

INSTITUTE
OF
HYDROLOGY

RAINFALL-RUNOFF PROCESSES
OVER URBAN SURFACES

TEXTBASE

FOR
REFERENCE ONLY

Proceedings of an International
Workshop held at IH, April 1978

Edited by

C H R KIDD

This Report describes the scope and findings of an urban hydrology workshop held at the Institute in April 1978. All the workshop participants contributed to the report and we should like to record our thanks for their support, both during their time at Wallingford and in the editing of the subsequent drafts.



REPORT NO 53

September 1978

NOT TO BE
TAKEN AWAY

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plate 1

WORKSHOP PARTICIPANTS

Back row (l. to r.) Jan van den Berg, Russell Mein, Gerard Ven,
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Front row (l. to r.) Per Jacobsen, John Packman, Jan Falk, Jan
Niemcynowicz, Wolfgang Neumann

1. INTRODUCTION

The high cost of the provision of storm drainage in urban areas places a heavy premium on the efficient hydraulic design of flow conduits. A mathematical model for the conversion of rainfall input to a design flow at some point in a sewer system is one component of the essential design process, and since there is great potential in construction cost savings, much energy has been expended over the last few years towards the development of such models.

The rainfall-runoff process in a fully-sewered urban catchment may be conveniently divided into two: an above-ground phase comprising principally hydrological phenomena and a below-ground phase which is primarily hydraulic. In the past, the testing of models against field-data has required simulation of both these phases, and it has never been certain as to whether a correct answer has been achieved by two correct simulations or two incorrect, but compensating, simulations. For this reason, the logical step in the development of models has been to isolate the one phase from the other, and to collect field data at the interface between the two phases (or where the runoff goes underground). To this end, several research groups engaged in model development have embarked on data collection programmes at this phase boundary.

An international workshop was held at the Institute during April 1978 to bring together these several groups for a concentrated attack on both the analysis of the data from the different research catchments and to review the performance of several different modelling strategies. The workshop participants (see Plate 1) were as follows:

| | |
|------------------|---|
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| G A Ven | Lelystad |
| | The Netherlands |
| J Falk | Department of Water Resources Engineering |
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| | Sweden |
| P Jacobsen | Department of Sanitary Engineering |
| | Technical University of Denmark |
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Since mathematical models are usually components of design methods, it is in this context that a modelling philosophy has to be examined. The chief division in modelling approach is between a distributed, physically-based approach and a lumped-parameter approach. For design applications, the requirement for catchment data must be kept to a minimum, and thus the first approach becomes less attractive. In general, physically-based models tend to be more time-consuming (and thus expensive) in terms of computational effort, although this argument is becoming less persuasive as computers become more powerful. For these reasons, design considerations generally dictate that a lumped-parameter approach is adopted. However, since the model does not relate directly to reality, it then becomes necessary to collect rainfall and runoff data from a variety of subcatchments in order to relate model parameters to those catchment characteristics for which information is readily accessible.

It follows that the more catchments that are instrumented the more confident will be the relationships derived between parameters and catchment characteristics. It was on the premise that most of these relationships will be largely independent of climatic variations that such a workshop could take place and happily, universally-applicable relationships have been developed from data from the Netherlands, Sweden and the UK.

The terms of reference of the workshop were based on the division of the rainfall-runoff process into three parts, as follows:

- (a) the determination of the runoff volume from a given rainfall input - this is a runoff volume submodel.
- (b) the distribution of the net rainfall (equal in total to the runoff volume) in time - this is obtained from a loss submodel.
- (c) the routing of the net rainfall to an inlet of the sewer system - this is a surface routing submodel.

Part (a) is the one area of the model which depends to a large extent on climatic conditions and differences in engineering practice. An example of the latter is the Dutch practice of paving parts of their urban areas (parking lots, footpaths) with bricks, which results in infiltration losses of up to 40%; this contrasts with practices in most other countries of paving with asphalt or concrete. (Even here there is considerable evidence from observations and from various measurements taken that the rate of infiltration through asphalt surfaces varies considerably depending on the proportion of sand in the mix.)

The workshop confined its activities to the development of mathematical models relevant to parts (b) and (c) above. There were several loss models and surface routing models well known in the literature and which seemed relevant to this exercise, but because the field data relates to both part (b) and part (c), certain arbitrary decisions had to be made in the choice of which submodels to use. A rigorous examination of all possible combinations of loss and surface routing models would be extremely time-consuming and not necessarily productive. Different loss models were studied in combination with arbitrarily chosen surface routing models. Following this phase of the work, a suitable loss model was selected and used for the comparison of various surface routing models.

Another key decision was the choice of optimisation method. Optimisation is the process by which "best" values of model parameters are determined according to some criterion of how well the modelled output matches the observed data. Alternative optimisation strategies and objective functions were examined, as described later.

Both the investigation of loss models and of optimisation methods engaged the workshop participants for approximately the first third of the time available. These two items are dealt with in Sections 3 and 4 in this report. The remaining time was spent in the comparison of a variety of surface routing models using a common loss model and a common optimisation method; this work appears in Section 5.

The final section of the report is a summary of findings and draws general conclusions from the workshop.

2. DATA

Data from subcatchment experiments in the Netherlands, Sweden and the UK have been converted to a common format to give a comprehensive data base of some 20 catchments, details of which are given in Table 1.

Data from the three different sources have much in common. Firstly, the particular environment of individual surface water inlets has necessitated the development of custom-built instrumentation and monitoring schemes for the measurement of runoff. Secondly, due to the swift response time of the runoff, relatively fine temporal discrimination is necessary; and, thirdly, following on from this latter point, good rainfall-runoff synchronisation is essential. These three points have been considered separately by the three experimental programmes, and a brief description of the three programmes demonstrates how the problems were solved in each case.

The Institute has been engaged in an extensive data collection programme since 1975, following the development of a meter for the measurement of discharge through a road gully (Blyth & Kidd, 1977). Fifteen road gullies have been instrumented in a total of six sub-catchments in Bracknell, Southampton, Stevenage and Wallingford. A typical example of the type of data collection system in use is shown in Figure 1, an experimental system (2 gully meters, 0.1 mm Rimco tipping bucket rain gauge and Microdata logger) installed in School Close, Stevenage. Data are recorded at half-minute intervals on magnetic tape cassettes, and processed to a selected event record on the Institute's Univac 1108 computer.

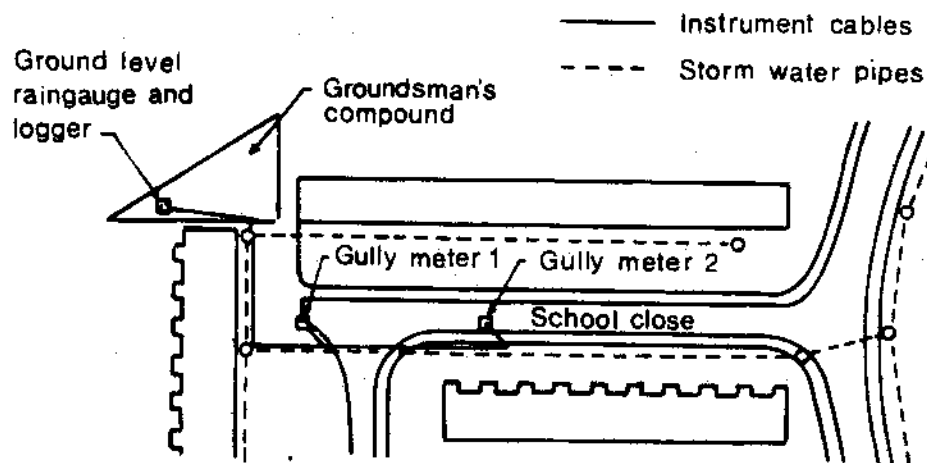


FIGURE 1 Experimental system at School Close, Stevenage, UK

2.2 Urban catchment research at Lelystad in the Netherlands

The IJsselmeerpolders Development Authority has set up a hydrological research project at Lelystad, one of the new towns in the recently constructed polders in the Lake IJssel. The town is built on about one metre of sand, and a subsurface drainage system controls the groundwater level in this sand overlaying a nearly impermeable subsoil. The urban area is very flat and the slope of the pavement is due to artificial camber only; roofs in the housing area are flat. A housing area and a parking lot have been instrumented as experimental catchments with physical characteristics as follows:

| catchment | gross area (ha) | covered area(%) | nature of covered area | | |
|--------------|--------------------|--------------------|------------------------|----------------|---------------|
| | | | roofed (%) | asphalt (%) | bricks (%) |
| housing area | 2.0 | 44 | 30 | 32 | 38 |
| parking lot | 0.7 | 99.6 | -- | 45 | 55 |

The storm water sewer systems in the catchments have a certain volume of permanent static storage which provides a quick response when it starts to rain and facilitates the calculation of the inflow into the storm drains (Van den Berg, 1976).

A central data logging system measures continuously the following elements:

- rainfall (ground level raingauge);
- storm water discharge (rectangular Thomson V-notch);
- subsurface drainage discharge (electromagnetic flow meter);
- groundwater level.

Recording takes place only when there is a significant change in the value of an element.

2.3 Swedish data collection programme

The Department of Water Resources Engineering at the University of Lund initiated an urban hydrology research project with a laboratory model in 1972 (Gottschalk & Niemczynowicz, 1975). These experiments led to the construction of an apparatus for runoff measurements in gutter inlets (Falk & Niemczynowicz, 1978, Lindh, 1976 and Arnell *et al.*, 1977). The principle of the gauge is shown in Figure 2. During the

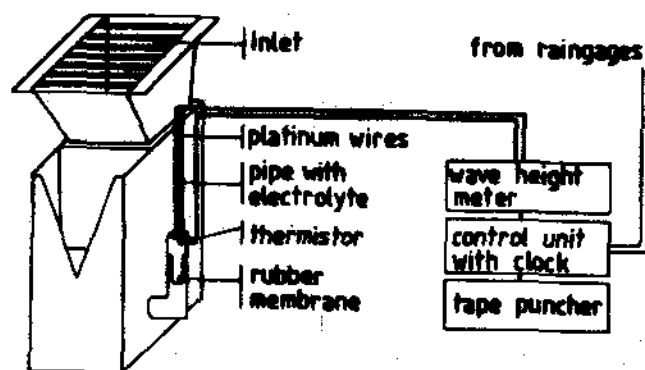


FIGURE 2 Swedish runoff measurement system

three years 1975-1977 a total of nine small subcatchments in the Klostergarten suburb of Lund have been monitored. The runoff data collection system is shown in Figure 3. Rainfall is measured with a tipping bucket raingauge (bucket size .02 mm). Data are recorded on a central logger at one minute intervals on paper tape. Since all the instruments are connected to the same clock, there is an absolute synchronization of recording time for rainfall and runoff.

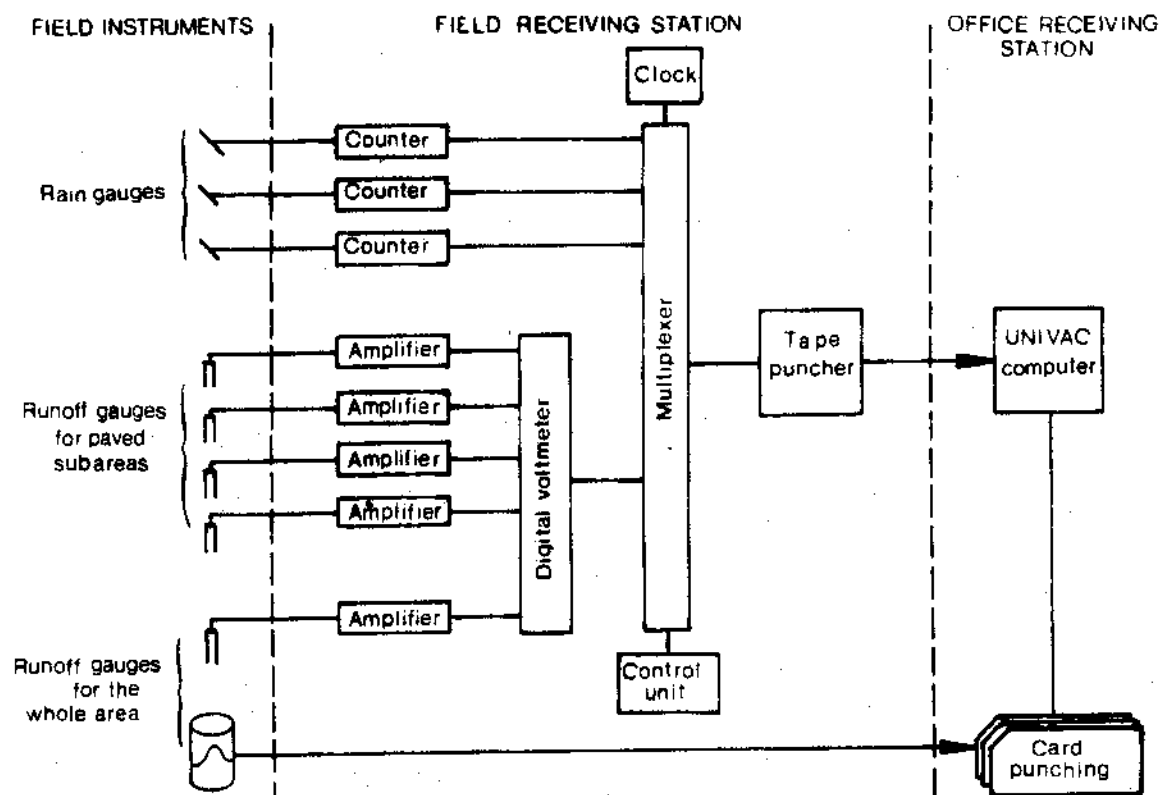


FIGURE 3 Klostergarten data collection system

2.4 The data base

Table 1 summarises the data used in the workshop. As mentioned above, runoff from the Dutch catchments relates to a number of inlets, whereas the area for the other catchment is for one inlet only. Figure 4 shows the distribution of the catchments in a slope-area matrix. It is suggested that further data collection should attempt to fill in the less populated areas of this matrix, since any attempt to establish relationships involving both these catchment characteristics will be hampered by the internal relationship shown in the figure.

A custom-built data-base (involving random-access files) was devised for the convenient and efficient access of these data. This data base and its management package is described in Appendix A.

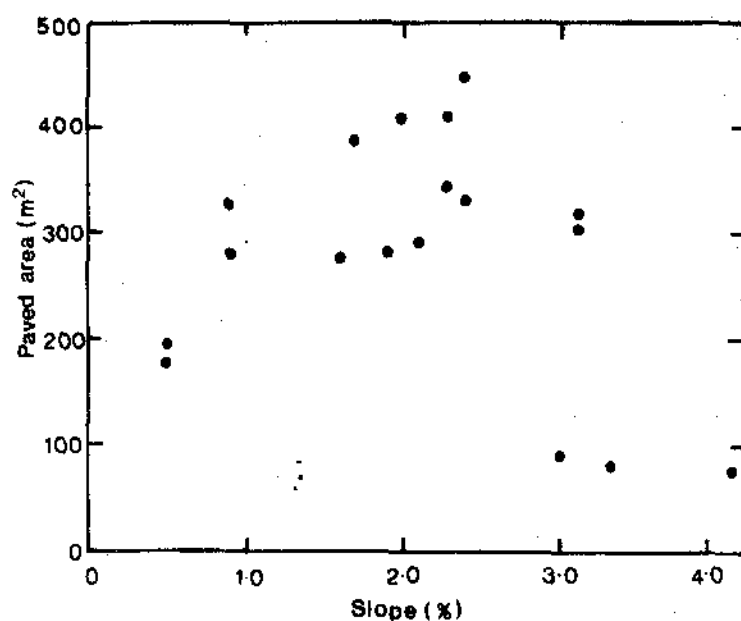


FIGURE 4 Distribution of catchment characteristics

TABLE 1 Summary of data used in the workshop

| CATCHMENT NUMBER | CATCHMENT NAME | ORIGIN | TOTAL AREA (m ²) | PAVED AREA (m ²) | SLOPE (%) | NO. OF EVENTS | YEARS OF DATA |
|---------------------|-----------------------|--------|------------------------------------|------------------------------------|--------------|------------------|------------------|
| 301 | LELYSTAD HOUSING AREA | NETH. | 20000 | 8800 | .5 | 10 | 69-75 |
| 311 | LELYSTAD PARKING LOT | NETH. | 7000 | 7000 | .5 | 10 | 69-75 |
| 2032 | ENNERDALE TWO | U.K. | 358 | 320 | 3.1 | 6 | 76-77 |
| 2033 | ENNERDALE THREE | U.K. | 90 | 90 | 3.0 | 9 | 76-77 |
| 2042 | BISHOPDALE TWO | U.K. | 591 | 450 | 2.4 | 11 | 76-77 |
| 2051 | HYDE GREEN ONE | U.K. | 485 | 346 | 2.2 | 7 | 76-77 |
| 2052 | HYDE GREEN TWO | U.K. | 844 | 417 | 2.0 | 8 | 76-77 |
| 2061 | SCHOOL CLOSE ONE | U.K. | 459 | 283 | 1.7 | 11 | 76-77 |
| 2062 | SCHOOL CLOSE TWO | U.K. | 717 | 393 | 0.9 | 11 | 76-77 |
| 4175 | LUND 1:75 | SWEDEN | 291 | 291 | 2.1 | 11 | 75 |
| 4176 | KLOSTERGARDEN 1:76 | SWEDEN | 326 | 326 | 0.9 | 11 | 76 |
| 4177 | KLOSTERGARDEN 1:77 | SWEDEN | 335 | 335 | 2.3 | 13 | 77 |
| 4276 | KLOSTERGARDEN 2:76 | SWEDEN | 82 | 82 | 3.3 | 11 | 76 |
| 4277 | KLOSTERGARDEN 2:77 | SWEDEN | 78 | 78 | 4.1 | 12 | 77 |
| 4376 | KLOSTERGARDEN 3:76 | SWEDEN | 306 | 306 | 3.1 | 11 | 76 |
| 4377 | KLOSTERGARDEN 3:77 | SWEDEN | 413 | 413 | 2.3 | 13 | 77 |
| 4476 | KLOSTERGARDEN 4:76 | SWEDEN | 277 | 277 | 1.6 | 10 | 76 |
| 4477 | KLOSTERGARDEN 4:77 | SWEDEN | 279 | 279 | 1.9 | 13 | 77 |

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A modelling package was developed prior to the workshop, to enable a variety of modelling assignments to be performed using the above data base. Incorporated in this package were the models currently being used by the different participants in their various projects. The package operates in three basic modes which are simulation, optimisation and error mapping (error response surfaces may be obtained for different pairs of model parameters). A fuller description of the package is given in Appendix B, and the package components will become clearer as this report progresses.

3. OPTIMIZATION

Ideally, mathematical models for rainfall-runoff behaviour should only have parameters which have direct physical significance and which are measurable, for then the values of the parameters can be determined from the physical characteristics of the catchment. Many models, however, have one or more parameters which are conceptual in nature (ie they do not have direct physical representation and cannot be measured). Models with such conceptual parameters are only useful for design if the parameter values can be related to physical characteristics. To achieve this the best value of each conceptual parameter for a number of catchments and events must be determined; a number of such values permits the determination of relationships with catchment characteristics.

The 'best' value of a model parameter is that which produces the closest agreement, subject to some chosen criterion, between observed and modelled output. The trial and error procedure used to find the best value is called optimization. There are three separate factors which must be considered:

- (a) the optimization method itself, ie the way in which parameter values are varied systematically in the search for the best value;
- (b) the way in which the optimization method is applied, that is whether to optimize on individual events or on groups of events;
- (c) the objective function, or measure of goodness-of-fit, by which the model performance is judged.

3.1 Optimization method

Many optimization methods are available, ranging from simple univariate schemes, in which one parameter value is varied at a time, to multivariate schemes in which all parameters are varied together in a way

designed to decrease the computation required to find the best parameter values.

For this study the number of parameters to be optimized was small, usually one or two, and the Rosenbrock scheme (1960) was considered to be suitable. This scheme is a multivariate method which varies all parameters together in accordance with a strategy based on the slope of the response surface.

A comparison of the parameter value obtained using the Rosenbrock method with those obtained by Falk and Niemczynowicz (1978) using a graphical method showed that both methods gave broadly similar results. Because computer optimization was more convenient, the Rosenbrock method was used for the remainder of the study.

3.2 Application of optimization method

There is a choice between obtaining the best parameter value for each single event or finding some kind of 'global' value by optimizing on several events together. The former has the advantage of enabling any relationships between 'best' parameter value and storm characteristics to be investigated but suffers from the disadvantage that the values obtained may have errors due to data problems with individual storms. By combining events the variability of the estimates of the parameter value(s) is reduced and a smoother response surface is obtained.

A comparative study of the results obtained using both types of application is given in Section 3.4.

3.3 Objective functions

The use of an automatic optimization method requires the user to choose an objective function which measures the goodness of fit. Six different functions were investigated for this study:

- (i) Integral Square Error (ISE). This is a version of the common least squares function but scaled to a dimensionless measure of the fit.

$$ISE = \{ \sum (Q_o - Q_m)^2 / \sum Q_o \} \times 100\% \quad (3.1)$$

where Q_o = observed discharge

Q_m = discharge predicted by model

and the summation is over the hydrograph at intervals equal to the time step.

- (ii) Biassed Integral Square Error (BISE) which is an attempt to put more weight on the goodness-of-fit near the peak value.

$$\text{BISE} = \{ \sqrt{\sum (Q_o^2 - Q_m^2)} / \sum Q_o \} \times 100\% \quad (3.2)$$

(iii) Error in Peak Estimate (PEAK)

$$\text{PEAK} = \{ (|P_o - P_m|) / P_o \} \times 100\% \quad (3.3)$$

where P_o = observed peak discharge

P_m = modelled peak discharge

(iv) Error in predicted Volume (VOL). This is only a useful measure if the volumes are not constrained to be equal.

$$\text{VOL} = \{ (|V_o - V_m|) / V_o \} \times 100\% \quad (3.4)$$

where V_o = volume of observed runoff

V_m = volume of predicted runoff

(v) Time to Peak (TTP) gives a measure of the accuracy of the modelled lag times.

$$\text{TTP} = (|TP_o - TP_m|) \text{ time increments} \quad (3.5)$$

where TP_o = observed time to peak

TP_m = modelled time to peak

(vi) Partial Integral Square Error (PISE), an objective function which was intended to be a compromise between the ISE and PEAK functions. It is defined as:

$$\text{PISE} = \sqrt{\sum (Q_o - Q_m)^2 / \sum Q_o} \times 100\% \text{ for } Q_o \geq P_o / 2 \quad (3.6)$$

which is the same as Eq. 3.1 but over a restricted range for which the observed discharge is at least one half the magnitude of the observed peak discharge.

Part of the study was an investigation of the relative merits of several of these objective functions.

3.4 Comparison of optimization schemes

Three catchments were chosen (1 Dutch, 1 Swedish, 1 British) for the study of optimisation methods. One model was used throughout - the

constant proportional loss submodel combined with the Nash Cascade surface routing submodel (this choice was necessarily arbitrary). Four objective functions were used for comparison as follows:

| | | |
|-------------------------------|--------|--------------|
| Integral Square Error | (ISE) | Equation 3.1 |
| Biased Integral Square Error | (BISE) | Equation 3.2 |
| Partial Integral Square Error | (PISE) | Equation 3.6 |
| Peak Error | (PEAK) | Equation 3.3 |

Each objective function was applied to the first seven events from each catchment (a) on all events combined to give a global best fit (Suffix C) (b) on all events individually, then taking the average of the parameter values (Suffix AV).

Thus for each catchment, eight sets of parameter values were obtained (for each objective function on both combined and individual events). Using each of these eight parameter sets, the remaining events in each catchment were simulated (three events from the Dutch catchment and four each from the British and Swedish catchments).

