CSA FOR LAST GRIDPOINT IS CALCULATED USING EQ. 24            ***
C       *** IN CASE IT+1 > ITHYB (NO RAINFALL)                           ***
 100    A(J,NK(J),2)=WLAX25(ALTO(J),AOTO(J),A(J,1,2),(ITHYB-1)*DELTAT,
       1ALFA(J),EXP(J),AL(J),AL(J),(IT-1)*DELTAT)

 900    QOUT(J)=ALFA(J)*A(J,NK(J),2)**EXP(J)
        IF (IT.GT.ITHYB) GO TO 990

C       *** ARRAY A IS REARRANGED. ***
        DO 910 JX=1,NK(J)
                A(J,JX,1)=A(J,JX,2)
 910    CONTINUE

 990    RETURN
        END
```

```
        QLNEW=0.0
        IF (OVSEG1(J).NE.0.) QLNEW=QOUT(OVSEG1(J))
        IF (OVSEG2(J).NE.0.) QLNEW=QLNEW+QOUT(OVSEG2(J))

        IF (OVSEG1.EQ.0.AND.OVSEG2.EQ.0.AND.IT+1.GT.ITHYB) THEN
C               *** ANALYTICAL SOLUTION IF  *NO LATERAL INFLOW* AND ***
C               ***                         *IT+1 > ITHYB           ***
 100            A(J,NK(J),2)=WLAX25(ALTO(J),AOTO(J),A(J,1,2),(ITHYB-1)*
               1DELTAT,ALFA(J),EXP(J),AL(J),AL(J),(IT-1)*DELTAT)
        ELSE
C               *** NUMERICAL SOLUTION IN CASE OF LATERAL INFLOW OR ***
C               *** IF IT+1 <= ITHYB.                               ***

C               *** CROSS-SECTIONAL AREAS OF FLOW ARE CALCULATED FOR THE ***
C               *** GRIDPOINTS AT THE END OF THE TIMEINTERVAL            ***
                DO 10 JX=2,NK(J)
                        A(J,JX,2)=WLAX28(J,JX,1,NSEG,NX,NT2,A,QLOLD(J),
     1                  QLNEW,ALFA(J),EXP(J),DX(J),DELTAT,NK(J))
 10             CONTINUE

C               *** SOME VARIABLES ARE REARRANGED. ***
                QLOLD(J)=QLNEW
                DO 910 JX=1,NK(J)
                        A(J,JX,1)=A(J,JX,2)
 910            CONTINUE
                IF (IT+1.EQ.ITHYB) THEN
                        AOTO(J)=A(J,1,2)
                        ALTO(J)=A(J,NK(J),2)
                ENDIF

        ENDIF

        QOUT(J)=ALFA(J)*A(J,NK(J),2)**EXP(J)
        RETURN
        END
```

DOF URBAN*REINOUT(1).WLAX2B

```
C     THIS FUNCTION SOLVES EQ. 8 OR 9 FROM CHAPTER 1 OF
C     THE APPLICATION OF A KINEMATIC WAVE MODEL TO URBAN SURFACE
C     BY RJMG AND CHR.
C     FOR SEGMENT J, GRIDPOINT JX AND TIME (IT+1)*DELTAT.
C     IF 1 < JX < NX EQ. 8 IS SOLVED AND
C     IF    JX=NX   EQ. 9.
C     IF JX <= 1 OR JX > NX THE CALCULATION GOES WRONG (NO CHECK IS
C     DONE TO PREVENT THIS !!!)
C     IT IS PREVENTED THAT THE RESULT WLAX2B BECOMES NEGATIVE,
C     BY MAKING IT EQUAL TO ZERO IN SUCH CASES.
C
C     EXPLANATION OF VARIABLES:
C     NEWAR         NEWLY CALCULATED CROSS-SECTIONAL AREA OF FLOW (M**2)
C     S             ALFA FROM EQ. 2.
C     N             EXPONENT N FROM EQ.2.
C     F1...F6       TEMPORARY VARIABLES TO PREVENT A VERY LONG
C                   FORTRANSTATEMENT.
C     J             NUMBER OF THE SEGMENT FOR WHICH THE CALCULATION IS PERFORMED.
C     JX            NUMBER OF THE GRIDPOINT CONCERNED.
C     IT            NUMBER OF THE TIMEINTERVAL.
C     NSEG,NX,NT    DIMENSIONS OF A.
C     NX            NUMBER OF GRIDPOINTS FOR SEGMENT J.
C
C     OTHER VARIABLES HAVE THE SAME MEANING AS EXPLAINED IN THE
C     SUBROUTINE KWAVE.
C     NONE OF THE FUNCTION PARAMETERS ARE CHANGED DURING THE EXECUTION.
C
C     PROGRAMMED BY RJMG; APRIL '77.
C
C     ****************************************************************
C
      REAL FUNCTION WLAX2B(J,JX,IT,NSEG,NX,NT,H,QLOLD,QLNEW,
     1S,N,DELTAX,DELTAT,NX)