For each of these independent events on each catchment using each of the parameter sets, a table was drawn up of the value of each objective (error) function (for example, see Table 2 below).

TABLE 2 Error summary for catchment 4376, event 400120

| Optimisation technique used to obtain parameter set | Peak Error % | | ISE % | | BISE % | | PISE % | |
|---|--------------|------|-------|------|--------|------|--------|------|
| | Value | Rank | Value | Rank | Value | Rank | Value | Rank |
| ISE _C | 1.00 | 1 | 2.65 | 2 | 7.23 | 3 | 7.06 | 3 |
| ISE _{AV} | 2.05 | 3 | 2.88 | 4 | 7.51 | 4 | 8.09 | 5 |
| BISE _C | 1.40 | 2 | 2.59 | 3 | 7.15 | 2 | 6.48 | 2 |
| BISE _{AV} | - 3.89 | 4 | 2.96 | 5 | 7.80 | 5 | 7.93 | 4 |
| PISE _C | 4.78 | 5 | 2.36 | 1 | 6.77 | 1 | 5.53 | 1 |
| PISE _{AV} | - 6.11 | 6 | 3.13 | 6 | 8.04 | 6 | 9.05 | 6 |
| PEAK _C | 23.93 | 8 | 6.20 | 8 | 13.31 | 8 | 16.72 | 8 |
| PEAK _{AV} | 16.60 | 7 | 4.44 | 7 | 10.56 | 7 | 13.36 | 7 |

Each parameter set was ranked according to the value obtained for each error function. (It is interesting to note that values of the Partial Integral Square Error (PISE) are for each parameter set greater than values of the total Integral Square error (ISE), implying that absolute errors are not uniformly distributed over the hydrograph and that larger errors occur in the region of the peak).

The table above shows that the $PISE_C$ parameter set gave a better fit than the other parameter sets in terms of ISE, BISE and PISE, but that the ISE_C gave the better fit in terms of observed peak discharge. The $PEAK_C$ and $PEAK_{AV}$ gave consistently poorer results.

Because the results as in Table 2 varied from storm to storm, the error values and ranks were summed for each catchment, from which it appeared that the parameter sets from $PISE_C$, $BISE_C$ and ISE_C gave the better results in terms of each of the error functions. For fitting to individual events and averaging, the parameter sets from ISE_{AV} and $BISE_{AV}$ were better than $PISE_{AV}$ and $PEAK_{AV}$.

These results were confirmed by summing the rank sums from three catchment tables, as shown in Table 3.

TABLE 3 Rank of error (summed over each catchment and each event) for each parameter set

| Parameter Set from | Peak Error | ISE | BISE | PISE |
|--------------------|------------|------|------|------|
| ISE_C | 36.5 | 34.5 | 35 | 33 |
| ISE_{AV} | 40 | 53 | 52 | 48 |
| $BISE_C$ | 35 | 32 | 33 | 38 |
| $BISE_{AV}$ | 56 | 54 | 45 | 51 |
| $PISE_C$ | 36 | 19.5 | 32 | 25 |
| $PISE_{AV}$ | 77 | 68 | 68.5 | 68 |
| $PEAK_C$ | 55 | 73.5 | 72 | 68 |
| $PEAK_{AV}$ | 60.5 | 61.5 | 58.5 | 64 |

It is apparent from this table that the $PISE_C$ parameter set gave the best result a greater number of times, with the $BISE_C$ parameter set second, the ISE_C parameter set third, and the ISE_{AV} set fourth. Thus, it would appear that the best value of a parameter to be used for prediction for other events on the same catchment is one determined by using the PISE objective function on combined events. However, if values of the best parameter(s) obtained on individual events are required (to identify any possible relationship between parameters and storm characteristics, for example) then the ISE function would seem to be the best one to use.

For this study, it was decided to carry out parameter optimization for the surface routing submodels on individual events using the ISE

objective function (i) to examine the variability of each parameter and (ii) to study the relationships between parameter value and storm characteristics. For the comparison of loss models described in the next section, optimisation was done on combined events. The latter work was done prior to the availability of the above results, but it is very unlikely that it should make any difference to the conclusions of Section 4.

4. COMPARISON OF LOSS MODELS

The volume of runoff from rural and urban catchments alike is always less than the volume of rainfall input because of the losses incurred in the translation process. Such losses include the physical phenomena of depression storage (including surface wetting), infiltration and evaporation. For urban areas particularly, the extent of the loss will depend as much on local engineering practices as on the meteorological conditions pertaining. For instance, a yearly water balance calculated for a Dutch parking lot shows that 40% of the incoming rainfall infiltrates the paved surface, probably due to the fact that about 80% of the area is laid with bricks (Van den Berg, 1978). Conversely, data from the Swedish catchments used in the workshop analysis indicate very low values of losses, 95% or more of which is explained by depression storage, no doubt because of the extensive use of asphalt within these catchments. Clearly, any formulae proposed for predicting net rainfall or runoff coefficients would have to take into account local practices.

An example of such a formula is that proposed by Stoneham & Kidd (1977) following analysis of data from several urban catchments in the UK.

$$\text{PRO} = 0.92\text{PIMP} + 53\text{SOIL} + 0.65\text{UCWI} - 33.6 \quad (4.1)$$

where
 PRO = percentage of runoff volume
 PIMP = percentage of impervious area
 SOIL = a soil index (NERC, 1975)
 UCWI = an urban catchment wetness index

4.1 Analysis

Regression analysis has been applied to catchment area and to both rainfall and runoff data for prediction of net rainfall volume. For example, the net rainfall for a Dutch parking lot with 55% brick cover and the remaining 45% of asphalt was estimated from volume and duration T of rainfall (Van den Berg, 1977).

$$q = 0.75P - 0.46\ln T - 0.44 \quad (4.2)$$

Depression storage has been assumed to be comparable for all catchments and hence estimated in a similar fashion for each. Rainfall and runoff volumes were plotted and the intercept of the rainfall axis with the regression line considered to represent depression storage (Figure 5). These particular analyses had been done previously for data from the different sources (Kidd, 1978; Falk & Niemczynowicz, 1978).

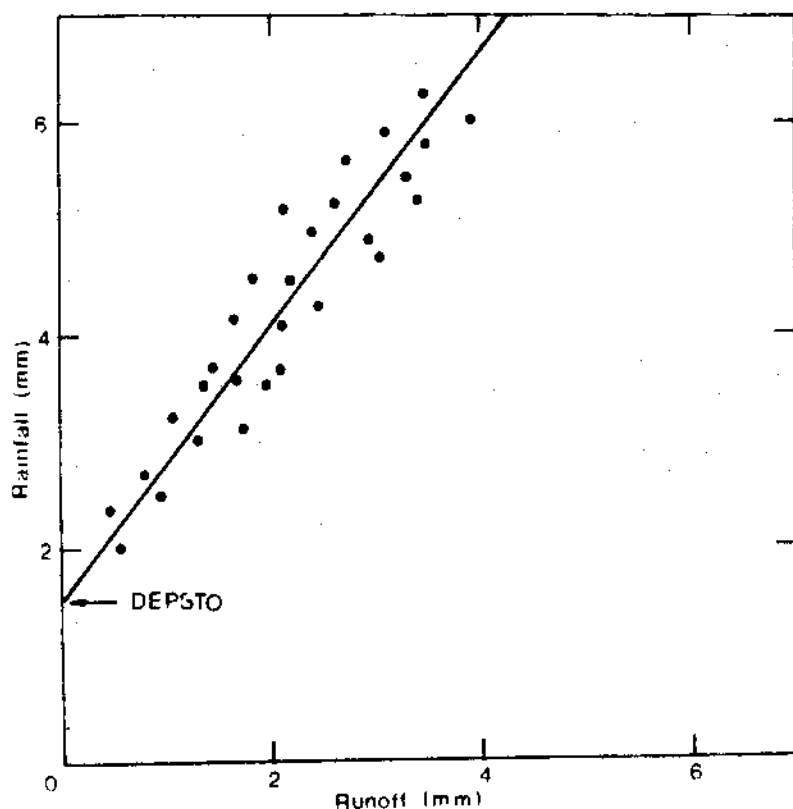


FIGURE 5 Estimation of depression storage

Values for depression storage (DEPSTO) range from 0.13 to 1.5 mm for the catchments in the workshop's data bank and have been related to catchment slope. Regression analysis gives:

$$\text{DEPSTO} = 0.77 \text{ SLOPE}^{-0.49} \quad (4.3)$$

with a correlation coefficient of 0.85, as shown in Figure 6 which also includes data from an Australian catchment (Langford & Turner, 1973).

4.2 Choice of loss model

The loss model determines both the proportion of the gross rainfall which becomes runoff (ie the net rainfall) and its distribution with

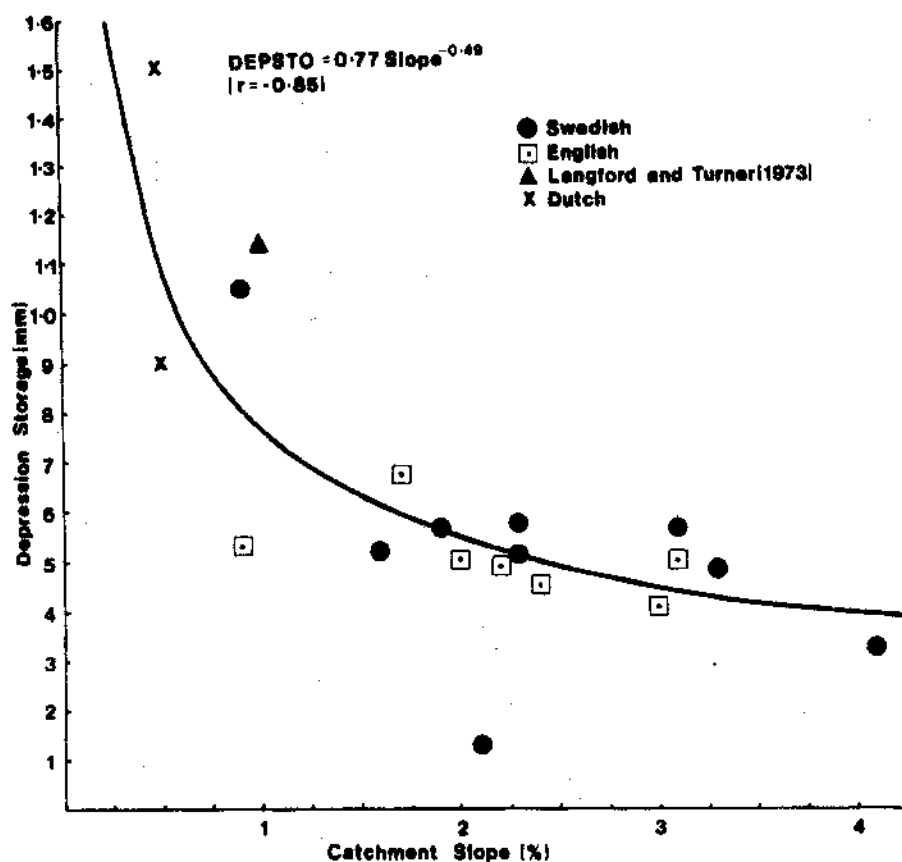


FIGURE 6 Relationship between depression storage and catchment slope

time. If the volume of net rainfall and the volume of runoff are constrained to be equal (ie volumes 'forced') then the function of the loss model is limited to the prediction of the temporal distribution of the net rainfall.

Eight loss models were considered for the study. A brief outline of these is given in Appendix C. After subjective analysis of the eight, it was concluded that only three different types of model were required to represent the others. These three, which were then used in the loss model component of the study, were:

- (i) The constant proportional loss model, in which the depression storage (DEPSTO) is first subtracted, the remaining losses being distributed as a fixed proportion of the remaining rainfall period (see Figure 7). This model has two parameters: DEPSTO and the runoff coefficient. If the volumes are to be forced, and DEPSTO is predicted from equation 4.3, then the runoff coefficient is fixed.
- (ii) The Phi-index model, in which the depression storage (DEPSTO), is first subtracted, after which loss takes place at a constant rate (Φ) for the remainder of the

FIGURE 7

Constant proportional loss model

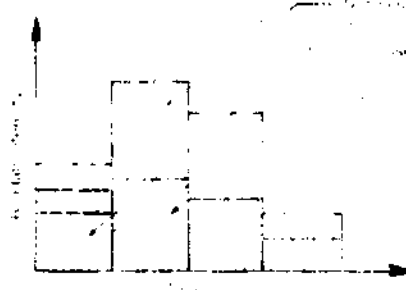


FIGURE 8

Phi-index model

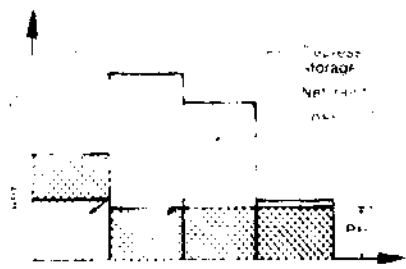


FIGURE 9

Variable proportional loss model



storm (see Figure 8). The two parameters of this model are DEPSTO and Φ , Φ being fixed by volume forcing if DEPSTO is calculated from equation 4.3.

- (iii) The variable proportional loss model, which uses a Horton-type equation to allow for a higher proportion of loss at the beginning of the storm than at the end (see Figure 9). The fraction of loss is given by:

$$Z_j = ZE + (ZO - ZE)e^{-\alpha r_j} \quad (4.4)$$

where

$$\begin{aligned}
 Z_j &= \text{fraction of loss in interval } j \\
 ZI &= \text{fraction of loss at start of storm} \\
 ZE &= \text{fraction of loss at end of storm} \\
 r_j &= \frac{\sum_{i=1}^j p_i}{P} \\
 p_i &= \text{rainfall in interval } i \\
 P &= \text{total rainfall} \\
 \alpha &= \text{constant}
 \end{aligned}
 \tag{4.5}$$

Thus, this model is a 3-parameter model, the parameters being ZE, ZO and α . (Note that DEPSTO is not a parameter for this model). Work in the Netherlands (Van den Berg, 1977) suggests that ZO and ZE can be estimated by:

$$ZO = 2(1 - \frac{Q}{P}) \quad \text{if } P \leq 2Q \tag{4.6}$$

$$\text{or } ZO = 1 \quad \text{if } P > 2Q \tag{4.7}$$

$$ZE = 0.5(1 - \frac{Q}{P}) \tag{4.8}$$

where Q is the total volume of runoff

If the volumes are forced and ZO and ZE are determined in this way, then α has a value of 2.02.

4.3 Comparison of loss models

The three loss models were compared using the non-linear reservoir as the surface routing model. The exponent (n) was fixed at 0.67, the value of DEPSTO found from equation 4.3, and the parameter k optimized for each model and each catchment on the combined events with the ISE objective function. Using this value of k the ISE objective function was then evaluated for each event.

The results are given in Tables 4 - 6.

Examination of the values given in these tables shows a tendency for the ISE objective function to decrease as the event duration increases. A reconsideration of the objective function led to the conclusion that a more uniform value would be obtained with

$$\sqrt{\frac{\frac{\sum (\text{observed} - \text{Predicted})^2}{n}}{\frac{\sum (\text{Observed})}{n}}} = \frac{\text{Root mean square}}{\text{Mean ordinate}}$$

TABLE 4 Loss model comparison for catchment 301
Values given are for the ISE objective function

| Loss Model | Event (Duration in minutes) | | | | | | | | | |
|--------------|--------------------------------|----------------|-----------------|----------------|----------------|----------------|----------------|-----------------|-----------------|----------------|
| | 300001 (119) | 300002 (59) | 300003 (119) | 300004 (89) | 300005 (59) | 300006 (89) | 300007 (59) | 300008 (119) | 300009 (104) | 300010 (59) |
| C.P.L. | 4.3 | 12.2 | 2.9 | 9.1 | 6.5 | 7.0 | 12.2 | 6.9 | 5.0 | 8.9 |
| Φ Index | 3.9 | 12.4 | 3.8 | 9.5 | 6.9 | 6.9 | 13.2 | 6.4 | 5.9 | 8.8 |
| V.P.L. | 2.7 | 13.4 | 2.3 | 7.7 | 6.0 | 6.4 | 10.3 | 6.2 | 4.6 | 8.8 |

TABLE 5 Loss model comparison for catchment 2042
Values given are for the ISE objective function

| Loss Model | Event (Duration in minutes) | | | | | | | | | | |
|--------------|--------------------------------|----------------|----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|--------------|
| | 204001 (35) | 204017 (40) | 204018 (55) | 204019 (130) | 204020 (30) | 204021 (50) | 204022 (50) | 204023 (35) | 204026 (60) | 204030 (20) | 204031 22 |
| C.P.L. | 3.6 | 4.7 | 6.2 | 1.6 | 9.2 | 7.4 | 6.1 | 3.0 | 6.0 | 13.7 | 14.2 |
| Φ Index | 4.5 | 4.3 | 4.1 | 1.6 | 9.4 | 7.4 | 6.0 | 3.0 | 6.2 | 14.8 | 14.9 |
| V.P.L. | 4.3 | 5.6 | 6.9 | 1.5 | 7.7 | 6.6 | 5.9 | 3.0 | 7.1 | 14.4 | 12.4 |

TABLE 6 Loss model comparison for catchment 4476
Values given are for the ISE objective function

| Loss Model | Event (Duration in minutes) | | | | | | | | | |
|--------------|--------------------------------|-----------------|----------------|-----------------|----------------|----------------|-----------------|----------------|-----------------|-----------------|
| | 400102 (78) | 400105 (193) | 400107 (58) | 400111 (108) | 400113 (58) | 400115 (78) | 400116 (193) | 400117 (73) | 400118 (113) | 400120 (113) |
| C.P.L. | 3.8 | 4.8 | 11.9 | 12.0 | 3.4 | 5.9 | 1.8 | 4.0 | 3.4 | 3.5 |
| Φ Index | 4.1 | 4.7 | 12.0 | 9.9 | 6.5 | 6.1 | 1.7 | 6.7 | 4.2 | 3.5 |
| V.P.L. | 4.1 | 4.8 | 11.0 | 11.0 | 2.8 | 5.9 | 1.7 | 4.9 | 3.7 | 3.5 |

which is equivalent to multiplying the ISE function by \sqrt{n} where n is the number of ordinates taken. Since the values were calculated at one minute intervals, n is equal to the duration of the runoff event (minutes).

After the above multiplication of the values given in Table 6 an analysis of variance was performed for the comparison of the models over all events on all three catchments. The analysis of variance table and results are given in Table 7 for the assumed statistical model

$$ISE_{mce} \sim N (\{\mu + \alpha_m + \beta_c + \gamma_e + |\alpha\beta|_{mc}\}, \sigma^2)$$

where ISE_{mce} = integral square error objective function dependent on model, catchment and event.

μ = overall mean value of ISE

α_m = bias due to models

β_c = bias due to catchments

γ_e = bias due to events

$|\alpha\beta|_{mc}$ = bias due to interaction of models and catchments

σ^2 = variance of the residual error.

TABLE 7 Analysis of variance calculations for loss models

(a) Analysis of variance table

| Source | Degrees of freedom | Sum of Squares | Mean Square | F ratio | Significance level |
|----------------------------|--------------------|----------------|-------------|---------|--------------------|
| Models | 2 | 264 | 132 | 3.95 | 5% |
| Catchments | 2 | 8589 | 4294 | 128.5 | 1% |
| Models x Catchments | 4 | 241 | 60 | 1.8 | NS |
| Events | 28 | 43153 | 1541 | 46.1 | 1% |
| Residual (Models x events) | 56 | 1872 | 33.4 | | |
| | 92 | 54119 | | | |

$$\text{Standard error of difference between 2 models} = \frac{2 \times 33.4}{56} = 1.09$$

$$\text{From tables of the t-distribution: } t_{5\%, 56} = 2.004$$

$$\therefore \text{Least significant difference between any 2 model means at the 5\% level} = 2.004 \times 1.09 = 2.19$$

(b) Table of model means

| Model | Overall Mean |
|--------------|--------------|
| C.P.L. | 52.55 |
| Φ Index | 54.75 |
| V.P.L. | 50.69 |

Examination of these results shows that both the variable proportional loss (V.P.L.) model and constant proportional loss model (C.P.L.) are significantly better than the Φ -index model at the 5% level of significance. Despite the lower value of mean objective function for the V.P.L. model the difference between it and the C.P.L. model is not significant.

The test above assumes that the error terms are normally distributed. A parameter-free test (Friedman, 1937) gave the same result.

5: COMPARISON OF SURFACE ROUTING MODELS

This section involved the participants for the major part of the duration of the workshop. The aim was to perform analyses using the different surface routing models available, and thus to draw some conclusions about their relative merits. It was necessary then to treat each of the models in the same way and conclusions from Sections 3 and 4 were used as a basis for these analyses. In this respect, the constant proportional loss submodel was used in all cases and optimisation achieved by deriving optimum parameter values for each individual event and taking the arithmetic mean for a given catchment. As before, volumes have been forced.

The different models have from one to three parameters. Where there is more than one parameter, a high degree of intercorrelation is generally observed (Van den Berg, 1977; Kidd, 1978) which makes optimisation of more than one parameter difficult; and, in this case, it is unlikely that realistic relationships between model parameters and catchment characteristics could be obtained for more than one parameter of a given model. For this reason, it was decided to fix all but one parameter for each model. As such, while different models have different numbers of parameters, they all have the same number of degrees of freedom during the optimisation process.

There was a total of eighteen catchments for which the analyses could be undertaken. Of these eighteen, four catchments (301, 2052, 4276, 4477) were withdrawn from the optimisation process and used for independent testing. It was decided that the main comparative test should be

based on the performance on the independent catchments of each model (incorporating the parameter v. catchment characteristics relationship derived from the other fourteen). The common treatment of the first six models is described below. Model 7 (Unit Hydrograph) was treated somewhat differently and is dealt with under a separate heading.

5.1 Common basis for comparison

Models 1 to 6 were individually studied using the following steps:

- (a) Examination of the sensitivity and interrelationship between model parameters, using the package mapping routine on three catchments (311, 2061, 4177) on:
 - (i) the 7th event
 - (ii) events 1 to 6 globally.

Typical examples are shown for each model (Objective function 1 was used). The sensitivity of a parameter was defined as the slope of the error surface in the region of the optimum (in the direction of the relevant parameter axis). This permitted an approximate estimate to be made of the relative sensitivity of two parameters. Where appropriate, these are quoted but are probably only accurate to within an order of magnitude.
- (b) Based on the results of step (a), fixed values were taken for all but one model parameter (where appropriate).
- (c) For each of the fourteen catchments (4 independent ones excluded);
 - (i) the free parameter was optimised for each event,
 - (ii) the arithmetic mean of the best individual event parameter values was calculated to give the optimum value for the catchment,
 - (iii) the coefficient of variation was calculated.
- (d) The relationship between the free parameter and catchment characteristics was examined, and a suitable regression equation generated. Suitable catchment characteristics were deemed to be catchment slope (SLOPE), overland flow length (LENGTH) and paved area (AREA).
- (e) The generalised model (incorporating the parameter estimation derived in step(d)) was applied to the four independent catchments.

The following six sections refer to each of models 1 to 6 in turn. The models are described and the salient points arising out of steps (a) to (e) above are presented. There are inevitably small deviations from the above scheme but none are significant. Section 5.8 describes the comparison of the models based on the analyses of step (e). Section 5.9 describes the analyses performed for model 7, the unit hydrograph

model. Section 5.10 describes some further analyses performed to examine the degree of nonlinearity of the rainfall-runoff process.

5.2 Model 1: Linear Reservoir

General description

The linear reservoir model is given by the following equations:

$$S = KQ \quad - \quad \text{dynamic equation} \quad (5.1)$$

$$\frac{dS}{dt} = I - Q \quad - \quad \text{continuity equation} \quad (5.2)$$

where S is the storage in the system
 Q is the outflow
 I is the inflow
 t is time
 k is a parameter of the model

The single linear reservoir is certainly one of the simplest of all conceptual models (especially in terms of the cost of computer time) with just one parameter to be determined, the storage coefficient k. It has been used for the simulation of surface runoff by several researchers - for instance by Viessman (1966), Watt & Kidd (1975), Neumann and Marr (1976) and Neumann (1977). The parameter k has a fixed value for a given catchment and for a certain rainfall event. There are, however, two difficulties to be resolved in connection with this model: firstly, can a one-parameter model simulate a measured hydrograph sufficiently well? Secondly, can a linear model ever represent a non-linear process satisfactorily? (There is no doubt that the rainfall/runoff process is non-linear - what we do not know exactly is the degree of non-linearity).

For design purposes, where no measurements exist with which to calibrate a model, some information is needed about the model parameter (some relationship with respect to catchment characteristics). There is, however, one way of deriving k analytically. Simplifications of the St Venant equations lead to the kinematic wave theory, a theory which is relevant to the overland flow problem. The time taken to reach the steady state (ie the time occupied by the rising limb of a hydrograph) was studied by Morgali and Linsley (1965). They derived the expression:

$$t_{\max} = \frac{1.45 \times L^{0.6}}{k_{\text{str}}^{0.6} \times S^{0.38} \times i^{0.4}} \quad \text{expressed in minutes}$$

L = length of flow in m, $k_{str.}$ = roughness in $m^{1/3}/s$

S = slope

i = intensity of rainfall in mm/min

By comparing the rising limbs of the kinematic model and the single linear storage model it is possible to obtain the storage coefficient

$$k = \frac{0.63 \times L^{0.60}}{k_{str}^{0.6} \times S^{0.38} \times i^{0.4}} \quad (5.3)$$

This model can therefore be applied to any surface, since the catchment characteristics of length of flow, roughness and slope are integrated with the parameter k . Further, the non-linear behaviour of the rainfall-runoff process is respected through the inclusion of parameter i for rainfall intensity. The model is said to be pseudo-linear.

An alternative to this approach was tested. This incorporates the k v. i relationship implied above, as follows:-

$$k = C I^{-.4} \quad (5.4)$$

where I is defined as the average rainfall intensity over the most intense 10 minutes. C now becomes the model parameter, and is to be optimised and regressed on catchment characteristics in the same way as the other models. The model was programmed by using equation 5.4 and then applying it as a special case to model 2 (with $n = 1$). This of course involves slightly more computation than is strictly necessary.

Sensitivity of model parameter

The optimisation was carried out using the first six events globally on catchments 2061 and 4376, as shown in Figure 10. Two optima are

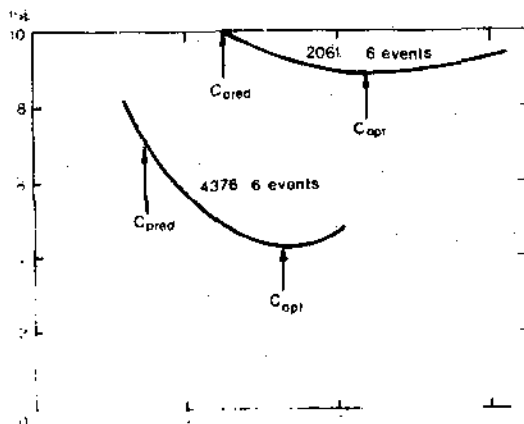


FIGURE 10

Sensitivity of parameter C

compared to the value which would have been obtained using equation 5.2. Much of the literature on this topic suggests that differences of about 30 seconds in the value of k are of little importance. Such an assumption would seem reasonable as will be verified later on.

A second sensitivity test was carried out on all the nine Swedish catchments with the C-Value optimized for each seventh event. A comparison is given by calculating the objective function I (ISE) for each individually optimised parameter C and then for a catchment-fixed C derived from the kinematic equation. In this equation the roughness was estimated as Strickler $k_{str} = 70$. Results of this test are shown in Table 8. The optimized C-value is always higher than the predicted, on average by about 67%, and the objective function is naturally also always better, by about 35%.

TABLE 8 Sensitivity test on parameter C

| Catchment No. | C-Value, obtained by | | Object. Function I for | |
|---------------|----------------------|------------|--------------------------|--------------|
| | Optimisation | Prediction | Optimis. C . | predict. C |
| | (eqn. 5.4) | (eqn. 5.3) | | |
| 4175 | 0.8 | 0.8 | 5.74 | 5.74 |
| 4176 | 3.0 | 2.0 | 7.88 | 9.50 |
| 4177 | 1.5 | 1.5 | 2.73 | 2.73 |
| 4276 | 0.8 | 0.6 | 5.46 | 5.65 |
| 4277 | 1.1 | 0.5 | 7.41 | 10.05 |
| 4376 | 1.5 | 0.7 | 3.58 | 4.84 |
| 4377 | 2.0 | 0.9 | 2.98 | 7.50 |
| 4476 | 2.0 | 0.9 | 2.10 | 2.60 |
| 4477 | 1.8 | 0.85 | 3.83 | 7.00 |
| Average | 1.61 | 0.97 | 4.6 | 6.2 |

As with the other models, a dependence was sought between optimized C-parameters and the rainfall-intensity on three catchments. Since no significant correlation could be found, it is to be supposed that the model-implied relationship between the storage coefficient and the rainfall intensity has some genuine meaning.

Optimisation of parameter C

Model parameters for each event on all but the independent catchments were optimised to obtain a mean value for parameter C for every catchment and to find a regression relationship between such averaged parameters and catchment characteristics.

The best regression equation is given by:

$$C_{\text{Regr}} = 1.43 \text{ SLOPE}^{-.40} \text{ LENGTH}^{.22} \quad (5.5)$$

This equation explains 59% of the variance in C_{Regr} (correlation coefficient 77). An interesting comparison between C_{Regr} & C_{Kinem} is shown in Figure 11 where C_{Kinem} is plotted for $K_{\text{Str}} = 50$ (or Manning's $n = .020$). It is interesting to note that the exponent of the slope in equation 5.5 compares very favourably with the theoretical value in equation 5.3, although that of LENGTH is somewhat lower.

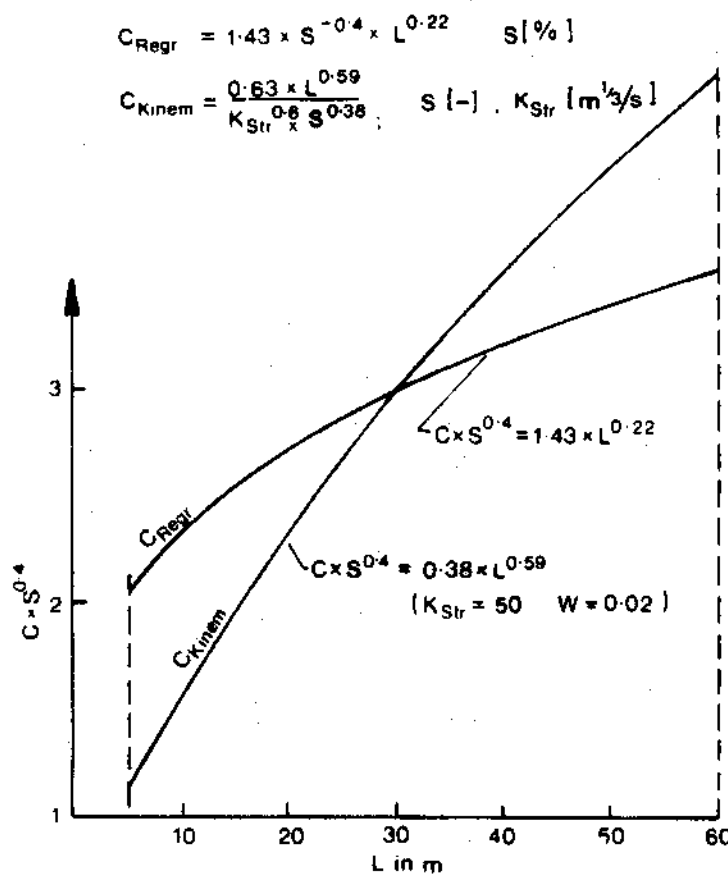


FIGURE 11 Comparison of C_{Regr} and C_{Kinem}

Figure 12 demonstrates equation 5.5, and individual values for each catchment are also shown. It will be noted that catchment 4176 is something of an outlier. This is observed for all models but because there is no reasonable physical explanation as to why the response of this particular catchment should be so damped, there is no justification for leaving it out; ignoring it would of course considerably improve the fit of the regression model (equation 5.5).

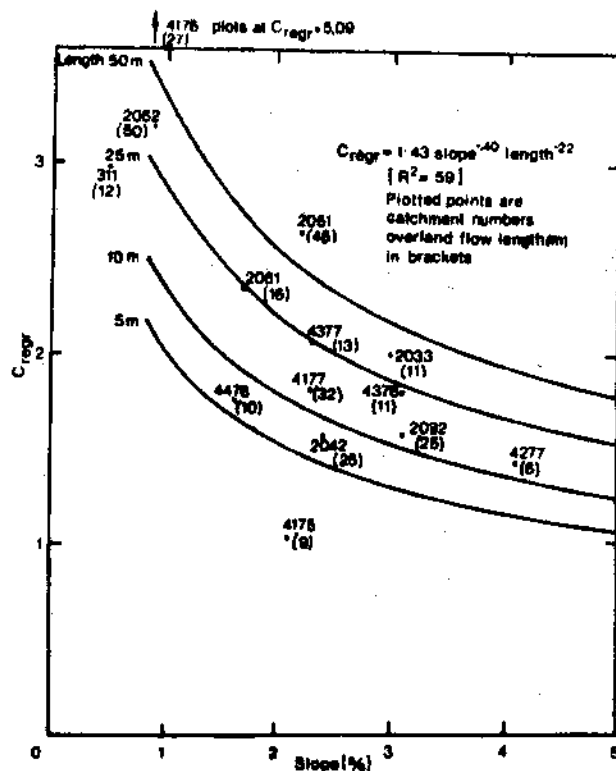


FIGURE 12 Relationship between C_{Regr} (Lin Res) and catchment characteristics

| Catchment No. | $C_{\text{Regression}}$ | | $C_{\text{Kinematic}}$ | |
|---------------------|-------------------------|-----------------|------------------------|-----------------|
| | Obj. F.1 (ISE) | Obj. F.3 (PEAK) | Obj. F.1 (ISE) | Obj. F.3 (PEAK) |
| 301 (10 events) | 7.0 | 20.2 | 8.1 | 34.3 |
| 2052 (8 events) | 8.8 | 26.4 | 9.1 | 25.1 |
| 4276 (11 events) | 4.9 | 31.4 | 5.3 | 23.0 |
| 4477 (13 events) | 6.6 | 22.7 | 7.9 | 16.9 |

It seems that C_{Regr} , evaluated from measured rainfall-runoff data, is better, at least with respect to objective function 1, but perhaps the C_{Kinem} fits the peak better. In conclusion, it is difficult to say which one should be used, or even if the single linear storage is at all satisfactory, a point which will be examined further in the comparison of all the models. In the model comparison (5.8), results for the model using C_{Regr} only are shown.

5.3 Model 2: Non-linear reservoir

General description

The non-linear reservoir model is given as follows:

$$S = kQ^n \quad \dots \quad \text{dynamic equation} \quad (5.6)$$

$$\frac{dS}{dt} = I - Q \quad \dots \quad \text{continuity equation} \quad (5.7)$$

The previous case of the linear reservoir is a special version of this model where parameter $n = 1$. Unlike the linear model, however, the above system has no analytical solution for general values of n and requires a numerical scheme. Examples of the use of this model have been given by Kidd (1976, 1978) and Van den Berg (1978). The model is programmed by performing a Newton-Raphson iteration on a finite-difference formulation of equation 5.6 and 5.7 combined.

Sensitivity of model parameters

Error surface mapping showed the marked inter-correlation between n and k which has been previously observed (Kidd, 1978). Figure 13 is an example of just such an error surface. As in the case of the Non-linear Reservoir with Time Lag (model 3, next section), the two parameters were found to be approximately equally sensitive to perturbations around the optimum. Also based on the same theoretical considerations (see section 5.4), it was decided to adopt a fixed value of n of .67.

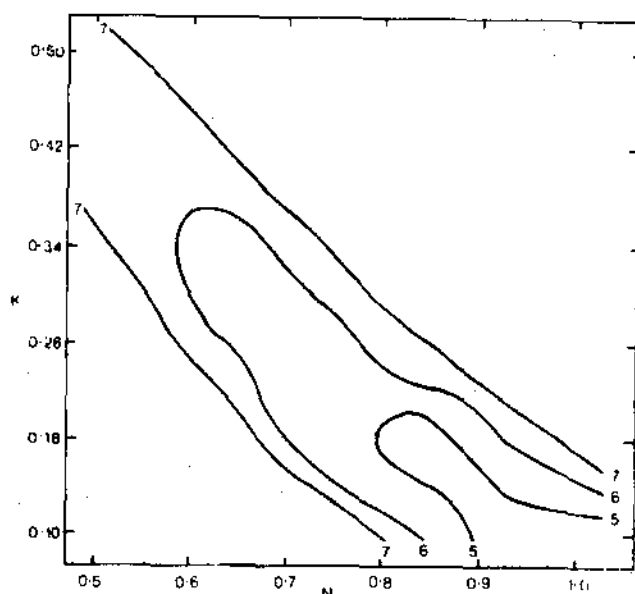


FIGURE 13 Error surface for nonlinear reservoir on catchment 311

Optimisation of parameter k

Optimum values of k were derived for the fourteen catchments, results of which are shown in Table 9. Compared with the Non-linear Reservoir with Time Lag model, the mean value of k was a little higher (to offset the effect of τ), and the coefficient of variation slightly lower (.26 against .29).

TABLE 9 Optimised values of k on 14 catchments

| Catchment | Mean | CV |
|-----------|------|-----|
| 311 | .219 | .28 |
| 2037 | .113 | .23 |
| 2033 | .173 | .37 |
| 2042 | .131 | .18 |
| 2051 | .167 | .36 |
| 2061 | .377 | .34 |
| 2062 | .214 | .26 |
| 4175 | .107 | .19 |
| 4176 | .091 | .17 |
| 4177 | .152 | .29 |
| 4277 | .132 | .19 |
| 4376 | .155 | .21 |
| 4377 | .171 | .22 |
| 4476 | .149 | .15 |
| Mean | .175 | .26 |

The best regression relationship was considered to be:

$$k = .172 \text{ SLOPE}^{-.362} \text{ LENGTH}^{.068} \quad (5.8)$$

The correlation coefficient is .70 ($R^2 = .49$). Figure 14 demonstrates the form of this relationship.

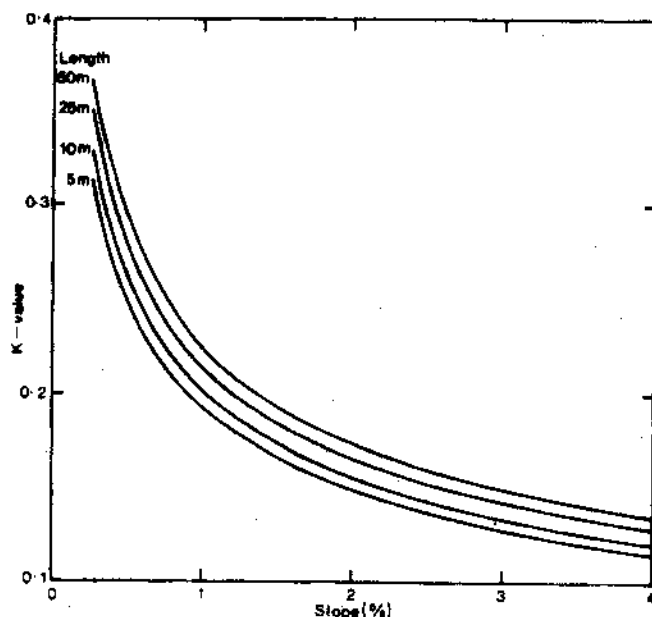


FIGURE 14

Relationship between K(NLR) and catchment characteristics

Independent Catchments

Equation 5.8 was used to predict suitable routing constants for the four independent catchments (see Table 10).

TABLE 10 Predicted k-values for independent catchments

| Catchment | Predicted k |
|-----------|----------------|
| 301 | .261 |
| 2052 | .168 |
| 4276 | .129 |
| 4477 | .159 |

5.4 Model 3: Non-linear reservoir with time-lag

General description

This is a three parameter model (k , n and τ), with the storage equation expressed as

$$S_{t-\tau} = kQ_t^n \quad (5.9)$$

and the continuity equation as

$$\frac{dS}{dt} = I - Q \quad (5.10)$$

The model may be compared with the non-linear reservoir version (model 2 above) and for $\tau=0$ the two models should be equal. In fact this is not the case due to different approaches in applying the finite differences scheme. Using the time lag makes the model less time consuming to solve (it turns the finite difference scheme from implicit to explicit) and, for this reason, a comparison analysis was extended to also include optimization on peak values. The model has been used for studies on Swedish subcatchments (Falk & Niemczynowicz, 1978).

Sensitivity of model parameters

Three catchments (311, 2061 and 4177) were used for analysis of the relationship between the model parameters n and k . The parameters show a strong internal correlation. When mapping the error surface

of n and k using the ISE objective function globally on six events, the optimum value for n tended to approach one (see Figure 15 and 16). The figures also show the same error surface mapping but with the peak objective function, indicating an n value of 0.7. For this part of the analysis the time lag τ was set to one minute.

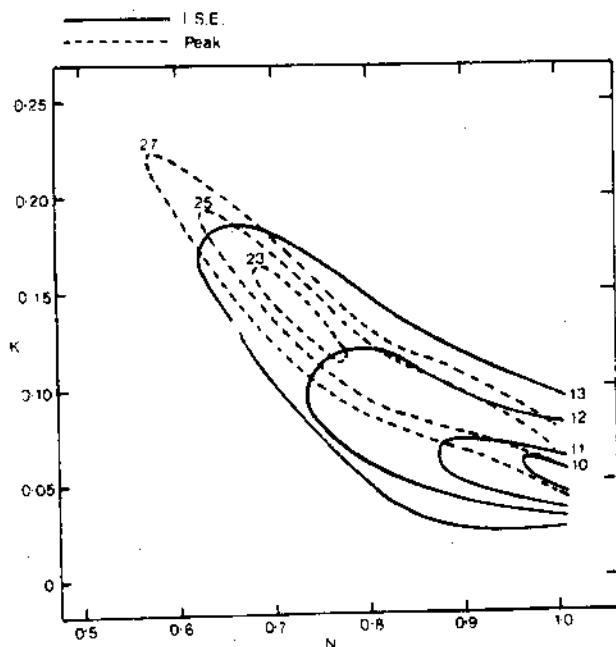


FIGURE 15

Error surface for NLR with
TL on catchment 4177

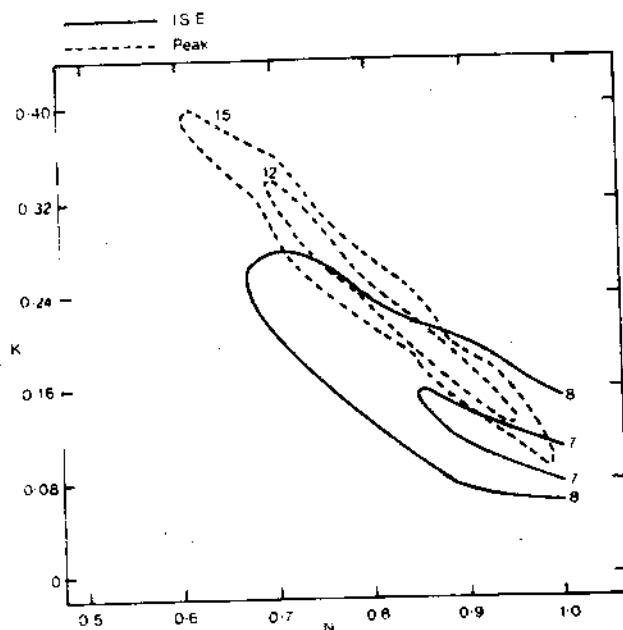


FIGURE 16

Error surface for NLR with
TL on catchment 311

The relative sensitivity of parameters n and k was investigated by means of the gradient of the error surface. For catchments 311 and

4177, n is about three times more sensitive to a change around the optimum. This is the case for the error surface for least square as well as for peak optimization. However, for surface 2061 the situation is the opposite, for here k is twice as sensitive as n . It may be concluded that they are approximately equally sensitive.

As parameters n and k are internally correlated, it was decided that one of them should be fixed. Accordingly, n was fixed at 0.67, a value which has a theoretical justification in that it can be derived from the Chezy equation. The error function was calculated for two τ values, one minute and two minutes; the integral square error almost doubled when increasing τ from one to two minutes. The value of τ was thus fixed at one minute, a value which has been used before for the Swedish catchments (Falk and Niemczynowicz, 1978).

Optimisation of parameter k

With the exception of the four independent catchments, model parameter k was optimized on all individual events for all catchments. The optimization was performed twice, for both the objective functions of integral square error and peak estimation. The mean k values and their coefficients of variation were calculated for each catchment, and Table 11 shows these values. Using the ISE optimization the average k value for all catchments equals 0.158 with a coefficient of variation of 0.29. For optimization on peak values the averages are 0.167 and 0.38 respectively, indicating that the scatter in k values within the events for a catchment is higher when optimizing on peaks. This is to be expected since data errors will be larger for a single value than for a whole event.

TABLE 11 Optimised values of k on 14 catchments

| Catchment | Optimization on OF1 | | Optimization on OF3 | |
|-------------|---------------------|-----|---------------------|-----|
| | Mean | CV | Mean | CV |
| 311 | .220 | .31 | .297 | .28 |
| 2032 | .106 | .10 | .089 | .21 |
| 2033 | .140 | .46 | .147 | .36 |
| 2042 | .115 | .17 | .137 | .16 |
| 2051 | .159 | .47 | .178 | .68 |
| 2061 | .156 | .38 | .180 | .50 |
| 2062 | .189 | .31 | .202 | .48 |
| 4175 | .092 | .26 | .089 | .28 |
| 4176 | .368 | .38 | .345 | .46 |
| 4177 | .136 | .30 | .144 | .40 |
| 4277 | .115 | .24 | .121 | .37 |
| 4376 | .137 | .23 | .149 | .39 |
| 4377 | .149 | .30 | .176 | .31 |
| 4476 | .127 | .20 | .088 | .42 |
| Mean values | .158 | .29 | .167 | .38 |

Model parameter k was regressed on three catchment characteristics - slope, area and length - for all the catchments used for parameter optimization. When using the k value obtained from the ISE objective function the best equation is:

$$k = 0.149 \text{ SLOPE}^{-0.406} \text{ LENGTH}^{0.0893} \quad (5.11)$$

where SLOPE is the average slope on the catchment in %
and LENGTH is the average length of the catchment in metres.

The multiple correlation coefficient is 0.75 ($R^2 = .56$)

The same calculation was carried out for k values derived using the fit of the peak value as objective function. The best equation then becomes:

$$k = 0.155 \text{ SLOPE}^{-0.458} \text{ LENGTH}^{0.0998} \quad (5.12)$$

Here the multiple correlation coefficient is 0.70 ($R^2 = .50$)

It must be stressed that these equations, as may be seen, mostly rest on the slope, as has been indicated before (Falk and Niemczynowicz 1978). It may appear surprising that the area does not affect k , although its effect will be taken up by the length of flow. For the small catchments examined, it was found that the rapid process of runoff is governed more by slope than by area. Figure 17 demonstrates equation 5.11 above.