      DIMENSION A(NSEG,NX,NT)
      REAL NEWAR,N

      DEFINE FUN1(X1,X2,S,N)=S*(X1**N-X2**N)
      DEFINE FUN2(X1,X2,S,N)=N*S*(X1**(N-1)+X2**(N-1))

      F5=FUN1(A(J,JX,IT),A(J,JX-1,IT),S,N)
      IF (JX.EQ.NX) GO TO 10

C     *** EQ. 8 IS SOLVED IF JX < NX. ***
      F1=FUN1(A(J,JX+1,IT),A(J,JX-1,IT),S,N)
      F2=FUN2(A(J,JX+1,IT),A(J,JX,IT),S,N)
      F3=FUN1(A(J,JX+1,IT),A(J,JX,IT),S,N)
      F4=FUN2(A(J,JX,IT),A(J,JX-1,IT),S,N)

      F6=-F1/(2*DELTAX)+QLOLD
      F7=.5*F2*(F3/DELTAX-QLOLD)
      F8=.5*F4*(F5/DELTAX-QLOLD)

      NEWAR=A(J,JX,IT)+DELTAT*F6
      NEWAR=NEWAR-DELTAT*DELTAT/2.*(F7-F8)
     *+DELTAT*(QLNEW-QLOLD)/DELTAT)

      GO TO 100

C     *** EQ. 9 IS SOLVED IF JX=NX. ***
   10 NEWAR=A(J,JX,IT)+DELTAT*((QLNEW+QLOLD)/2-F5/DELTAX)

C     *** IT IS PREVENTED THAT WLAX2B BECOMES NEGATIVE. ***
  100 WLAX2B=0.0
      IF (NEWAR.GE.0.0) WLAX2B=NEWAR

      RETURN
      END
```

```
DOE-LREAN*REINOUT(1).GLIN3
C     THIS FUNCTION SEARCHES THE RAINFALL FOR CALCULATIONSTEP IT IN
C     THE ARRAY GL WHICH CONTAINS THE RAINFALL DATA FOR IT*NT SECOND
C     TIMEINTERVALS, WHEN THE CALCULATION IS DONE FOR DT*IT*NT SECOND
C     TIMEINTERVALS.
C     IT SEARCHES ELEMENT (IT-1)*DT+1 FROM THE 1-DIMENSIONAL ARRAY GL
C     WHICH HAS NT ELEMENTS AND MULTIPLIES THIS BY FAC.
C     IF NOT 0 < (IT-1)*DT+1 <= NT THE RESULT IS ZERO.
C     DT HAS TO MEET THE FOLLOWING CONDITIONS:
C            1 DT <= 1.
C            2 THERE MUST EXIST AN INTEGER N FOR WHICH IS VALID THAT
C              N*DT = 1.
C     DT IS NOT CHECKED ON THESE CONDITIONS.
C     NONE OF VALUES OF THE FUNCTION-ARGUMENTS WILL BE CHANGED DURING
C     THE EXECUTION.
C
C     PROGRAMMED BY RJMG APRIL '79.
C
C     ***********************************************************
C
      REAL FUNCTION GLINT(NT,GL,DT,IT,FAC)
      DIMENSION GL(NT)


      IF (DT.NE.1.) GO TO 30
      GLINT=BOUND(NT,GL,IT)
      GO TO 999

   30 GLINT=BOUND(NT,GL,IFIX((IT-.99)*DT+1.))

  999 GLINT=GLINT*FAC
      RETURN

C7

C     THIS FUNCTION SEARCHES ELEMENT IT FROM THE 1-DIMENSIONAL ARRAY
C     GL WHICH HAS NT ELEMENTS.
C     IF NOT 0 < IT <= NT THE FUNCTION IS MADE ZERO.
C     NONE OF THE VALUES OF THE FUNCTION-ARGUMENTS WILL BE CHANGED
C     DURING AN EXECUTION.
C
C     PROGRAMMED BY RJMG APRIL '79.
C
C     ***********************************************************
C
      REAL FUNCTION BOUND(NT,GL,IT)
      DIMENSION GL(NT)

      BOUND=0.0
      IF (IT.GE.1.AND.IT.LE.NT) BOUND=GL(IT)

      RETURN
      END
```

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