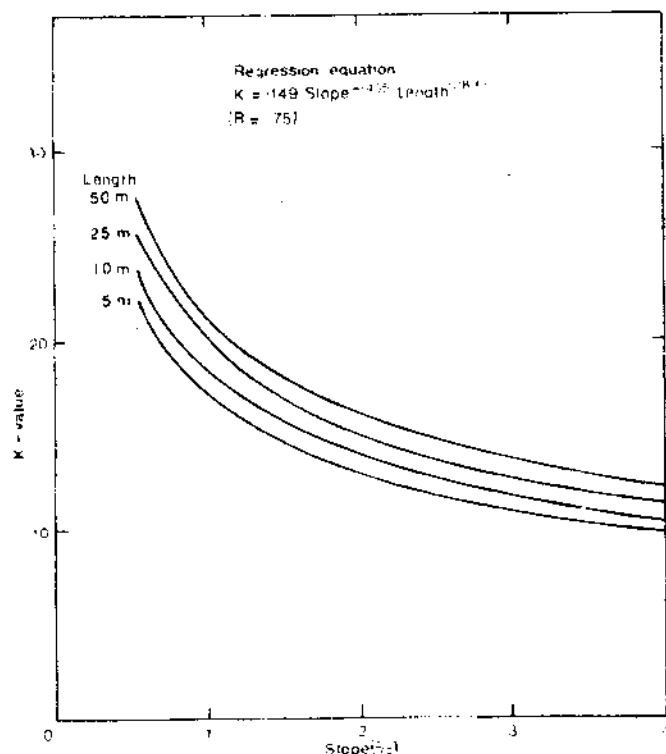


FIGURE 17

Relationship between
 k (NLR with TL) and
 catchment characteristics

Independent catchments

Equations 5.11 and 5.12 were used for determining the routing constant k for four catchments held back from the previous analysis (see Table 12).

Although the analyses were performed for k from both equations (similar results were obtained), only results using equation 5.11 are given in section 5.8.

TABLE 12 k values for independent catchments

| Catchment | k from ISE (eqn 1) | k from peak (eqn 2) |
|-----------|----------------------|-----------------------|
| 301 | 0.246 | 0.273 |
| 2052 | 0.152 | 0.158 |
| 4276 | 0.111 | 0.111 |
| 4477 | 0.141 | 0.146 |

5.5 Model 4: Nash Cascade

General description

The Nash Cascade is a linear model with two parameters, n and k . The parameter n is dimensionless and k has the dimension of time. For integer n the model can be physically represented by a series of n equal linear reservoirs, each with a reservoir constant k (Figure 18). The generalised Nash model allows for non-integer values for n , and for n less than unity. The impulse function is:

$$u(o,t) = \frac{1}{k\Gamma n} \cdot \left(\frac{t}{k}\right)^{n-1} e^{-t/k} \quad (5.13)$$

with Γn is the gamma function.

For $n > 1$ the impulse response has a peak at $t = (n-1)k$ and $u(0,0) = 0$

For $n = 1$ the impulse response reduces to:

$$u(o,t) = \frac{1}{k} e^{-t/k}$$

and $u(0,0) = 1/k$

For $0 < n < 1$, equation 5.13 holds, but the function becomes a faster than exponential decay from $u(0,0) = \infty$.

In each case, the lag time of the impulse response equals $n k$.

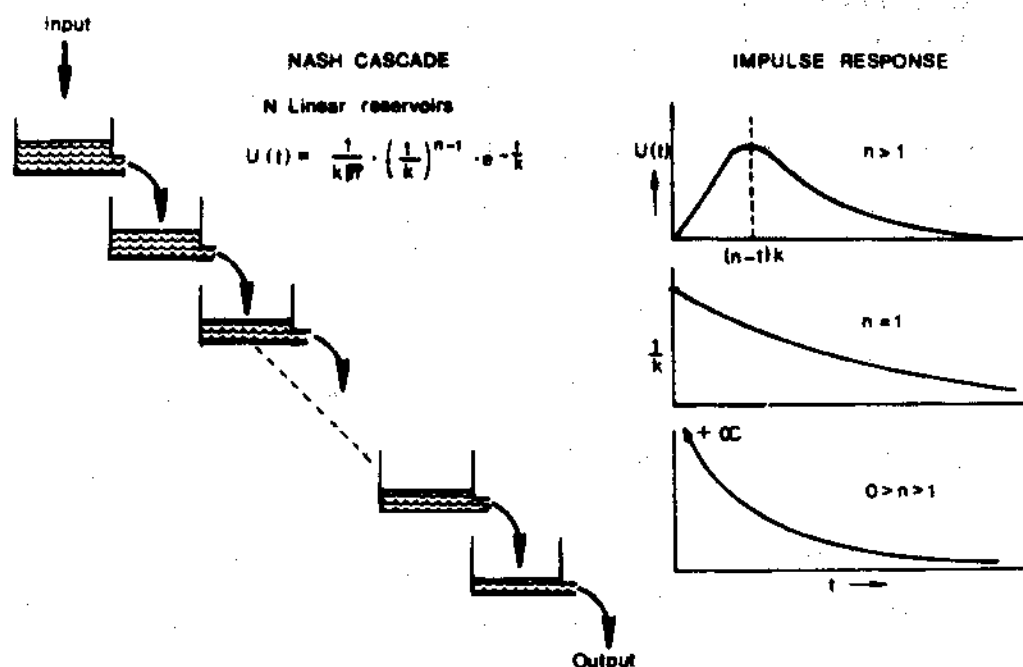


FIGURE 18 Representation of the Nash model

Sensitivity of model parameters

The fact that the lag time equals $n k$ indicates that the parameters are internally correlated. The relationship between n and k was analysed on four catchments (311, 2061, 2033, 4277), and a strong internal correlation was confirmed with error mappings on the first six events globally and the 7th event individually. An example of the error mapping of objective function 1 (ISE) on event 203016 of catchment 2033 is given in Figure 19.

The sensitivity of the model parameters was studied with the help of the mapping routine. For the four catchments the average optimum value for n and k was calculated for the first six events. For objective function 1 (ISE) the average n value was 1.3; the average k value was .05. For objective function 3 (PEAK) the average n value was about 2.6; the average k value was about .03. The sensitivity of the model parameters depends on the chosen objective function. For ISE, n was slightly more sensitive than k . For PEAK, n was less sensitive than k . In both cases, the optimum value of k seemed to be less variable than n between events and between catchments. The parameter k was fixed with a value of .05 (= 3 minutes).

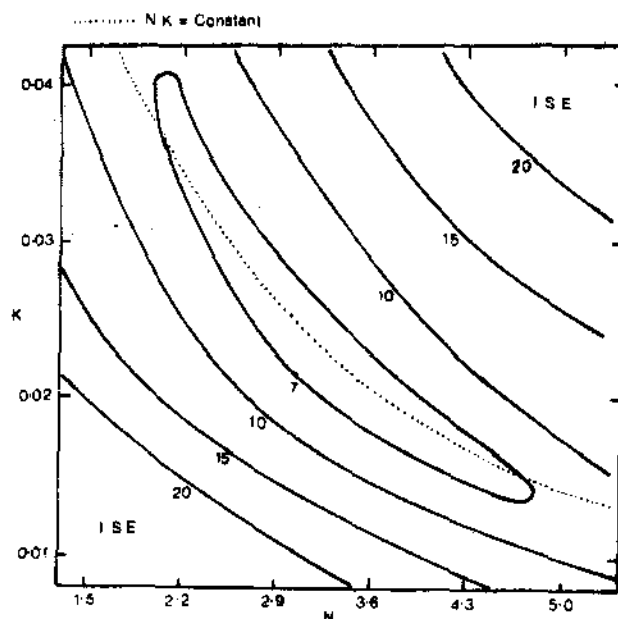


FIGURE 19

Error surface for Nash model
on catchment 2033

Optimisation of parameter n

With the exception of four independent catchments, the parameter n was optimised for individual events for all 14 catchments. The results are shown in Table 13.

TABLE 13 Optimised values of n on 14 catchments

| Catchment | Mean | Coefficient of variation |
|-----------|------|-----------------------------|
| 311 | 1.19 | .19 |
| 2032 | 1.22 | .25 |
| 2033 | 1.71 | .35 |
| 2042 | 1.10 | .16 |
| 2051 | 1.57 | .43 |
| 2061 | 1.00 | .26 |
| 2062 | 2.14 | .29 |
| 4175 | .90 | .23 |
| 4176 | 3.33 | .32 |
| 4177 | 1.29 | .28 |
| 4277 | 1.12 | .26 |
| 4376 | 1.27 | .26 |
| 4877 | 1.40 | .26 |
| 4477 | 1.35 | .24 |

Regression with catchment characteristics

Multiple regression analysis was carried out with the optimal mean n value.

Correlation matrix.

| | n | AREA | SLOPE | LENGTH | ln AREA | ln SLOPE | ln LENGTH |
|-----------|---|------|-------|--------|---------|----------|-----------|
| n | 1 | .208 | -.578 | .425 | .222 | -.652 | .461 |
| AREA | | 1 | -.401 | .859 | .972 | -.291 | .652 |
| SLOPE | | | 1 | .389 | .499 | .972 | -.439 |
| LENGTH | | | | 1 | .523 | -.386 | -.964 |
| ln AREA | | | | | 1 | -.377 | .631 |
| ln SLOPE | | | | | | 1 | -.406 |
| ln LENGTH | | | | | | | 1 |

The best regression equation was considered to be

$$n = 1.188 \text{ SLOPE}^{-.344} \text{ LENGTH}^{-.150} \quad (5.14)$$

This equation has a correlation coefficient of .71 ($R^2 = .50$)

Figure 20 demonstrates the form of this equation.

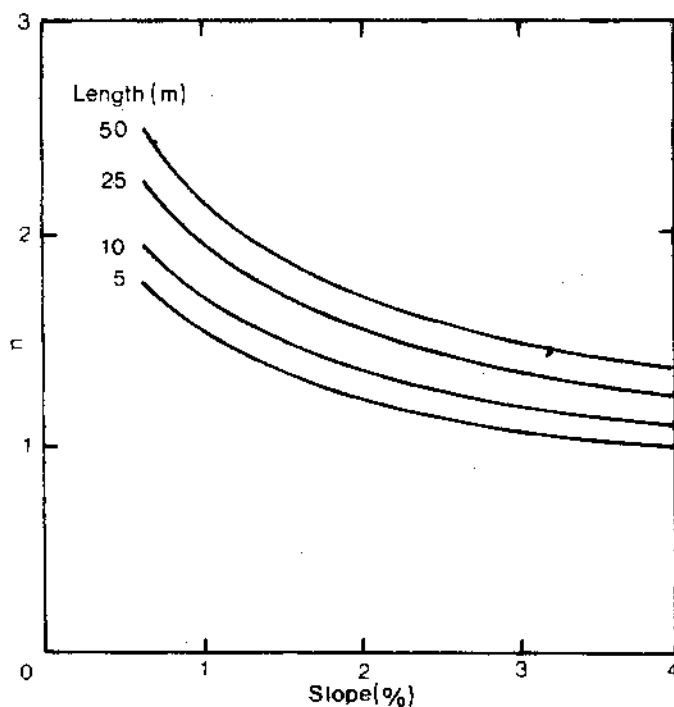


FIGURE 20

Relationship between
n (Nash model) and
catchment characteristics

Independent catchments

The multiple regression formula was used for determining the n parameter for the four independent catchments, values of which are as follows:

| Catchment | SLOPE | LENGTH | n |
|-----------|-------|--------|------|
| 301 | 0.5 | 12 | 2.19 |
| 2052 | 2.0 | 30 | 1.56 |
| 4276 | 3.3 | 8.6 | 1.09 |
| 4477 | 1.9 | 10.1 | 1.35 |

5.6 Model 5: Variable k Muskingum

General description

The Muskingum method originates from the US Army Corps of Engineers (1940), who used the method in flood studies in the Muskingum River Basin. Since then, the method has been successfully used as a flood routing method in several flood studies and, with refinements by Cunge (1969), the method has been used to calculate the runoff from urban areas in storm water pipelines (Bettess & Price 1976). However, there has been no attempt made to use the method to calculate the surface runoff in urban areas.

In order to introduce non-linearity into the method, the lag time k has been made variable, leading to the following basic equations:

$$S = k(\epsilon i + (1 - \epsilon)Q) \quad (5.15)$$

$$(k = C Q^{-n})$$

$$\frac{dS}{dt} = i - Q \quad (5.16)$$

where S is the storage volume;

i is the net rainfall intensity;

Q is the outflow from the catchment;

ϵ ($0 \leq \epsilon \leq 1$), C and n are parameters of the model.

In this form, the method contains three parameters: C , n and ϵ ; n governs the degree of non-linearity, while ϵ governs the degree of translation.

It may be noted that both the non-linear reservoir and the linear reservoir methods are special cases of the variable k Muskingum model introduced here. For $\epsilon = 0$ the relation between n_{Musk} and n_{NLR} is given by the equation:

$$n_{\text{Musk}} = -n_{\text{NLR}} + 1 \quad (5.17)$$

$n_{\text{Musk}} = 0$ is the linear reservoir model.

Sensitivity of model parameters

The sensitivity of the Muskingum model with respect to the three model parameters C , n and ϵ was analysed using data from three catchments (311, 2061 and 4177). The method of analysis was described in Section 5.1 and the results are summarised in Table 14.

TABLE 14 Optimised parameter values (using ISE) for three catchments

| Model Parameter | | Catchment No. | | |
|-----------------|-----------|---------------|-----------|-------------|
| | | 311 | 2061 | 4177 |
| C | 7th event | .09 | .09 | .07 |
| | combined | .08 | .09 | .07 |
| N | 7th event | .03 - .11 | .10 - .26 | .05 - .20 |
| | combined | - .03 - .00 | .16 - .60 | - .05 - .15 |
| ε | 7th event | .05 - .10 | .05 - .30 | .00 - .10 |
| | combined | .05 - .15 | .00 - .15 | .05 - .15 |

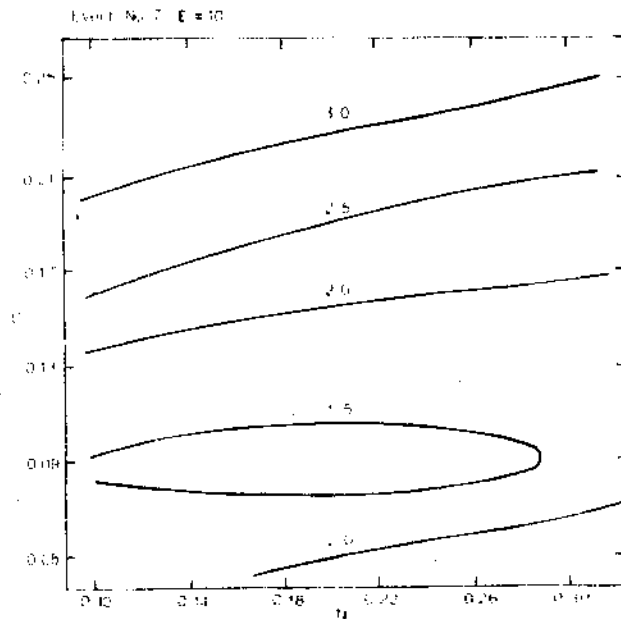


FIGURE 21

Error surface for Muskingum model on catchment 2061

Figure 21 is an example of the error surface mapping for the parameters C and n ($\epsilon = .10$). Both Table 14 and Figure 21 show that the Muskingum model is most sensitive with respect to the C-parameter. Using the sensitivity index described in section 5.2, the results are:

- * C is approximately 100 times as sensitive as ϵ
- * C is approximately 50 times as sensitive as n
- * n is approximately twice as sensitive as ϵ

Two parameters were fixed therefore and, based on the sensitivity analysis, ϵ and n were chosen with fixed values of 0.10 and 0.15 respectively.

Optimisation of parameter C

With the exception of the four independent catchments, model parameter C was optimised on individual events for all the remaining fourteen catchments using OF1 (ISE) as the basis for the optimisation. The mean C values and their coefficients of variation were calculated for each catchment. The results are summarised in Table 15.

Examination of the relationship between optimised C and storm characteristics suggested no significant trend.

TABLE 15 Optimised values of C on 14 catchments

| Catchment No | Mean C-value | Coefficient of variation |
|--------------|--------------|--------------------------|
| 311 | .11 | .27 |
| 2032 | .07 | .27 |
| 2033 | .10 | .50 |
| 2042 | .09 | .39 |
| 2051 | .12 | .42 |
| 2061 | .13 | .38 |
| 2062 | .15 | .31 |
| 4175 | .06 | .25 |
| 4176 | .24 | .28 |
| 4177 | .09 | .33 |
| 4277 | .08 | .27 |
| 4376 | .09 | .29 |
| 4377 | .11 | .31 |
| 4476 | .09 | .23 |

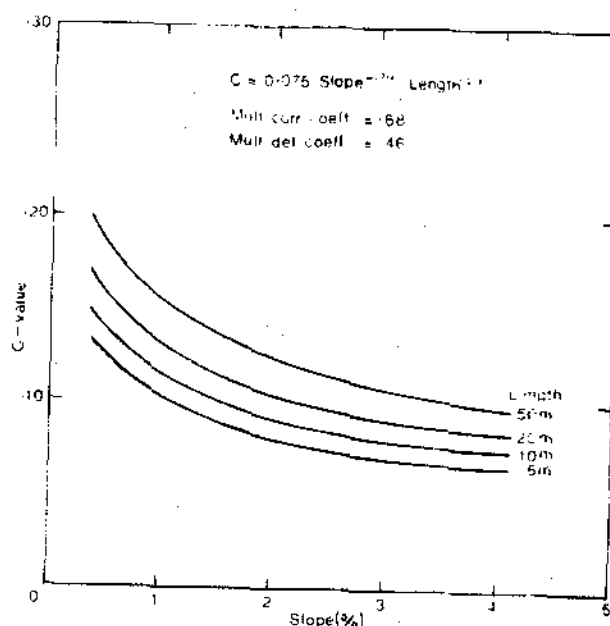


FIGURE 22

Relationship between C (Muskingum) and catchment characteristics

Model parameter C was regressed on three catchment characteristics, slope, length and area, for all the fourteen catchments used for parameter optimisation. The regression analysis showed that slope is the most important factor, and that the correlation with length was more significant than with area. The best equation based on fourteen catchments is:

$$C = 0.075 \text{ SLOPE}^{-.297} \text{ LENGTH}^{.176} \quad (5.18)$$

where SLOPE is the average slope(%) of the catchment, and LENGTH is the average length (m) of the catchment. The multiple correlation coefficient for the equation is 0.68 (multiple determination coefficient 0.46). Figure 22 demonstrates the form of this relationship.

Independent catchments

Equation (5.18) was used for predicting the model-parameter C for the four independent catchments. The C-values thus obtained may be summarised as follows:

| Catchment | Predicted C | Slope (%) | Length (m) |
|-----------|----------------|-----------|------------|
| 301 | .143 | 0.5 | 12.0 |
| 2052 | .112 | 2.0 | 30.0 |
| 4276 | .077 | 3.3 | 8.6 |
| 4477 | .094 | 1.9 | 10.1 |

The results of these simulations are dealt with in Section 5.8.

5.7 Model 6: Time of Entry

General description

The time of entry model is represented by Figure 23. Essentially, it is a time-area routing method in which the increase in area with time is linear, and the time for the full area to contribute is denoted the time of entry (TE). To derive the inlet hydrograph the rainfall excess is applied to the time area diagram. If, as is assumed here, the time area diagram is linear, the effect is the same as a moving average of the rainfall excess taken over time TE. Such a model is very simple to use and is incorporated in several existing design procedures with perhaps the best known being the TRRL Method (Watkins, 1962).

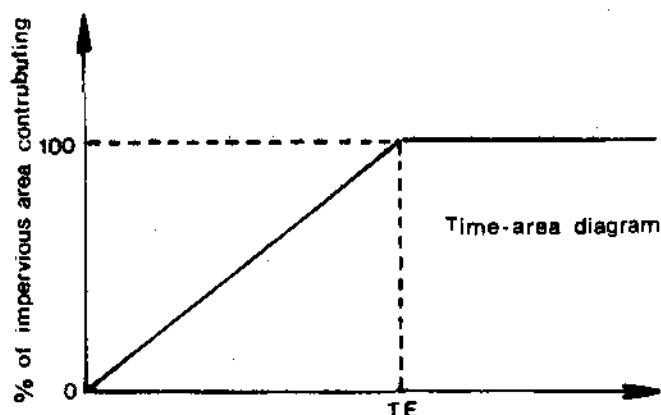


FIGURE 23

Representation of time
of entry model

Sensitivity of model parameter TE

As with the other models, the first investigation concerned the sensitivity of the parameter(s) - in this instance, TE. Since the time of entry model is a one parameter model, the values of the objective functions can be plotted directly against TE. Figure 24 shows the plot of the objective functions ISE and PEAK for six combined events on catchment 2061. It is apparent that although the PEAK function is more sensitive than the ISE function, both show a minimum value at the same TE value. This is an encouraging sign that the best value of TE has been obtained, particularly since the same result was obtained for the two other catchments tested (311 and 4177).

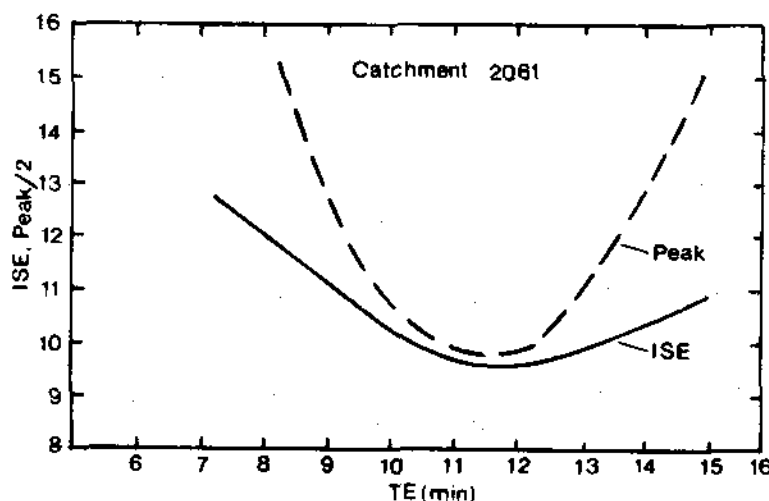


FIGURE 24

Sensitivity of
time of entry
parameter

Figure 24 suggests that the 'best' value of TE is of the order of 11-12 minutes, a much higher value than the 2-4 minutes usually adopted in design practice. Table 16 shows the TE values obtained for optimization on individual events for all of the test catchments. Again the 'best' values of TE are quite large, most being in the range 7-10 minutes. Other points to note from the table are that the coefficient of variation of TE with events is reasonably low, and that catchment 4176 again produces an anomalous result.

TABLE 16 Time of entry model - optimised values of 'TE' (min)

| CATCHMENT | EVENT NO. | | | | | | | | | | | | | | | mean s.d. S | C _v |
|-----------|-----------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----------------|----------------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | | |
| 311 | 8 | 10 | 9 | 11 | 7 | 9 | 12 | 8 | 7 | 7 | | | | | | 8.8 1.8 | 0.20 |
| 2032 | 7 | 7 | 7 | 8 | 4 | 7 | | | | | | | | | | 6.7 1.4 | 0.20 |
| 2033 | 8 | | 4 | 4 | 6 | 9 | 8 | 10 | 12 | | | | | | | 7.6 2.9 | 0.37 |
| 2042 | 7 | 8 | 7 | 7 | 9 | 7 | 7 | 10 | 7 | 7 | 7 | | | | | 7.6 1.0 | 0.14 |
| 2051 | 13 | 10 | 14 | 6 | 7 | 8 | 8 | | | | | | | | | 9.4 3.1 | 0.32 |
| 2061 | 9 | 7 | 10 | 14 | 11 | 14 | 7 | 12 | 14 | 14 | 12 | | | | | 11.3 2.7 | 0.24 |
| 2862 | 11 | 18 | 20 | 12 | 16 | 12 | 8 | 12 | 10 | 9 | 9 | | | | | 12.5 3.9 | 0.31 |
| 4175 | 11 | 6 | 6 | 6 | 5 | 5 | 7 | 7 | 9 | 6 | 4 | | | | | 6.6 2.0 | 0.30 |
| 4176 | 10 | 26 | 12 | 16 | 30 | 30 | 30 | 18 | 22 | 24 | 24 | | | | | 22.0 7.2 | 0.33 |
| 4177 | 9 | 10 | 6 | 12 | 6 | 6 | 7 | 8 | 8 | 8 | 9 | 9 | 8 | | | 8.2 1.9 | 0.23 |
| 4277 | 10 | 4 | 10 | 9 | 6 | 5 | 10 | 6 | 9 | 6 | 6 | 7 | | | | 7.3 2.2 | 0.29 |
| 4376 | 5 | 8 | 7 | 7 | 9 | 7 | 7 | 12 | 11 | 13 | 8 | | | | | 8.6 2.5 | 0.29 |
| 4377 | 9 | 9 | 7 | 14 | 7 | 7 | 8 | 9 | 8 | 11 | 11 | 10 | 9 | | | 9.2 2.0 | 0.22 |
| 4476 | 9 | 7 | 9 | 8 | 8 | 7 | 11 | 9 | 13 | 5 | | | | | | 8.6 2.2 | 0.26 |

A multiple regression of the TE values from Table 16 against catchment characteristics produced the following equation:

$$TE = 74.4 \text{ SLOPE}^{-0.27} \text{ LENGTH}^{0.13} \quad (5.19)$$

The correlation coefficient is .64 ($R^2 = .41$)

The form of the equation is not unexpected, since the slope and length are dominant factors in determining the time of concentration. A plot of the equation and data are shown in Figure 25.

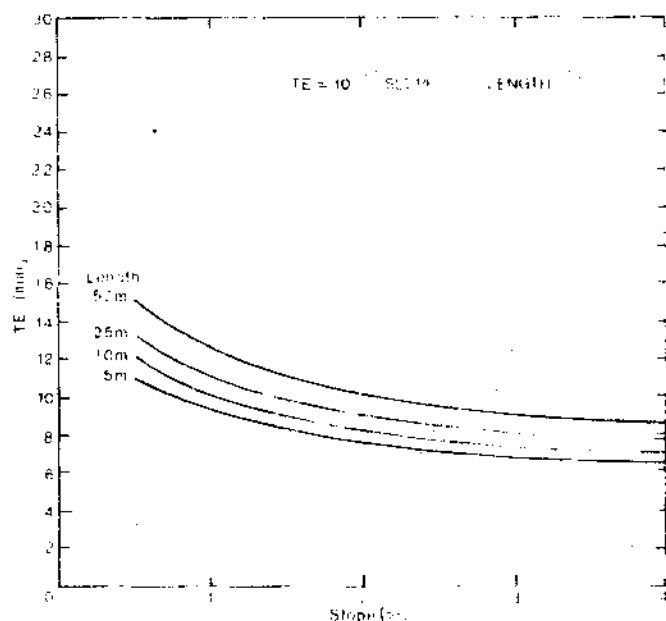


FIGURE 25

Relationship between
TE (time of entry) and
catchment characteristics

Independent catchments

Using equation 5.17 and the appropriate values of slope and length, the value of TE was predicted for the four independent catchments 301, 2052, 4276, and 4477. The values of TE obtained from the prediction equation are given as follows:

| <u>Catchment</u> | <u>Predicted TE</u> <u>(mins)</u> |
|------------------|--------------------------------------|
| 301 | 12 |
| 2052 | 10 |
| 4276 | 7 |
| 4477 | 8 |

With these corresponding values of TE the model was run for all of the events on each catchment and the objective functions calculated. The results obtained in these independent tests are tabulated with those from the other models in the next section.

5.8 Comparison of models 1 to 6

It was considered that the best objective method of model comparison was split-test analysis, with model simulation on catchments which had not contributed to the relationships between model parameters and catchment characteristics. For this purpose, one Dutch (301), two Swedish (4276 and 4477) and one British (2052) catchments were held back from the previous analyses.

Models were compared according to three objective functions, integral square error (ISE), peak estimation (PEAK) and time to peak (TTP). Values of these functions were generated for each event, and the results of these analyses are shown for each independent catchment in Tables 17 to 20. No clear-cut conclusions as to the models' relative performance are possible from scrutiny of these tables, and so an analysis of variance was performed on the results.

The analysis of variance calculations were performed on the ISE objective function multiplied by the storm duration (the reasons for the multiplication are outlined in Section 4 on the loss models). A summary of the analysis for the six models over the four catchments (42 events) is given below for the following statistical model.

$$ISE_{mce} \sim N(\{\mu + \alpha_m + \beta_c + \gamma_e + |\alpha\beta|_{mc}\}, \sigma^2)$$

where ISE_{mce} is the integral square error objective function dependent on model, catchment, and event

μ is the overall mean value of ISE

- α_m is the bias due to models
 β_c is the bias due to catchments
 γ_e is the bias due to model events
 $|\alpha\beta|_{mc}$ is the bias due to model and catchment interactions
 σ^2 is the variance of the residuals

| Source | Degrees of freedom | Total sum of squares | Mean square | F ratio | Significance level |
|-----------------------------------|--------------------|----------------------|-------------|---------|--------------------|
| Models | 5 | 5048 | 1010 | 9.64 | .1% |
| Catchments | 3 | 18194 | 6065 | 57.92 | .1% |
| Models x catchments | 15 | 8398 | 560 | 5.35 | .1% |
| Events within catchments | 38 | 181099 | 4766 | | |
| Models x events within catchments | 190 | 19895 | 104.7 | | |
| Total | 251 | | | | |

From the mean square of the residuals (models x events within catchments), the standard error of difference between 2 means =

$$\sqrt{\frac{2 \times 104.7}{42}} = 2.23$$

From statistical tables, the student-t value for 5% probability, 190 degrees of freedom = 1.98.

Hence the least significant difference between two model means at the 5% level is $2.23 \times 1.98 = 4.40$. Thus it is now possible to examine the mean values of the ISE objective function for significant differences between models.

| Model | Mean value of objective function over all events |
|------------------------------------|--|
| Linear Reservoir | 52.2 |
| Non-linear Reservoir | 52.1 |
| Non-linear Reservoir with Time Lag | 58.8 |
| Muskingum | 52.5 |
| Nash Cascade | 60.2 |
| Time of Entry | 63.4 |

TABLE 18 Independent model test on catchment 2052

| EVENT NR | LINEAR RESERVOIR | | | | | NON-LINEAR RESERVOIR | | | | | NON-LINEAR RESERVOIR WITH TIME LAG | | | | | MUSKINGUM | | | | | NASH CASCADE | | | | | TIME OF ENTRY | | | | |
|--------------------|------------------|-------|----|------|-------|----------------------|------|-------|----|------|------------------------------------|----|------|-------|----|-----------|-------|----|---|---|-------------------------|---|---|---|---|---------------|---|---|---|---|
| duration | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 |
| | c = 3.21 | | | | | k = | | | | | k = | | | | | e = 0.10 | | | | | k = 0.05 h ₂ | | | | | TE = 12 mins | | | | |
| | | | | | | n = 0.67 | | | | | k = | | | | | c = .143 | | | | | n = | | | | | | | | | |
| | | | | | | | | | | | (1 min) | | | | | n = 0.15 | | | | | | | | | | | | | | |
| Objective function | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 |
| 205004 | 5.8 | -42.1 | 7 | 5.4 | -40.9 | 4 | 8.5 | -70.8 | 6 | 5.5 | -42.1 | 4 | 8.6 | -62.0 | 6 | 10.0 | -83.7 | 3 | | | | | | | | | | | | |
| 205006 | 26.2 | 58.8 | 4 | 26.7 | 52.6 | 5 | 25.7 | 53.5 | 4 | 27.4 | 47.6 | 5 | 26.3 | 46.5 | 3 | 22.7 | 49.4 | -2 | | | | | | | | | | | | |
| 205008 | 15.0 | 35.3 | | 13.4 | 44.7 | | 13.2 | 40.7 | | 14.4 | 41.0 | | 16.0 | 24.9 | | 16.2 | 12.7 | | | | | | | | | | | | | |
| 205009 | 9.5 | 36.9 | 1 | 10.2 | 36.8 | | 9.4 | 30.2 | 1 | 10.0 | 34.8 | 1 | 8.0 | 29.7 | 0 | 15.6 | 45.0 | -5 | | | | | | | | | | | | |
| 205010 | 2.9 | -5.6 | 1 | 3.4 | -12.5 | 1 | 4.2 | -21.5 | 4 | 3.4 | -7.7 | 1 | 3.5 | -7.7 | 1 | 3.4 | -7.8 | 2 | | | | | | | | | | | | |
| 205013 | 6.0 | 28.7 | -4 | 6.4 | 20.0 | -3 | 5.3 | 11.2 | -4 | 6.3 | 21.1 | -3 | 5.1 | 18.0 | -4 | 6.0 | 6.0 | -3 | | | | | | | | | | | | |
| 205014 | 3.3 | 8.4 | 2 | 3.5 | -2.6 | 2 | 2.6 | -23.1 | 1 | 3.2 | 10.4 | 2 | 2.6 | 12.6 | 1 | 3.4 | 26.0 | 0 | | | | | | | | | | | | |
| 205015 | 1.4 | .4 | 2 | 1.6 | -8.0 | 3 | 1.6 | -18.0 | 2 | 1.5 | -.8 | 3 | 1.5 | -.7 | 2 | 2.3 | .8 | -1 | | | | | | | | | | | | |
| MEAN | 8.8 | 14.5 | | 8.8 | 11.9 | | 8.8 | .3 | | 9.0 | 13.0 | | 9.0 | 7.7 | | 10.0 | 6.1 | | | | | | | | | | | | | |
| STAND. DEV. | 8.3 | 31.2 | | 8.2 | 33.5 | | 7.8 | 41.2 | | 8.5 | 29.7 | | 8.4 | 32.9 | | 7.5 | 41.6 | | | | | | | | | | | | | |

Key:
 Objective function 1: Integral square error
 3: peak estimation
 5: time to peak

TABLE 19 Independent test on catchment 4276

| EVENT NR | LINEAR RESERVOIR | | | | | NON-LINEAR RESERVOIR | | | | | NON-LINEAR RESERVOIR WITH TIME LAG (1 min) | | | | | MUSKINGUM | | | | | NASH CASCADE | | | | | TIME OF ENTRY | | | | |
|--------------------|------------------|------|----|------|------|----------------------|------|------|----|------|--|----|------|------|----|----------------------|------|----|---|---|--------------------|---|---|---|---|---------------|---|--|--|--|
| | duration | | | | | k = n = 0.67 | | | | | c = n = 0.67 | | | | | c = .143 n = 0.15 | | | | | k = 0.05 hz n = | | | | | TE = 12 mins | | | | |
| | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | | | |
| Objective Function | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 400101 | 4.3 | 21.2 | 0 | 3.3 | 6.3 | 0 | 4.5 | -3.6 | -1 | 4.3 | 16.6 | 0 | 5.6 | 29.2 | 0 | 5.9 | 34.1 | -1 | | | | | | | | | | | | |
| 400102 | 2.8 | 13.9 | | 2.9 | 12.3 | | 2.6 | 6.6 | | 2.8 | 10.7 | | 2.8 | 10.8 | | 3.0 | -7.7 | | | | | | | | | | | | | |
| 400105 | 5.6 | 59.0 | 1 | 5.5 | 56.5 | 1 | 5.1 | 43.0 | 0 | 5.7 | 59.6 | 1 | 5.9 | 61.9 | 1 | 6.1 | 56.2 | -2 | | | | | | | | | | | | |
| 400107 | 10.5 | 35.9 | 0 | 10.1 | 32.8 | 0 | 10.0 | 15.0 | -1 | 10.8 | 40.6 | 0 | 10.9 | 40.7 | 0 | 12.5 | 38.1 | -1 | | | | | | | | | | | | |
| 400111 | 7.2 | 27.2 | 0 | 5.5 | 13.5 | 0 | 7.3 | -3.9 | -1 | 6.3 | 19.9 | 0 | 7.0 | 30.5 | -1 | 7.3 | 24.0 | -2 | | | | | | | | | | | | |
| 400113 | 2.5 | 14.2 | 1 | 3.2 | 18.4 | 1 | 3.6 | .7 | 1 | 2.9 | 15.5 | 1 | 2.4 | 11.3 | 1 | 2.2 | -2.7 | -1 | | | | | | | | | | | | |
| 400115 | 6.3 | 48.8 | -1 | 5.4 | 42.9 | -1 | 6.2 | 34.7 | -1 | 6.0 | 45.9 | -1 | 6.8 | 50.0 | -1 | 7.3 | 52.3 | -1 | | | | | | | | | | | | |
| 400116 | 1.6 | 14.9 | 0 | 1.5 | 6.0 | 0 | 1.6 | -2.0 | -1 | 1.5 | 7.6 | 0 | 1.5 | 11.5 | 0 | 1.4 | 7.6 | -1 | | | | | | | | | | | | |
| 400117 | 3.7 | 32.8 | 0 | 3.3 | 29.0 | 0 | 4.1 | 8.7 | -1 | 3.5 | 31.6 | 0 | 3.7 | 33.7 | 0 | 5.1 | 36.0 | -2 | | | | | | | | | | | | |
| 400118 | 2.8 | 26.1 | 0 | 2.2 | 14.3 | 0 | 2.6 | 4.5 | -1 | 2.3 | 15.0 | 0 | 2.4 | 17.9 | 0 | 3.5 | 26.1 | 0 | | | | | | | | | | | | |
| 400120 | 6.8 | 51.9 | -2 | 6.4 | 48.3 | -2 | 6.9 | 42.6 | -1 | 6.8 | 49.7 | -2 | 7.2 | 51.7 | -2 | 6.8 | 49.3 | -2 | | | | | | | | | | | | |
| MEAN | 4.9 | 31.4 | | 4.5 | 25.9 | | 5.0 | 13.3 | | 4.8 | 28.4 | | 5.1 | 31.7 | | 5.6 | 28.5 | | | | | | | | | | | | | |
| STAND. DEV. | 2.6 | 16.0 | | 2.4 | 17.9 | | 2.5 | 18.2 | | 2.7 | 17.9 | | 2.8 | 17.9 | | 3.1 | 21.7 | | | | | | | | | | | | | |

Key:

Objective function 1: integral square error

3: peak estimation

5: time to peak

TABLE 20 Independent test on catchment 4477

| EVENT NR | LINEAR RESERVOIR | | | | | NON-LINEAR RESERVOIR | | | | | NON-LINEAR RESERVOIR WITH TIME LAG (1 min) | | | | | MUSKINGUM | | | | | NASH CASCADE | | | | | TIME OF ENTRY | | | | |
|--------------------|------------------|------|----|------|-------|----------------------|------|-------|----|------|--|----|------|-------|----|-----------|-------|----|---|---|--------------|---|---|---|---|---------------|---|---|---|---|
| | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 |
| Objective Function | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 | 1 | 3 | 5 |
| 400202 | 10.8 | 26.6 | 1 | 11.5 | 30.0 | 1 | 4.0 | 9.6 | 0 | 11.5 | 24.1 | 1 | 8.7 | 25.3 | 0 | 15.8 | 36.6 | -3 | | | | | | | | | | | | |
| 400204 | 4.7 | 2.8 | -1 | 5.8 | - .6 | 1 | 7.2 | -17.0 | -1 | 4.9 | - 4.1 | -1 | 4.9 | - 5.4 | -1 | 4.8 | -10.1 | -1 | | | | | | | | | | | | |
| 400207 | 5.4 | 12.5 | 0 | 4.7 | 1.4 | 0 | 8.9 | -18.8 | -1 | 6.0 | 14.2 | 0 | 8.7 | 21.8 | -1 | 9.0 | 19.3 | -3 | | | | | | | | | | | | |
| 400211 | 5.2 | -9.6 | 1 | 5.7 | -27.6 | 1 | 5.9 | -46.7 | 0 | 5.8 | -34.1 | 1 | 5.7 | -33.8 | 0 | 6.6 | -45.5 | -2 | | | | | | | | | | | | |
| 400212 | 8.3 | 9.3 | 0 | 8.5 | - .2 | 0 | 11.0 | -23.1 | -1 | 8.1 | - 1.0 | 0 | 8.8 | - 1.3 | -1 | 9.8 | 6.7 | 0 | | | | | | | | | | | | |
| 400213 | 12.2 | 42.0 | 0 | 12.0 | 40.0 | 0 | 5.8 | 21.8 | 0 | 12.4 | 40.7 | 0 | 11.5 | 40.4 | -1 | 15.3 | 43.3 | -2 | | | | | | | | | | | | |
| 400215 | 3.9 | 16.9 | -1 | 3.4 | 6.6 | 1 | 4.1 | -14.1 | 0 | 4.0 | 16.1 | -1 | 5.5 | 19.1 | -2 | 5.8 | 12.8 | -2 | | | | | | | | | | | | |
| 400216 | 4.1 | 25.3 | -5 | 4.4 | 22.8 | -5 | 4.2 | 13.0 | 0 | 3.8 | 22.6 | -5 | 3.4 | 20.4 | 0 | 4.7 | 11.7 | -4 | | | | | | | | | | | | |
| 400218 | 2.5 | 34.8 | 0 | 2.5 | 35.3 | -2 | 2.7 | 24.0 | -1 | 2.7 | 37.4 | -2 | 2.9 | 37.0 | -2 | 3.4 | 29.0 | -4 | | | | | | | | | | | | |
| 400219 | 6.7 | 37.8 | 0 | 9.6 | 37.4 | 0 | 7.6 | 22.8 | -1 | 9.9 | 37.6 | 0 | 8.9 | 35.8 | 0 | 9.1 | 27.3 | 0 | | | | | | | | | | | | |
| 400223 | 8.4 | 12.6 | 1 | 7.8 | 4.3 | 1 | 6.3 | -21.0 | 0 | 7.4 | 15.2 | 1 | 6.8 | 19.6 | 0 | 11.2 | 29.0 | -3 | | | | | | | | | | | | |
| 400224 | 4.6 | 16.3 | 0 | 4.6 | 9.6 | 0 | 3.7 | - 2.8 | -1 | 4.4 | 11.3 | 0 | 3.8 | 13.7 | 0 | 5.7 | 23.1 | 1 | | | | | | | | | | | | |
| 400225 | 8.3 | 48.1 | 0 | 9.6 | 50.1 | 1 | 6.7 | 38.9 | -1 | 8.0 | 44.2 | 1 | 7.0 | 41.0 | 0 | 7.3 | 33.8 | -1 | | | | | | | | | | | | |
| MEAN | 6.6 | 21.2 | | 6.9 | 16.1 | | 6.0 | -1.0 | | 6.8 | 17.2 | | 6.7 | 18.0 | | 8.3 | 16.7 | | | | | | | | | | | | | |
| STAND. DEV. | 2.9 | 16.5 | | 3.1 | 21.9 | | 2.3 | 24.7 | | 3.0 | 21.7 | | 2.6 | 21.2 | | 3.9 | 23.4 | | | | | | | | | | | | | |

Key
Objective function: 1: Integral square error
2: peak estimation
3: time to peak

The figures above show that on the four test catchments, the linear reservoir, the non-linear reservoir, and the Muskingum models performed significantly better than the non-linear reservoir with time lag, the Nash cascade, and the time of entry model. The difference between these groups is significant at the 5% level at least. (They also show that the magnitude of the difference between models differs from catchment to catchment, since models \times catchments is significant). The analysis of variance assumes that the residuals are normally distributed. The same overall results were obtained however by applying a non-parametric test (Friedman, 1937).

A general conclusion that can be drawn from the analysis is that the non-linear models performed better than did the linear ones. This conclusion is reached by noting that the non-linear reservoir with time lag would have been in the top group except for a poor performance on one catchment, and that the linear model was operated in a non-linear way (the parameter depends on the size of the storm). The linear models (Nash cascade and time of entry) were in the bottom group of performers.

A similar analysis on the absolute value of the peak objective function showed that there was no significant difference between the performance of the models in this respect. Again, there was considerable variation between catchments.

5.9 Model 7: Unit Hydrograph

In addition to the six models available in the modelling package, a unit hydrograph analysis program was available. This program was not included in the main package because of the large number of model parameters (every unit hydrograph ordinate) needed to be determined, each exhibiting strong auto-intercorrelation. The program uses a quadratic programming subroutine from the Numerical Algorithms Group (NAG) library. Unit hydrographs are derived by solving the indeterminate equation set given by the usual convolution summation (with error);

$$q_i = \sum_{j=1}^i (x_{(i-j+1)} \cdot u_j) + \epsilon_i \quad 1 < i < n$$

subject to the criteria (i) $\sum_{i=1}^n \epsilon_i^2 = \text{minimum}$

$$\text{and} \quad (\text{ii}) \quad u_j \geq 0 \quad 1 < j < n$$

$$\sum_{j=1}^n u_j = 1$$

where q_i is measured outflow

r is measured rainfall

u is unit hydrograph ordinate

The advantage of the quadratic programming technique over the simpler matrix inversion technique is the inclusion of the second criterion allowing only positive unit hydrograph ordinates. The unit hydrographs derived should therefore be more realistic.

Unit hydrographs were derived for each event individually from each of the 14 test catchments. Unit hydrographs could not however be obtained from several events due to numerical scaling problems in the quadratic programming routine. In general, individual unit hydrographs were 'noisy', showing large variations about an expected smooth shape. This was partly due to the time interval of the data being too long (1 minute) in relation to catchment time to peak (of the order of 2 minutes). The long time interval made estimation of hydrograph shape in the region of the peak especially difficult. An attempt to produce a smooth shape by applying a moving average filter to the derived ordinates was abandoned because of the large attenuation of peak discharge produced - another effect of the small number of ordinates near the peak. A further cause of 'noisy' unit hydrograph shape was that no separation of response into slow (base) and quick (surface) response was made. As a result, residual non-zero flow at the end of the event was reflected in large values for the last few ordinates of the derived unit hydrographs and correspondingly lower values for earlier ordinates. While separation of the response would give large inconsistencies between the unit hydrograph and the other surface routing models, the other models specify conceptual responses which cannot yield high ordinates at the end of the recession. Furthermore, there is some justification for using a separate mechanism for the input forced response before the inflexion of recession limb and the free response afterwards. Because of their noisy shape, about a third of the derived unit hydrographs were rejected from further analysis.

The remaining unit hydrographs (about five for each catchment) were averaged after first synchronising peaks. The catchment-averaged unit hydrographs showed quite large variations in skewness between catchments, and as a result no single dimensionless unit hydrograph could faithfully fit all catchments. Thus a triangular unit hydrograph was fitted by eye to each catchment average unit hydrograph and the peak and time to peak measured (time base is determined from peak by the necessity of unit volume of discharge). Table 21 below gives the values obtained.

A regression analysis was performed on these data to determine suitable forms of equation for both Q_p and T_p . In order to allow fair comparison between this model and the other surface routing models, only one parameter (Q_p or T_p) should be determined from catchment characteristics, the other being determined from an internal relationship. (In fact, as will be seen, the effect of determining both parameters from catchment characteristics would be small). One catchment, 4176, showed a markedly

different response from the others so regression equations were determined both including and excluding that catchment.

For the internal relationship Q_p and T_p , the most suitable form was found to be a regression of $Q_p T_p / 2$ (ie the area under the rising limb) on Q_p . The equations derived for the 13 and 14 catchment case are given below:

$$14 \text{ catchments: } Q_p T_p = 1.182 Q_p + .256 \quad R^2 = .91 \quad (5.20)$$

$$13 \text{ catchments: } Q_p T_p = 1.131 Q_p + .276 \quad R^2 = .86 \quad (5.21)$$

TABLE 21 Optimised U^H characteristics.

| CATCHMENT NO. | TIME TO PEAK, T_p | PEAK, Q_p |
|---------------|---------------------|-------------|
| 311 | 2.35 | .200 |
| 2032 | 1.60 | .490 |
| 2033 | 2.00 | .380 |
| 2042 | 2.00 | .350 |
| 2051 | 1.90 | .260 |
| 2061 | 2.60 | .220 |
| 2062 | 2.20 | .280 |
| 4175 | 1.60 | .500 |
| 4176 | 3.20 | .116 |
| 4177 | 2.00 | .390 |
| 4277 | 1.60 | .370 |
| 4376 | 2.00 | .385 |
| 4377 | 1.70 | .375 |
| 4476 | 2.00 | .420 |

In both cases the coefficient of Q_p is significant at the 0.1% level. Inclusion of T_p as another independent variable improved the correlation coefficient, but produced unrealistic estimates of Q_p from T_p and vice versa.

For an external relationship for either Q_p or T_p , each parameter was regressed on catchment characteristics using both the 13 and 14 catchment data set. In each case, only catchment characteristic, slope (S), was found to be significant at the 5% level. The derived equations are given below:

$$14 \text{ catchments: } Q_p = .235 S^{.473} \quad R^2 = .69 \quad (5.22)$$

$$T_p = 2.334 S^{-.230} \quad R^2 = .68 \quad (5.23)$$

$$13 \text{ catchments: } Q_p = .272 S^{.335} \quad R^2 = .66 \quad (5.24)$$

$$T_p = 2.187 S^{-.169} \quad R^2 = .63 \quad (5.25)$$

To determine:

- (i) whether it was better to estimate Q_p from S and then T_p from Q_p , or alternatively to estimate T_p from S and Q_p from T_p ;
- (ii) the effect of catchment 4176; and
- (iii) the effect of using predicted rather than observed Q_p or T_p in the internal relationships;

graphs of Q_p predicted against Q_p observed and T_p predicted against T_p observed were prepared. These showed that the 13 catchment equations gave better correlation coefficients of predicted on observed values even when catchment 4176 was included. They also showed that predicting Q_p from S gave the highest correlation on Q_p observed, but the lower correlation of T_p predicted on T_p observed. It was therefore decided to predict T_p from S by equation (6) and Q_p from T_p by equation (5.25). Each equation giving a correlation coefficient of predicted on observed of 0.65 (note this is of similar order to that obtained from estimating both Q_p and T_p from S).

Using these equations, unit hydrographs were estimated for the four test catchments, and each of the test events was simulated. Table 22 gives a summary of the objective error functions. Comparison of these results with the other surface routing models shows that the unit hydrograph method as developed here performed far worse than any of the other models, particularly with regard to peak estimation. This indicates something is wrong with the analysis, since both the time of entry and the Nash cascade models are effectively unit hydrograph models, the time of entry model giving a rectangular unit hydrograph and the Nash cascade giving a smooth curved unit hydrograph. It would be expected, therefore, that a triangular unit hydrograph model would yield results intermediate between these other two. Indeed, if a triangular unit hydrograph model had been included in the package this probably would have been the case. However, it was only decided to approximate the catchment average unit hydrograph by a triangle at a late stage, so the triangular unit hydrograph was not considered earlier. The reason why this latter model has done so badly is explained therefore by the large number of events that had to be rejected from the catchment averaging process (for the other models each event produced a response even if the fit was not very good), and also by the long time interval of the data in relation to unit hydrograph time to peak - causing very unstable individual unit hydrographs.

5.10 Non-linearity of the surface routing process

It was decided to investigate the non-linearity of the rainfall-runoff

TABLE 22 Results of independent tests on UH model

| CATCH- MENT 101 | TRIANGULAR UNIT HYDROGRAPH | CATCH- MENT 2052 | TRIANGULAR UNIT HYDROGRAPH | CATCH- MENT 4274 | TRIANGULAR UNIT HYDROGRAPH | CATCH- MENT 4377 | TRIANGULAR UNIT HYDROGRAPH |
|----------------------------|----------------------------------|----------------------------|----------------------------------|----------------------------|----------------------------------|----------------------------|----------------------------------|
| EV. NO. OBJECTIVE FUNCTION | EV. NO. OBJECTIVE FUNCTION | EV. NO. OBJECTIVE FUNCTION | EV. NO. OBJECTIVE FUNCTION | EV. NO. OBJECTIVE FUNCTION | EV. NO. OBJECTIVE FUNCTION | EV. NO. OBJECTIVE FUNCTION | EV. NO. OBJECTIVE FUNCTION |
| 1 | 5.61 - 61 - 3 | 4 | 15.85 - 147 7 | 1 | 4.10 11 - 1 | 2 | 15.96 - 25 0 |
| 2 | 19.03 - 65 - 3 | 6 | 41.58 - 11 4 | 2 | 5.41 - 29 20 | 1 | 5.07 - 42 - 1 |
| 3 | 4.14 - 5 12 | 8 | 22.86 - 40 14 | 5 | 4.29 45 141 | 7 | 6.53 - 13 - 1 |
| 4 | 15.25 - 62 - 2 | 9 | 12.21 - 32 1 | 7 | 11.76 18 0 | 11 | 8.08 - 95 + 1 |
| 5 | 13.83 - 76 - 1 | 10 | 5.45 - 27 4 | 11 | 9.81 3 - 1 | 12 | 11.10 - 44 - 1 |
| 6 | 8.48 25 - 2 | 13 | 9.90 - 13 3 | 13 | 4.68 - 36 - | 13 | 10.65 - 2 - 1 |
| 7 | 22.46 - 11 - 2 | 14 | 4.89 - 32 1 | 15 | 4.80 23 - 1 | 15 | 4.67 - 9 0 |
| 8 | 9.93 - 77 - 5 | 15 | 3.30 - 28 2 | 16 | 1.37 - 6 0 | 16 | 5.24 - 10 0 |
| 9 | 9.25 - 60 - 3 | MEAN | 14.51 - 41.3 (41.3) | 17 | 5.10 - 14 0 | 18 | 1.64 5 0 |
| 10 | 13.88 - 19 - 4 | S.D. | 12.71 43.8 (43.8) | 18 | 3.93 - 13 0 | 19 | 3.11 - 1 - 1 |
| MEAN | 12.19 - 41.1 (46.1) | | | 20 | 5.57 41 - 1 | 23 | 12.48 - 27 0 |
| S.D. | 5.80 35.5 (27.8) | | | MEAN | 5.53 4.1 (21.9) | 24 | 13.10 - 63 0 |
| | | | | S.D. | 2.86 26.7 (14.3) | 25 | 12.41 + 25 - 3 |
| | | | | | | MEAN | 9.45 - 23.2 (27.8) |
| | | | | | | S.D. | 4.43 31.7 (27.4) |

TABLE 23 K, N values optimisation on individual events, Function No 1

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | Mean | C.V. | S.D. |
|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|------|------|------|------|-------|-------|
| 2042 K | .051 | .086 | .050 | .048 | .149 | .110 | .105 | .151 | .109 | .060 | .054 | | | | | |
| N | .940 | .753 | .869 | .976 | .552 | .719 | .718 | .689 | .682 | .984 | .969 | | | .807 | .185 | .148 |
| 2051 K | .235 | .103 | .258 | .056 | .060 | .136 | .078 | | | | | | | | | |
| N | .926 | 1.000 | .315 | .836 | .924 | .711 | .967 | | | | | | | .811 | 0.295 | .239 |
| 2061 K | .059 | .050 | .093 | .124 | .138 | .162 | .054 | .206 | .199 | .157 | .155 | | | | | |
| N | 1.000 | .880 | .736 | .838 | .519 | .999 | .962 | .428 | .808 | .889 | .838 | | | .810 | .229 | .186 |
| 2062 K | .235 | .234 | .166 | .196 | .150 | .078 | .118 | .149 | .162 | .149 | | | | | | |
| N | .686 | .917 | .752 | .668 | .803 | .866 | .998 | .675 | .569 | .614 | | | | .738 | .194 | .149 |
| 4175 K | .276 | .285 | .245 | .243 | .076 | .055 | .095 | .159 | .197 | .052 | .022 | | | | | |
| N | .309 | .353 | .408 | .330 | .608 | .808 | .617 | .470 | .553 | .805 | .979 | | | .567 | .391 | .272 |
| 4176 K | .112 | .214 | .099 | .185 | .383 | .245 | .507 | .149 | .334 | .233 | .321 | | | .253 | .049 | .124 |
| N | .862 | 1.000 | .950 | .877 | 1.000 | 1.000 | .722 | .784 | .810 | 1.000 | .845 | | | .913 | .105 | .096 |
| 4177 K | .077 | .082 | .045 | .095 | .039 | .031 | .042 | .063 | .093 | .236 | .147 | .138 | .174 | | | |
| N | .971 | 1.000 | .969 | 1.000 | 1.000 | .890 | .958 | .891 | .833 | .597 | .785 | .661 | .515 | .851 | 0.203 | 0.165 |
| 4277 K | .089 | .047 | .095 | .171 | .137 | .139 | .191 | .204 | .198 | .207 | .136 | .113 | | | | |
| N | .940 | .800 | .924 | .506 | .583 | .583 | .500 | .509 | .509 | .512 | .519 | .524 | | .617 | .273 | .169 |
| 311 K | .074 | .082 | .069 | .089 | 0.050 | 0.060 | .107 | .058 | .069 | 0.053 | | | | | | |
| N | .941 | 1.000 | 1.000 | 1.000 | .970 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | | | | .991 | .020 | .020 |
| OVERALL MEAN | | | | | | | | | | | | | | 0.79 | | |

process independently. Consideration of the theory of open channel flow, of which overland flow and gutter flow are typical manifestations, suggests that the process should indeed be non-linear. The application of certain steady-state flow formulae to the dynamic equation of, for instance, the non-linear reservoir, suggest various values for the non-linearity exponent ($n = .60$) for the Manning or Rutter Formula and $n = .67$ for the Chezy Equation. It is of interest to see to what extent the data supports these theoretical considerations.

The non-linear reservoir with time lag (surface routing model 2) was used in conjunction with the constant proportion loss submodel to give some insight into the best value of the non-linearity exponent n . Both surface routing parameters were optimised for each event on nine catchments, and a summary of the results are shown in Table 23. Note that there is considerable scatter in the best n -values (this is to be expected in the light of the strong interdependency between n and k observed in section 5.4), but that values seem to be in the range .6 to .9.

A data set was compiled consisting of all the optimum surface routing parameters for each of the 146 events together with catchment characteristics (slope, area, length) and storm characteristics (rainfall volume, duration and maximum 5-minute intensity). This data set is shown in Appendix D. A part of the correlation matrix is reproduced below in Table 24.

TABLE 24 Correlation matrix for 146-event data set

| | CATCHMENT CHARACTERISTICS | | | STORM CHARACTERISTICS | | |
|-------------------|---------------------------|-------|--------|-----------------------|-------|----------|
| | AREA | SLOPE | LENGTH | M5I | RFVOL | DURATION |
| C, LIN RES | .12 | - .55 | .35 | .12 | .08 | .04 |
| k, NON-LIN RES | .08 | - .48 | .23 | .10 | .09 | .13 |
| k, NLR with LAG | .07 | - .49 | .22 | .13 | .09 | .11 |
| n, NASH CASCADE | .17 | - .46 | .35 | .27 | - .18 | .05 |
| C, MUSKINGUM | .16 | - .50 | .34 | - .15 | - .10 | .06 |
| TE, TIME OF ENTRY | .12 | - .45 | .26 | - .21 | - .14 | .08 |

This table confirms that, for all models, the two most powerful controls of the optimised parameter are the slope and length of the catchment. After these two, a regression analysis of the model parameter on slope, length and M5I (the maximum 5-minute rainfall intensity) yields a Student's-t statistic that gives some insight into the significance of the control exerted by M5I. For the Nash Cascade and Time of Entry models (both linear), this analysis demonstrates that the M5I is significant to within the .1% level. For the remainder of the models (the linear reservoir may be seen as pseudo-linear), the M5I is not significant to within the 20% level. This fact reinforces the evidence in support of the non-linearity of the routing process. Of course, the M5I could be used in predictive mode to further improve the estimation

of model parameters if this is deemed desirable. The object of resorting to a non-linear model is to avoid the dependence of routing parameters on storm characteristics.

It was further felt that the models might fit larger events better than smaller events, because the depression storage part of the model would have less of a control on the result. The following analysis was performed for the Muskingum model only, on the grounds that a broadly similar result could be obtained for any of the other models.

For each catchment, a mean value of optimised Muskingum C was obtained; then, for each event, the variation about this mean was obtained by:

$$\text{VARIATION} = \frac{C - C_{\text{MEAN}}}{C_{\text{MEAN}}}$$

Thus, a value of this variation about the catchment mean was obtained for each of the 146 events in the above data set. These were plotted against rainfall volume (different symbols for each catchment), as shown in Figure 26. This figure does demonstrate that the variations seem to be decreasing with increasing rainfall volume, although there are many more points associated with the smaller events.

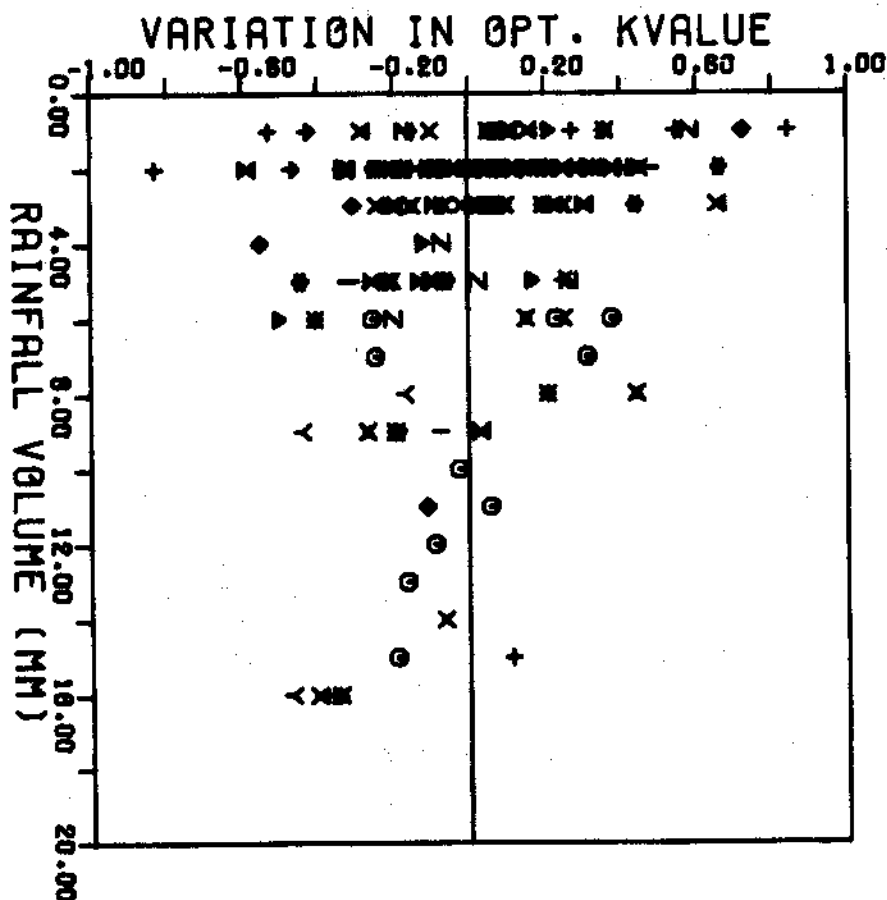


FIGURE 26 Variation in k value for all catchments

6. CONCLUSIONS

Data from three sources (Netherlands, Sweden and UK) have been collated into a data base, consisting of a total of 188 rainfall-run-off events on 16 urban subcatchments. A number of loss models and surface routing models were investigated using those data, and a number of optimisation schemes and objective functions were examined. The traditional least squares objective function (OF1) leaves something to be desired as far as single event simulation is concerned, but no better one could be found in the short time available. It was decided that the most appropriate optimisation scheme for a given catchment was to derive optimum model parameter(s) for each event and then to define the overall optimum as the arithmetic mean of the event values. Although better results could sometimes be obtained using a global optimum, the advantage of the adopted scheme is that it gives more information concerning the variability of best parameter value(s) from event to event.

A relationship was developed which related the depression storage to catchment characteristics. This regression relationship could be used for estimation on ungauged catchments.

Loss models were investigated first. It was considered that of the eight loss submodels available, three represented the range of models, and these (phi-index, constant proportional loss, variable proportional loss) were investigated in greater detail. It was found that the phi-index model was significantly inferior to the latter two (at the 5% level), but that no significant difference could be observed between these two. The constant proportional loss model was adopted due to its relative simplicity and its popularity in the relevant literature.

TABLE 25 Summary of results

| Model | Fixed Parameter Values | Regression Equation | Correlation Coefficient |
|------------------------|-------------------------------------|---|----------------------------|
| 1. LINEAR RESERVOIR | - | $C = 1.43 \text{ SLOPE}^{-.40} \text{ LENGTH}^{.22}$ | .77 |
| 2. NONLINEAR RESERVOIR | $N = .67$ | $K = .172 \text{ SLOPE}^{-.36} \text{ LENGTH}^{.068}$ | .70 |
| 3. NLR WITH TIME LAG | $N = .67$ $\tau = 1$ (minute) | $K = .149 \text{ SLOPE}^{-.41} \text{ LENGTH}^{.089}$ | .75 |
| 4. NASH CASCADE | $K = .05$ (hours) | $N = 1.19 \text{ SLOPE}^{-.34} \text{ LENGTH}^{.15}$ | .71 |
| 5. MUSKINGUM | $e = .10$ $n = .15$ | $C = .075 \text{ SLOPE}^{-.30} \text{ LENGTH}^{.18}$ | .68 |
| 6. TIME OF ENTRY | | $TE = 74.4 \text{ SLOPE}^{-.27} \text{ LENGTH}^{.13}$ | .64 |

Using the conclusions mentioned above, seven surface routing models were investigated. These were subjected to broadly the same analyses, the outcome of which was a relationship in each case between one model parameter and catchment characteristics (slope and overland flow length). These relationships showed a marked similarity, and all explained approximately the same percentage of the variance (between 40% and 55%) in the optimised parameter. The catchment slope exerts the stronger control of the two catchment characteristics in all cases. Table 25 shows a summary of the results of these analyses.

The performances of the models were compared by applying them to four independent catchments which had not been employed in the analyses described in the last paragraph. An analysis of variance on these results suggests that, in terms of overall fit (least squares), the linear reservoir, nonlinear reservoir and Muskingum models were superior to the others. In terms of peak estimation, there was nothing to choose between any of the models. The unit hydrograph model performed poorly compared to the other models in all respects.

In conclusion, a number of fully generalised models have been presented. Given a means of estimating the volume of runoff from a given rainfall event (which will vary according to climate and engineering practice), any of the surface routing models described could be used to generate an inlet hydrograph to a sewer system. Analyses have indicated that the choice of surface routing model is less critical than the manner in which it is utilised. In general, however, a nonlinear model will perform marginally better than a linear one.

ACKNOWLEDGEMENTS

The workshop participants would like to acknowledge the considerable assistance of the following IH staff: Robin Clarke for his help with the analyses of variance, Ian Makin for his organisation of the data and help with the analyses, and Mark Venn for his help and advice on computing matters. The visiting participants would like to extend their special thanks to Celia Kirby for arranging for their stay to be so pleasant.

The workshop was sponsored jointly by the Department of the Environment and the Institute of Hydrology.

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APPENDIX A : THE DATA BASE

The task of model building can often be simplified if the analyst is relieved of the responsibility of data management. Scientific analysis of the data can then proceed without the need to organise the input data into the form required for each particular process. Accordingly, a small data base was set up, and the data from the catchments under consideration were loaded into it prior to analysis. Data could then be retrieved in a standard form by reference to a single library subroutine. In addition, a data base access package (DAPPLE) was implemented, which performed all the routine functions, such as loading and listing the data, and general housekeeping.

1. THE DATA BASE

The data base is used to hold the rainfall/runoff data, together with information on catchment characteristics. It consists of a set of random access files, and uses a system of 'pointers' for optimum efficiency. There are three types of record on the data base, namely catchment records, rainfall and runoff records, and parameter records. The catchment records, which are used as a directory, are stored in ascending order of catchment number, and are accessed by a binary search algorithm. The contents of each type of record are as follows.

1.1 Catchment record

| | |
|---|---------------------------------|
| Catchment number | 1 integer |
| Catchment name | 36 alphanumeric characters |
| Total area of catchment (m^2) | 1 real value |
| Paved area within catchment (m^2) | 1 real value |
| Number of events for catchment | 1 integer |
| Time interval between successive observations (seconds) | 1 integer (normally 60 seconds) |

1.2 Rainfall and Runoff records (these have the same form)

| | |
|--------------------------------|---------------------------|
| Event number | 1 integer |
| Start time (in form hh.mm) | 1 real value |
| Date of event (in form ddmmyy) | 1 integer |
| Duration of event (in minutes) | 1 integer |
| Catchment wetness | 4 alphanumeric characters |
| Number of values | 1 integer |
| Rainfall/runoff observations | Up to 200 real values |

1.3 Parameter records

(a parameter record is used to hold information on segments within a particular catchment. A maximum of 10 segments are allowed).

| | |
|--------------------|-----------|
| Number of segments | 1 integer |
|--------------------|-----------|

Characteristics of first segment 10 values
 Characteristics of second segment 10 values

.
 .
 .
 .
 .
 .
 .

(up to a maximum of 10 segments.)

The segment characteristics consist of:

| | |
|---|--------------|
| Segment type (1 \equiv overland flow segment, 2 \equiv gutter flow segment) | 1 integer |
| Length of segment (m) | 1 real value |
| Longitudinal slope of segment | 1 real value |
| Channel side slopes | 1 real value |
| Number of longitudinal increments | 1 integer |
| Overland inflow segment number 1 | 1 integer |
| Overland inflow segment number 2 | 1 integer |
| Upstream inflow segment | 1 integer |

2. DAPPLE

The data base access package (DAPPLE) consists of a set of procedures to load, delete and examine items on the data base. Ease of use has been a primary consideration in the design of the system. The package may be used interactively, and requires a minimum of input from the user. Program flow is directed by commands of one or two words. A list of the available commands is given below.

| COMMAND | FUNCTION |
|------------------|---|
| DELETE | Delete record(s) from the data base |
| FINISH | Indicates the end of DAPPLE input |
| INITIALISE | Initialise the data base |
| LIST | List selected parts of the data base |
| LISTD | List the directory |
| LISTL | List the directory, and all the system pointers |
| LOAD | Load rainfall/runoff data onto the data base |
| PLOT | Plot selected events |
| PRINTING END | Controls the generation of hard copy output |
| PRINTING START | |
| STORE | Load catchment parameters onto the data base |
| STORE PARAMETERS | |
| SUMMARY | Summarise data for selected catchments |

2.1 DELETE

The form of the DELETE command is

```
DELETE
KATCH,ISTORM
```

where KATCH is the required catchment number.

If ISTORM > 0, event number ISTORM is deleted.

ISTORM = 0, all the records pertaining to catchment number KATCH are deleted.

ISTORM < 0, only the catchment parameters are deleted.

```
Example      DELETE
              214,600133
```

Event 600133 is deleted from catchment 214.

2.2 FINISH

No parameters are required for the FINISH command. The system is closed, and any cards input after the FINISH command will not be processed by DAPPLE.

2.3 INITIALISE

This command initialises the database, and should only be used by the person having the responsibility of the database. To prevent illegal use, a key must be input after the command. The command will only be actioned if the correct key is received.

Note: the database should only be initialised once, ie at the start of its life-time, otherwise any information that is on the database will be lost.

2.4 LIST

This command is used to list selected events or catchment parameters. The form of the command is

```
LIST
KATCH,ISTORM
```

where KATCH is the catchment of interest.

If ISTORM > 0, the event with number ISTORM is listed

ISTORM = 0, all events for the catchment are listed

ISTORM < 0, the catchment parameters are listed.

Example LIST
 215,400002

The rainfall and runoff data for event number 400002 from catchment 215 are listed.

If KATCH is set equal to zero, then according to the setting of ISTORM, data for all catchments will be listed.

Thus, if

KATCH = 0 and ISTORM 70, the event with number ISTORM is listed for each catchment having data for this event.

KATCH = 0 and ISTORM = 0, all events on the database are listed

KATCH = 0 and ISTORM < 0, the catchment parameters are listed for all catchments having them.

2.5 LISTD and LISTL

LISTD is used to list the directory. No parameters are required. The listing gives each catchment number, together with the catchment name, total and paved areas, and the events present for the catchment.

LISTL gives the same information as *LISTD*, but in addition, it includes the system pointers, for use in debugging.

2.6 LOAD

This command is used to load the catchment records, and the rainfall and runoff records. The data is presented as a series of events, with one rainfall record per event, and up to six runoff records per event, each runoff record corresponding to a different catchment.

The form of the command is

```
LOAD
MAX,NST,ITI
```

where MAX is the maximum number of runoff records in an event

NST is the stream number that the data will be on, e.g. if the data is following on cards, the NST should be set to 5. Note that 5 will be assumed if NST is set <0.

ITI is the time interval of the data. If set <0, 60 seconds will be assumed.

The data should be in the following format.

MCAT(1),MCAT(2), ,MCAT(MAX)
 CNAME(1)
 CNAME(2)
 :
 :
 :
 CNAME(MAX)
 NC,NEV,TA(1),PA(1),TA(2),PA(2), , TA(MAX),PA(MAX)
 CWET
 repeated RAIN(1),RUN(1,1),RUN(1,2),.....,RUN(1,NR)
 NEV RAIN(2),RUN(2,1)RUN(2,2),.....,RUN(2,NR)
 times :
 :
 :
 RAIN(IDUR+1),RUN(IDUR+1,1)RUN(IDUR+1,2),..... RUN(IDUR+1,NR)

where MCAT(I),I=1,..... MAX are the numbers of the catchments
 involved in the following data
 CNAME(I),I=1 ... MAX are the catchment names. 36 alphanumeric
 characters are allowed. If the catchments
 are already present on the database, these
 may be left blank

NC has no significance, and may be set to zero. (It is
 present to give compatibility with existing data formats)
 NEV is the number of events in the following data
 TA(I),I=1,...MAX are the total areas of the catchments involved
 PA(I),I=1,...MAX are the paved areas of the catchment involved.
 If the catchments are already present on the database,
 these areas may all be set to zero.

ISTORM is the event number of the following event
 START is the starting time, in the form hh.mm,
 e.g.: 21.40
 IDATE is the date, in the form ddmmyy,
 e.g. 141277
 IDUR is the duration of the event in minutes
 NCAT(I),I=1,...NR are the catchment numbers corresponding to
 the following runoff records. They must
 be a subset of the values MCAT(J),J=1,... MAX.
 NCAT(I),I=NR+1,... MAX should be set to zero
 CWET is the antecedent catchment wetness conditions
 (4 alphanumeric characters)

RAIN(I),RUN(I,J) are the rainfall and runoff values. The length of each series should be IDUR+1 (format used is (F7.3,6F6.3), again to give compatibility with existing formats)

2.7 PLOT

This command may be used to plot the rainfall and runoff data from selected events on the Calcomp plotter. The form the command is

```
PLOT
KATCH,ISTORM
```

where KATCH is the catchment number of interest.

If ISTORM > 0, the event with number ISTORM is plotted

ISTORM = 0, all events present for catchment KATCH are plotted.

If KATCH is set to zero, then depending on the setting of ISTORM, data from all events will be plotted.

Thus, if

KATCH = 0 and ISTORM > 0, rainfall and runoff values are plotted for each catchment having data for event ISTORM

KATCH = 0 and ISTORM = 0, the data from all events on the database are plotted.

2.8 PRINTING commands

If DAPPLE is being used from a terminal, then this facility allows print output to be directed to the site line printer, rather than to the terminal. This can be particularly useful when listings of series are required.

The two commands are

```
PRINTING START
```

```
and PRINTING END
```

Any print produced between these two commands will be directed to the line printer. Note that PRINTING END can be omitted if the next instruction is the FINISH command.

2.9 STORE

This command is used to load the catchment parameters onto the database. The command may be specified as either

```
STORE
```

```
or STORE PARAMETERS
```


The command and the required data has the form

```

STORE
KATCH
ISEG
NS(1),SL(1),SS(1),CHW(1),SLOPE(1,1),SLOPE(2,1),NL(1),IS1(1),IS2(1),
                                IUS(1)
NS(2),SL(2),SS(2),CHW(2),SLOPE(1,2),SLOPE(2,2),NL(2),IS1(2),IS2(2),
                                IUS(2)
.
.
.
US(ISEG),SL(ISEG),SS(ISEG),CHW(ISEG),SLOPE(1,ISEG),SLOPE(2,ISEG),
                                IS1(ISEG),IS2(ISEG),
                                IUS(ISEG)

```

| | |
|-------------|--|
| where KATCH | is the appropriate catchment number |
| ISEG | is the number of segments |
| NS(I) | are the segment types |
| SL(I) | are the segment lengths |
| SS(I) | are the segment slopes (longitudinal) |
| CHW(I) | are the channel bottom widths |
| SLOPE(1,I) | is side slope 1 |
| SLOPE(2,I) | is side slope 2 |
| NL(I) | are the number of longitudinal increments |
| IS1(I) | are the first overland inflow segment numbers |
| IS2(I) | are the second overland inflow segment numbers |
| IUS(I) | are the upstream inflow segments |

2.10 SUMMARY

This command gives a summary of the data available for a given catchment. The required input is

```

SUMMARY
KATCH

```

where KATCH is the required catchment number.

If KATCH is set to zero, a summary is given of the data available for all catchments on the data base.

A summary for a given catchment consists of the catchment name and number, and the total and paved areas. Then for each event present for the catchment, the event number is given together with the starting time and date and the duration, the catchment wetness, and the rainfall and runoff volumes.

APPENDIX B : URBAN HYDROLOGY MODELLING PACKAGE

1. INTRODUCTION

The Urban Hydrology Modelling Package (UHMP) has been developed at the Institute for the specific purpose of performing modelling assignments on rainfall-runoff data from urban subcatchments. It interacts with a custom-built data base system, which retrieves rainfall and runoff data for any number of specified events on a given catchment. A variety of different modelling assignments can be performed, using a comprehensive range of models. The data specifications, modelling assignments and model options are discussed under separate headings, which are followed by details of how to feed the run specifications into the package. Finally, there is a section which describes the structure of the package which, although not a prerequisite to the running, is included since a passing knowledge is helpful.

2. DATA SPECIFICATIONS

The data base is set up such that up to 18 events on any catchment may be stored. There is a limit of 2000 ordinates per event. (Any time interval data may be used). Only one catchment may be nominated in any single package run, but any number of events up to the limit of 18. Precise details on how to feed in this information may be found under the run specifications section.

3. MODELLING ASSIGNMENTS

There are three modes in which the package may be run: simulation, optimisation and error surface mapping.

Simulation: Specified events are modelled using up to 10 specified model parameters.

Optimisation: Certain model parameters may be designated to be allowed to float within specified limits. A Rosenbrock optimisation routine is used to determine the set of optimum parameter values according to the minimisation of some designated objective function. There is a further option to optimise the model on individual specified events or to derive a global optimum on combined events.

Error surface mapping: The interrelationship of certain specified parameters may be investigated by plotting a 6 x 6 grid of objective function values on two axes relating to the parameters specified. Contours may then be drawn by hand to obtain an error response surface. If 2 parameters are specified then 1 such plot will result, if 3 then 3 plots will result, if 4 then 6 plots will result, and so on. As with the optimisation case, mapping may be done on individual events or on combined events.

Assignment option: Choice of assignment option is determined by specifying as follows:

- 1 = simulation run
- 2 = optimisation run (on individual events)
- 3 = optimisation run (on combined events)
- 4 = mapping run (on individual events)
- 5 = mapping run (on combined events)

Objective functions: There are 6 choices of objective functions, which are designated by the following options:

| Option | Objective Function |
|--------|--|
| 1: | Integral Square Error, least squares $\rightarrow \left \sum (Q_{OBS} - Q_{MOD})^2 \right ^{1/2} / \sum Q_{OBS} \times 100\%$ |
| 2: | biased least squares $\rightarrow \left \sum Q_{OBS}^2 - Q_{MOD}^2 \right ^{1/2} / \sum Q_{OBS} \times 100\%$ |
| 3: | peak estimation $\rightarrow \left PEAK_{OBS} - PEAK_{MOD} \right / PEAK_{OBS} \times 100\%$ |
| 4: | volume estimation $\rightarrow \left VOL_{OBS} - VOL_{MOD} \right / VOL_{OBS} \times 100\%$ |
| 5: | time to peak $\rightarrow TTP_{OBS} - TTP_{MOD}$ |
| 6: | As 1, but only over the range $Q_{OBS} > \frac{1}{2} \times PEAK_{OBS}$ |

Option 1 is a traditional least squares function. Option 2 is similar but applies more weight to the high flows than to the low flows of a given hydrograph. In practice, these two functions are very highly correlated. Options 3 and 4 relate to hydrograph peak and hydrograph volume respectively. Further options could easily be accommodated.

4. MODEL OPTIONS

A number of different model options may be specified. The rainfall-runoff model is divided into two, a loss submodel (choice of 8) and a surface routing submodel (choice of 6). A detailed description of the particular models may be found elsewhere. There are three options which determine the model used, 2 related to the loss submodel and 1 to the surface routing model:

Loss model:

| Option | | No. of Paras. | Parameter Names |
|--------|--|------------------|---------------------|
| 1 | CONSTANT CONTRIBUTING AREA | 2 | DEPSTOG <u>AREA</u> |
| 2 | CONSTANT PROPORTIONAL LOSS (PAVED AREA) | 2 | DEPSTOG, <u>CP</u> |
| 3 | CONST. PROPORTIONAL LOSS (TOTAL AREA) | 2 | DEPSTOG, <u>CP</u> |

| Option | | No. of Paras. | Parameter Names |
|--------|-------------------------------------|------------------|--------------------------------------|
| 4 | PHI INDEX (PAVED AREA) | 2 | DEPSTOG, <u>PHI</u> |
| 5 | PHI INDEX (TOTAL AREA) | 2 | DEPSTOG, <u>PHI</u> |
| 6 | VARIABLE CONTRIBUTING AREA | 3 | DEPSTOG, <u>AO</u> , TANTHETA |
| 7 | VARIABLE PROP. LOSS (PAVED AREA) | 3 | <u>ALPHA</u> , <u>ZO</u> , <u>ZE</u> |
| 8 | VARIABLE PROP. LOSS (TOTAL AREA) | 3 | <u>ALPHA</u> , <u>ZO</u> , <u>ZE</u> |

A second option is used to define whether or not the volumes are forced. If the observed runoff is used to determine the volume of net rainfall, then one of the parameters above becomes redundant. The underlined one relating to each option is used such that its value ensures that the net rainfall volume equals that of the runoff volume, and is thus not entered into the run specifications if the volume forcing option is selected:

| Option | |
|--------|--------------------|
| 0 | Volumes not forced |
| 1 | Volumes are forced |

Surface routing submodel:

| <u>Option</u> | <u>Model Name</u> | <u>No. of Paras.</u> | <u>Parameter Names</u> |
|---------------|----------------------|--------------------------|------------------------------|
| 1 | NONLINEAR RESERVOIR | 2 | KVALUE, NVALUE |
| 2 | KINEMATIC WAVE | 2n | CHEZYC, EXPON for n elements |
| 3 | TIME OF ENTRY | 1 | TE |
| 4 | VARIABLE K MUSKINGUM | 3 | CVALUE, NVALUE, EPSLN |
| 5 | NASH CASCADE | 2 | KVALUE, NVALUE |
| 6 | NL RES WITH TIME LAG | 3 | KVALUE, NVALUE, TAU |
| 7 | LINEAR RESERVOIR | 1 | CVALUE |

The linear reservoir model is a special case of option 1 or 5. Further options could be easily accommodated.

5: RUN SPECIFICATIONS

Input to the package is as follows:

| CARD | ITEM | NAME | FORMAT | COLUMNS |
|--------|---|----------|--------|---------|
| 1 | Catchment number | KATCH | I4 | 1-4 |
| | Number of events to be modelled | NOEV | I3 | 5-7 |
| | Plot option (0=no plot, 1=plot) | IPLOT | I2 | 8-9 |
| | Objective function (see See section 3) | IOBJF | I2 | 10-11 |
| | Modelling assignment (see section 3) | JOPTI | I2 | 12-13 |
| | No. of parameters | NPAR | I2 | 14-15 |
| | Maximum no. of iterations of opt. routine | MITTS | I2 | 16-17 |
| 2 | Event number (up to NOEV) | IEV(K) | 10I7 | 1-70 |
| 3 | Event numbers (if NOEV > 10) | IEV(K) | 10I7 | 1-70 |
| 4 | Loss submodel option (see Section 4) | IOPTI | I2 | 1-2 |
| | Loss submodel name | ALOSS | A25 | 3-27 |
| | Surface routing submodel option (see Section 4) | IOPT2 | I2 | 28-29 |
| | Surface routing submodel name | ASURF | A20 | 30-49 |
| | Volume forcing option | IOPT3 | I2 | 50-51 |
| 5+ | Parameter number | NP | I5 | 1-5 |
| NPAR+4 | Optimisation order (EO for fixed value) | IP(NP) | I5 | 6-10 |
| | Parameter name | NAME(NP) | AB | 11-18 |
| | Starting or fixed parameter value | P(NP) | F10.3 | 19-28 |
| | Minimum parameter value | PMIN(NP) | F10.3 | 29.38 |
| | Maximum parameter value | PMAX(NP) | F10.3 | 39.48 |

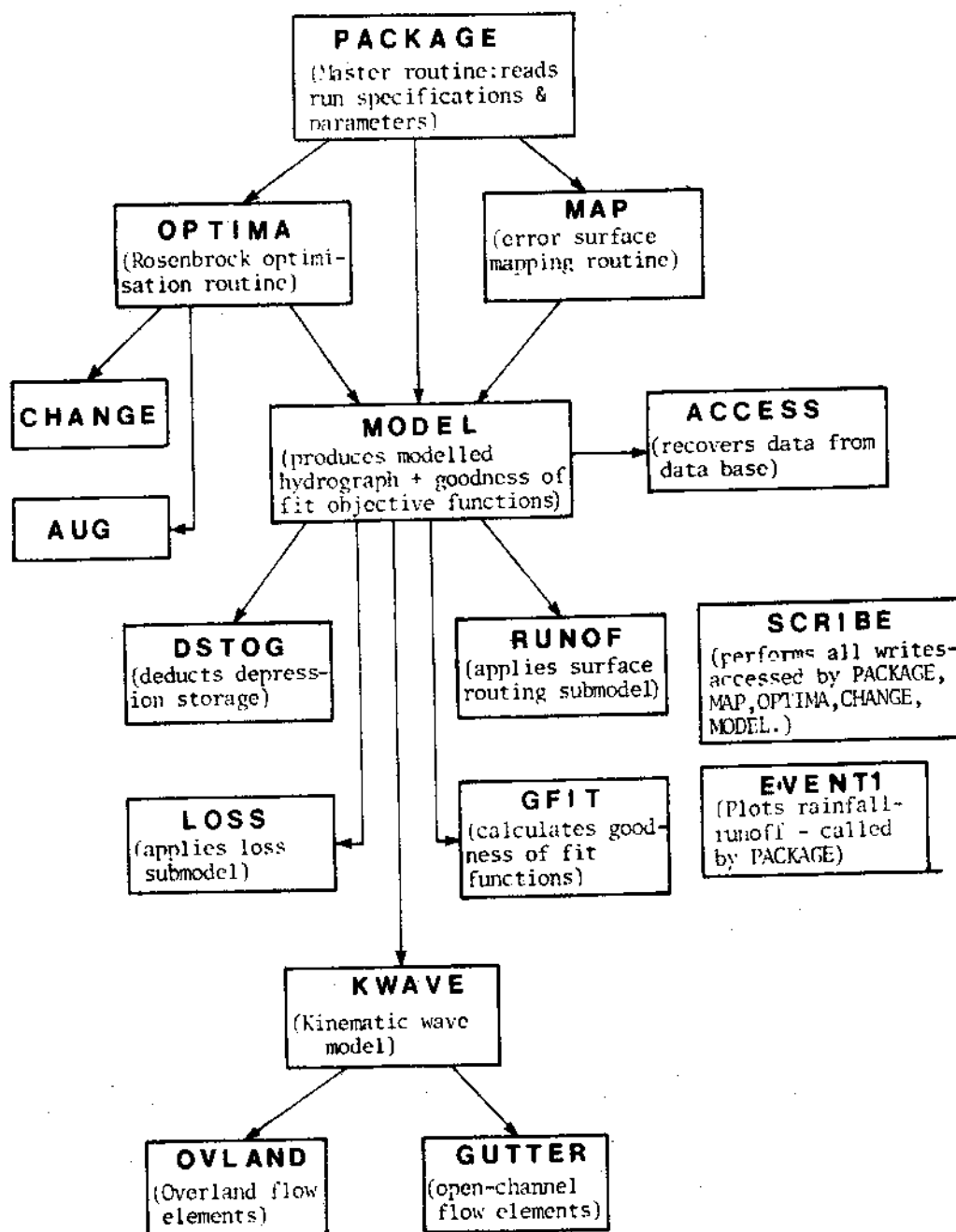
Parameters must be entered in the order in which they are to be used. Definition of depression storage is mandatory. It should be entered as the first parameter, and be set equal to zero if no depression storage is required. Where there are more several parameters for a given submodel they must be entered in the order specified in Section 4.

6: PACKAGE STRUCTURE

The structure of the package is illustrated in Figure A1. The routine PACKAGE is the master routine. Subroutines OPTIMA and MAP are called according to the modelling assignment selected. MODEL is called by 1 of these three routines as appropriate, and MODEL calls

ACCESS when data retrieval is required. OPTIMA has 2 subroutines (AUG & CHANGE), & MODEL calls upon 7 subroutines (DSTOG, LOSS, RUNOF, GFIT, KWAVE, OVLAND and GUTTER). All line-printer assignments are performed by SCRIBE, and EVENT1 is an optional routine which produces a plot of observed and modelled rainfall and runoff on the CALCOMP plotter. In the figure, the direction of the arrows indicates which of 2 routines has called the other. Comment cards have been liberally distributed in each routine.

FIGURE A1 URBAN HYDROLOGY MODELLING PACKAGE



APPENDIX C : URBAN RAINFALL RUNOFF MODELS

INTRODUCTION

The transformation of rainfall (input) to runoff (output) is accomplished by the application in series of two sub-models; the LOSS sub-model and the SURFACE ROUTING sub-model.

Within the computer package there exist a number of these sub-models, each one associated with a particular option number. For any run, the values assigned to these option numbers specify which LOSS sub-model and which SURFACE ROUTING sub-model are to be used for the rainfall/runoff transformation.

LOSS SUB-MODELS

The loss sub-model defines (a) net rainfall in mm/hr and (b) contributing area in m^2 .

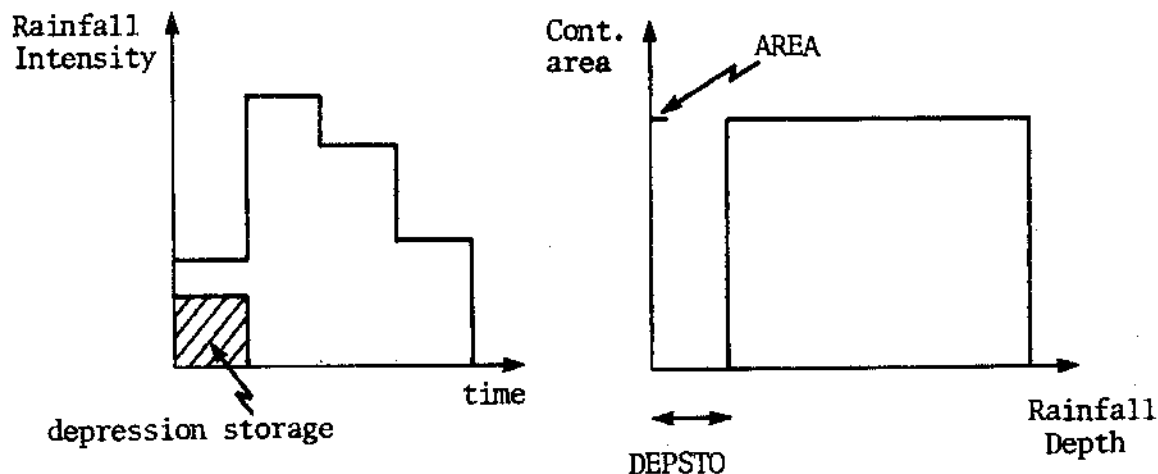
Some sub-models are subdivided according to the definition of contributing area, the methods for calculating the net rainfall being identical. This division relates to whether the contributing area is equal to the paved area or the total area of the subcatchment. The sub-division is only relevant to the British data, since paved and total area are the same for the Dutch and Swedish data.

The analysis is further divided according to whether or not volumes are to be forced. For any storm event, rainfall and runoff are observed and, hence, total rainfall and total runoff are both known. This knowledge will permit one of the parameters of the loss sub-model to be determined rather than simply assigning a predicted value to it. Within each sub-model there exists a 'switch' option which controls whether volumes are to be forced or whether parameter values are to be assigned.

Below is a short description of each sub-model. The sub-models are numbered according to their option numbers. There follows a table which summarises the models and the parameters required for each.

1. CONSTANT CONTRIBUTING AREA

Net rainfall is rainfall less depression storage; contributing area is defined, and is constant through the event.

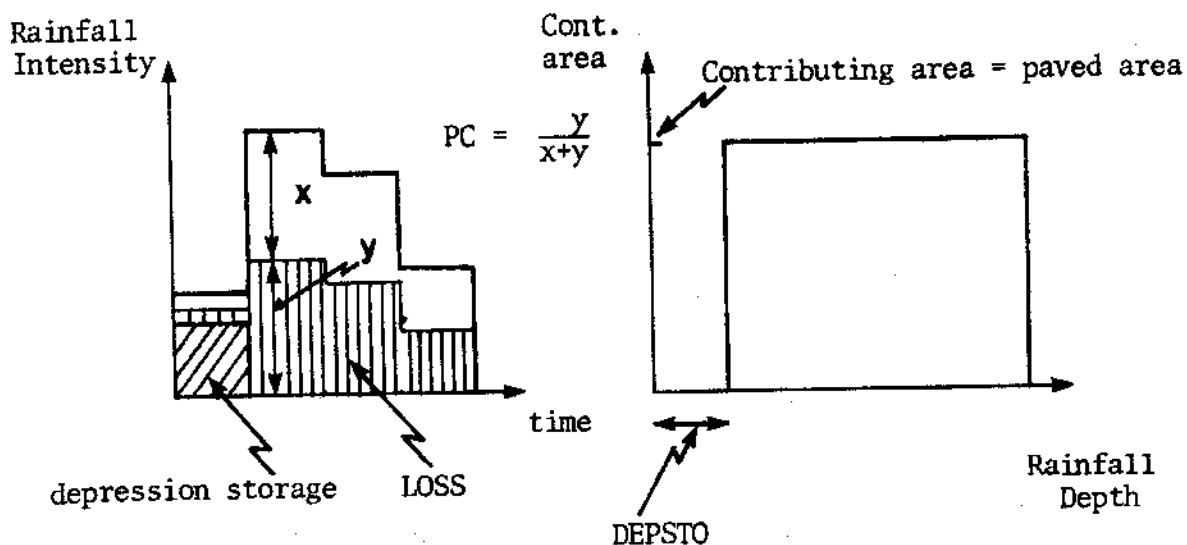


2 parameter model: AREA, DEPSTO

When volumes are forced: $AREA = ROVOL / (RFD - DEPSTO)$

2. CONSTANT PROPORTIONAL LOSS (A)

Net rainfall is some constant proportion of the rainfall input, and contributing area is constant.



Net rainfall = (Rainfall - depression storage) x (1.0 - PC)

2 parameter model : PC, DEPSTO

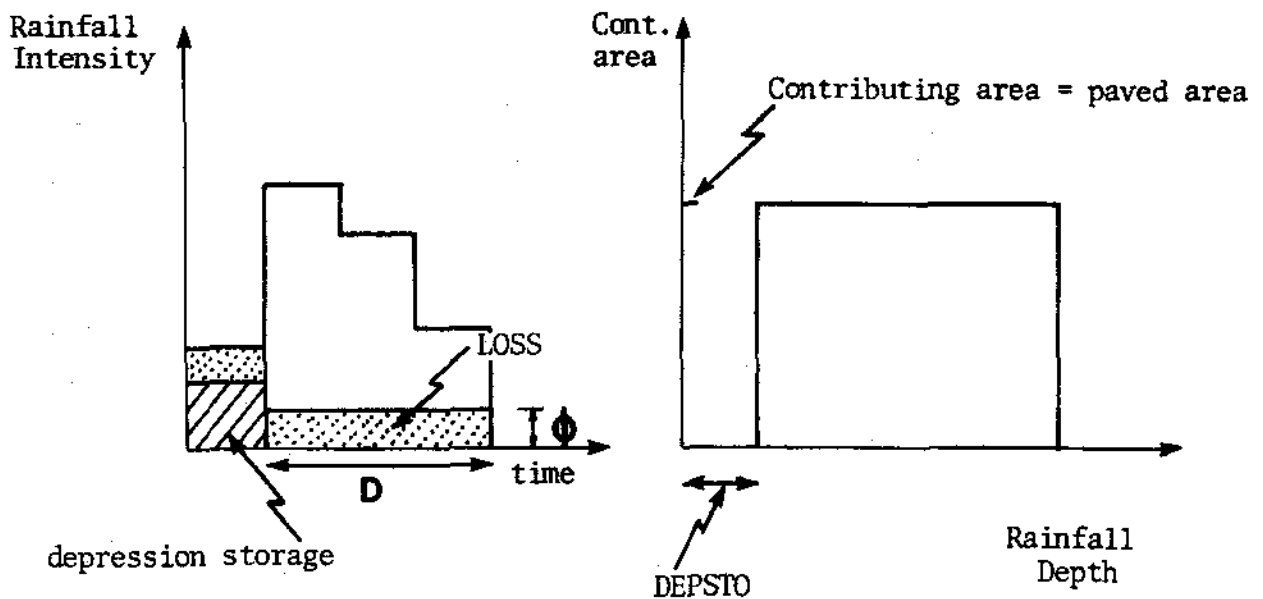
When volumes forced: $PC = 1.0 - ROVOL / ((RFD - DEPSTO) \times \text{Cont. area})$

3. CONSTANT PROPORTIONAL LOSS (B)

Same as 2 except that contributing area = total area

4. PHI INDEX (A)

Net rainfall is rainfall input less some allowance which remains constant through an event; contributing area is also constant.



$$\text{Net rainfall} = \text{Rainfall} - \text{depression storage} - \phi$$

2 parameter model : PHI, DEPSTO

$$\text{When volumes forced: } \text{PHI} = \frac{1}{D} \left[\text{RFD} - \text{DEPSTO} - (\text{ROVOL} / \text{Cont. area}) \right]$$

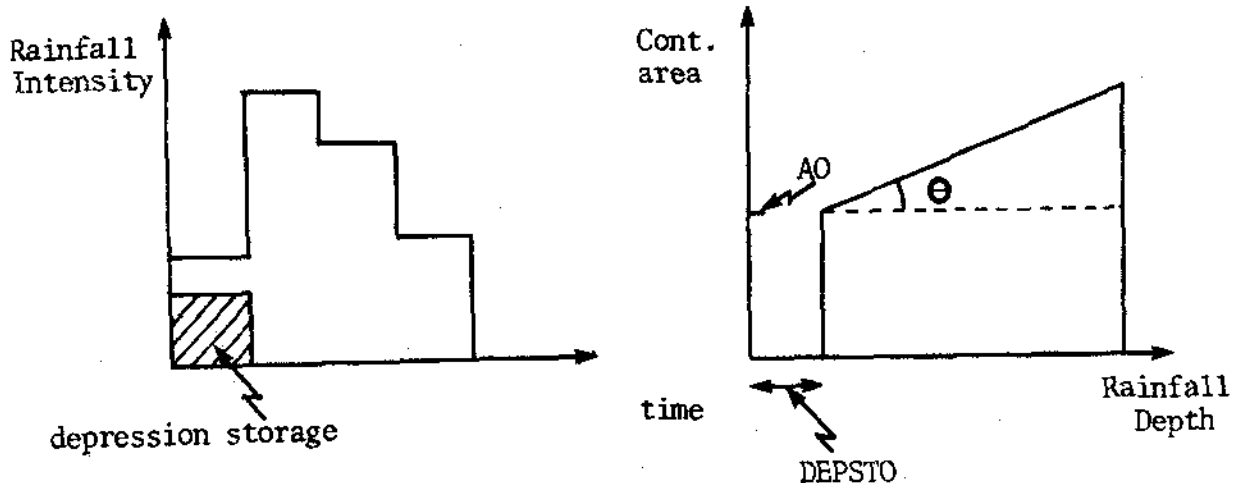
where D = duration of net rainfall

5. PHI INDEX (B)

Same as 4 except that contributing area - total area.

6. VARIABLE CONTRIBUTING AREA

This model allows the runoff volume to be equivalently greater towards the end of the event than near the beginning. Net rainfall is rainfall less depression storage, and contributing area increases through the event.

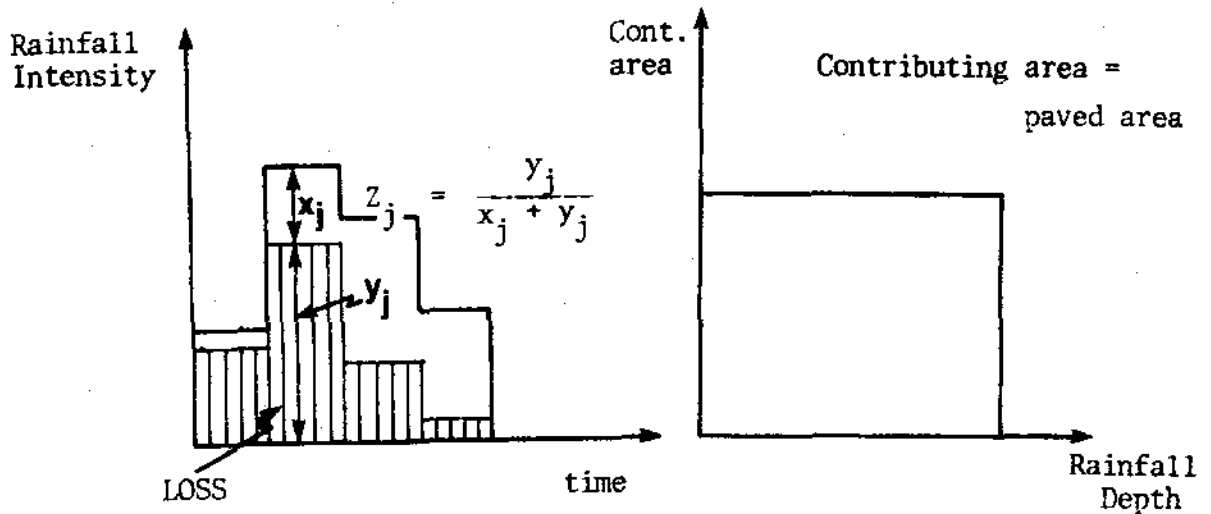


3 parameter model: $AO, \theta, DEPSTO$

When volumes are forced: $AO = \frac{ROVOL}{(RFD - DEPSTO)} - \frac{(RFD - DEPSTO) \tan \theta}{2}$

7. VARIABLE PROPORTIONAL LOSS (A)

This model also allows the response volume to increase through the events; contributing area remains constant, while proportional loss decreases through the event.



$$\text{Fraction of loss, } z_j = ZE + (ZO - ZE)e^{-\alpha r_j}$$

where ZO = fraction of loss at start of rainfall
 ZE = fraction of loss at end of rainfall

$$\text{and } r_j = \frac{\sum_{i=1}^j p_i}{P} \quad \text{where } p_i \text{ is amount of rainfall in } i\text{th period } \Delta t$$

and P is the total rainfall.

α is a constant

$$\text{Net rainfall} = \text{Rainfall} \times (1.0 - z_j)$$

3 parameter model : ZO, ZE, α

When volumes are forced, α is defined.

8. VARIABLE PROPORTIONAL LOSS (B)

Same as for 7 except that contributing area = total area

| OPTION NUMBER | SUB.MODEL NAME | NO. OF 'PARAMETERS' | PARAMETERS |
|------------------|--------------------------------|------------------------|---------------------------|
| 1 | CONSTANT CONTRIBUTING AREA | 2 | DEPSTO, <u>CCA</u> |
| 2 | CONSTANT PROPORTIONAL LOSS (A) | 2 | DEPSTO, <u>PC</u> |
| 3 | CONSTANT PROPORTIONAL LOSS (B) | 2 | DEPSTO, <u>PC</u> |
| 4 | PHI INDEX (A) | 2 | DEPSTO, <u>PHI</u> |
| 5 | PHI INDEX (B) | 2 | DEPSTO, <u>PHI</u> |
| 6 | VARIABLE CONTRIBUTING AREA | 3 | DEPSTO, <u>AO</u> , THETA |
| 7 | VARIABLE PROPORTIONAL LOSS (A) | 3 | <u>ALPHA</u> , ZO, ZE |
| 8 | VARIABLE PROPORTIONAL LOSS (B) | 3 | <u>ALPHA</u> , ZO, ZE |

*Those parameters underlined indicate those parameters calculated when volumes are forced. In the computing package, parameters should be specified in the order shown and before the surface routing parameters.

SURFACE ROUTING SUB-MODELS

These sub-models define the runoff in litres/sec.

With the one exception of the Kinematic Wave sub-model, all the surface routing sub-models are lumped models. Surface routing is, in all cases, accomplished by some combination of translation and storage routing.

Below is a brief description of each sub-model. The sub-models are numbered according to their option numbers. There follows a table which summarises the models and the parameters required.

1. NON-LINEAR RESERVOIR

Storage Equation : $S = KQ^n$

Continuity Equation : $\frac{dS}{dt} = I - Q$

Routing Equation: $I - Q - nKQ^{n-1} \frac{dQ}{dt} = 0$

2 parameter model: n. K. All storage routing, no translation. Linear reservoir is special case with $n = 1$; K is then time lag (hours).

2. KINEMATIC WAVE

Derivative of the full St. Venant Equations - ignores diffusion and acceleration terms in dynamic equation - justified where lateral inflow predominates.

The Chezy equation is assumed to be a satisfactory example for the dynamic equation:

e.g. $V = Ch^n$ for overland flow

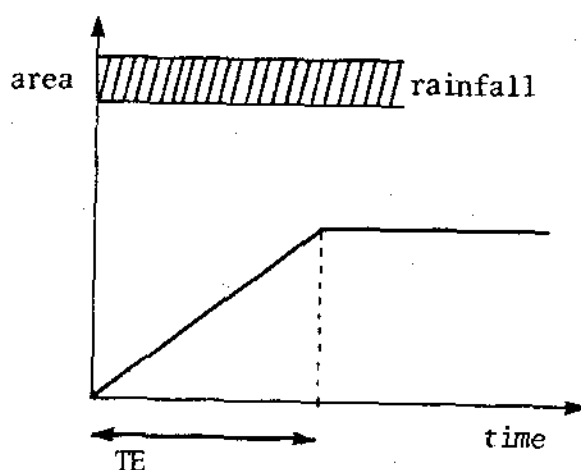
where C is the Chezy constant

and n is the exponent ($= \frac{1}{2}$ for the Chezy eqn)

The subcatchment may be divided into up to 10 elements either of overland flow or of open-channel gutter flow. Parameters of the model are the Chezy constant and exponent for each element. The disadvantage of the model is that much data (slopes, lengths, dimensions for each element) and runtime are required, but it should simulate better due to its being a distributed model. Computer package effectively limits up to 4 elements; so far only tried where division into elements closely matches reality.

3. TIME OF ENTRY

Traditional model used in design in the U.K. Equivalent to routing the input through a linear time-area diagram of time base equal to the time of entry:



1 parameter model: TE Pure translation, no dynamic storage

4. VARIABLE K MUSKINGUM

Introduced because nonlinear reservoir seems to peak early for British catchments. Therefore some translation required.

$$\text{Storage Equation: } S = K(\epsilon I + (1-\epsilon)Q)$$

$$\text{Continuity Equation: } \frac{dS}{dt} = I - Q$$

The lagtime K is assumed to vary with the outflow in the following manner

$$K = C \left(\frac{1}{Q}\right)^n$$

3 parameter model: C, n, ϵ

where ϵ governs degree of translation, n governs degree of non-linearity

NB/NON-LINEAR RESERVOIR is a special case with $n = -n$
and $\epsilon = 0$

5. NASH CASCADE

2 parameter linear model. Storage routing through n equal linear reservoirs each with routing constant equal to K.

Lag time of response = nK

The impulse response of the Nash model is:

$$u(o,t) = \frac{1}{K\Gamma(n)} \cdot e^{-t/K} \cdot (t/K)^{n-1}$$

where $\Gamma(n)$ is the gamma function of n

6. NON-LINEAR RESERVOIR WITH TIME LAG

$$\text{Storage Equation: } S_{t-\tau} = KQ_t^n$$

$$\text{Continuity Equation: } \frac{dS}{dt} = I - Q$$

3 parameter model : K, n, τ .

Advantages are a) easier to program than non-linear reservoir
b) introduces some element of translation

| OPTION NO. | SUB-MODEL NAME | NO. OF PARAS | PARAMETERS |
|---------------|---------------------------|-----------------|---|
| 1 | NON-LINEAR RESERVOIR | 2 | KVALUE, NVALUE |
| 2 | KINEMATIC WAVE | 2 x M | CHEZYC, EXPON for each of M elements |
| 3 | TIME OF ENTRY | 1 | TE |
| 4 | VARIABLE K MUSKINGUM | 3 | C VALUE, NVALUE, EPSLN |
| 5 | NASH CASCADE | 2 | KVALUE, NVALUE |
| 6 | NONLIN RES. WITH TIME LAG | 3 | KVALUE, NVALUE, TAU |

Parameters must be entered into the package in the order shown, and after the loss submodel parameters.

APPENDIX D : OPTIMISATION DATA MATRIX

| CATCHMENT CHARACTERISTICS | | | | | STORM CHARACTERISTICS | | OPTIMUM MODEL PARAMETERS | | | | | | TE |
|---------------------------|------------------------------|-----------|---------------------|---|-----------------------|----------------|--------------------------|-------------|------------|--------------|------------|---------------|------|
| Catchment number | Paved area (m ²) | Slope (%) | Overland length (m) | | Rainfall volume (mm) | Duration (min) | MSI (mm/h) | C (LIN RES) | K (NL RES) | K (NLR + TL) | N (NASH C) | C (MUSKINGUM) | |
| * 311 175 1.0 12 * | 175 | 1.0 | 12 | * | 15 | 78 | 35 | * 2.45 | .163 | .174 | .99 | .088 | 8 * |
| * 311 196 1.0 12 * | 196 | 1.0 | 12 | * | 6 | 12 | 41 | * 3.63 | .265 | .255 | 1.27 | .134 | 10 * |
| * 311 175 1.0 12 * | 175 | 1.0 | 12 | * | 12 | 75 | 21 | * 2.41 | .159 | .154 | 1.15 | .091 | 9 * |
| * 311 196 1.0 12 * | 196 | 1.0 | 12 | * | 7 | 58 | 46 | * 4.08 | .283 | .276 | 1.43 | .143 | 11 * |
| * 311 175 1.0 12 * | 175 | 1.0 | 12 | * | 6 | 24 | 41 | * 2.12 | .157 | .136 | .90 | .081 | 7 * |
| * 311 196 1.0 12 * | 196 | 1.0 | 12 | * | 12 | 41 | 62 | * 2.84 | .204 | .202 | .91 | .099 | 9 * |
| * 311 175 1.0 12 * | 175 | 1.0 | 12 | * | 6 | 10 | 59 | * 4.74 | .326 | .371 | 1.63 | .150 | 12 * |
| * 311 196 1.0 12 * | 196 | 1.0 | 12 | * | 11 | 81 | 52 | * 2.66 | .260 | .227 | 1.02 | .115 | 8 * |
| * 311 175 1.0 12 * | 175 | 1.0 | 12 | * | 10 | 59 | 51 | * 2.70 | .214 | .228 | 1.00 | .106 | 7 * |
| * 311 196 1.0 12 * | 196 | 1.0 | 12 | * | 7 | 16 | 54 | * 2.30 | .158 | .163 | 1.00 | .082 | 7 * |
| * 2032 320 3.1 25 * | 320 | 3.1 | 25 | * | 5 | 14 | 31 | * 2.25 | .149 | .125 | 1.25 | .080 | 7 * |
| * 2032 320 3.1 25 * | 320 | 3.1 | 25 | * | 4 | 21 | 29 | * 1.49 | .113 | .097 | .78 | .060 | 7 * |
| * 2032 320 3.1 25 * | 320 | 3.1 | 25 | * | 2 | 20 | 3 | * 1.54 | .108 | .111 | 1.40 | .076 | 7 * |
| * 2032 320 3.1 25 * | 320 | 3.1 | 25 | * | 2 | 26 | 10 | * 1.72 | .112 | .099 | 1.50 | .079 | 8 * |
| * 2032 320 3.1 25 * | 320 | 3.1 | 25 | * | 6 | 45 | 26 | * .95 | .070 | .104 | .68 | .034 | 4 * |
| * 2032 320 3.1 25 * | 320 | 3.1 | 25 | * | 1 | 10 | 12 | * 1.57 | .125 | .097 | 1.42 | .083 | 7 * |
| * 2033 70 3.0 11 * | 70 | 3.0 | 11 | * | 15 | 36 | 47 | * 3.35 | .242 | .221 | 1.70 | .111 | 8 * |
| * 2033 90 3.0 11 * | 90 | 3.0 | 11 | * | 1 | 5 | 14 | * 3.82 | .262 | .223 | 1.61 | .184 | 24 * |
| * 2033 70 3.0 11 * | 70 | 3.0 | 11 | * | 2 | 11 | 15 | * .33 | .030 | .036 | .87 | .017 | 4 * |
| * 2033 90 3.0 11 * | 90 | 3.0 | 11 | * | 1 | 19 | 11 | * .43 | .076 | .055 | .88 | .047 | 1 * |
| * 2033 70 3.0 11 * | 70 | 3.0 | 11 | * | 2 | 13 | 11 | * 1.54 | .117 | .103 | 1.00 | .080 | 6 * |
| * 2033 90 3.0 11 * | 90 | 3.0 | 11 | * | 1 | 13 | 9 | * 2.21 | .152 | .130 | 1.84 | .109 | 9 * |
| * 2033 70 3.0 11 * | 70 | 3.0 | 11 | * | 2 | 27 | 9 | * 1.47 | .133 | .119 | 1.21 | .094 | 9 * |
| * 2033 90 3.0 11 * | 90 | 3.0 | 11 | * | 1 | 23 | 8 | * 2.10 | .200 | .162 | 1.72 | .127 | 10 * |
| * 2033 70 3.0 11 * | 70 | 3.0 | 11 | * | 2 | 21 | 9 | * 2.75 | .201 | .199 | 2.70 | .139 | 12 * |
| * 2042 450 2.4 25 * | 450 | 2.4 | 25 | * | 5 | 13 | 35 | * 2.04 | .142 | .123 | .94 | .076 | 7 * |
| * 2042 450 2.4 25 * | 450 | 2.4 | 25 | * | 3 | 31 | 9 | * 1.53 | .110 | .102 | 1.31 | .079 | 9 * |
| * 2042 450 2.4 25 * | 450 | 2.4 | 25 | * | 3 | 25 | 17 | * 1.30 | .105 | .085 | .90 | .064 | 7 * |
| * 2042 450 2.4 25 * | 450 | 2.4 | 25 | * | 14 | 123 | 15 | * 1.59 | .141 | .119 | .96 | .079 | 7 * |
| * 2042 450 2.4 25 * | 450 | 2.4 | 25 | * | 1 | 16 | 7 | * 1.59 | .140 | .119 | 1.39 | .094 | 9 * |
| * 2042 450 2.4 25 * | 450 | 2.4 | 25 | * | 2 | 46 | 13 | * 1.40 | .145 | .125 | 1.10 | .096 | 7 * |
| * 2042 450 2.4 25 * | 450 | 2.4 | 25 | * | 3 | 23 | 16 | * 1.79 | .141 | .120 | 1.21 | .083 | 7 * |
| * 2042 450 2.4 25 * | 450 | 2.4 | 25 | * | 1 | 20 | 7 | * 2.05 | .174 | .157 | 1.57 | .115 | 10 * |
| * 2042 450 2.4 25 * | 450 | 2.4 | 25 | * | 2 | 39 | 11 | * 1.58 | .137 | .113 | 1.18 | .088 | 7 * |
| * 2042 450 2.4 25 * | 450 | 2.4 | 25 | * | 1 | 14 | 11 | * 1.40 | .118 | .117 | 1.09 | .076 | 7 * |
| * 2042 450 2.4 25 * | 450 | 2.4 | 25 | * | 1 | 13 | 9 | * 1.08 | .088 | .089 | .98 | .089 | 7 * |
| * 2051 345 2.2 45 * | 345 | 2.2 | 45 | * | 1 | 16 | 9 | * 4.75 | .266 | .302 | 1.02 | .204 | 13 * |
| * 2051 346 2.2 45 * | 346 | 2.2 | 45 | * | 1 | 5 | 8 | * 2.19 | .144 | .124 | 2.19 | .125 | 10 * |
| * 2051 345 2.2 45 * | 345 | 2.2 | 45 | * | 2 | 25 | 9 | * 3.43 | .195 | .133 | 2.78 | .154 | 14 * |
| * 2051 346 2.2 45 * | 346 | 2.2 | 45 | * | 4 | 32 | 16 | * 1.29 | .084 | .082 | .89 | .053 | 6 * |
| * 2051 345 2.2 45 * | 345 | 2.2 | 45 | * | 3 | 35 | 10 | * 1.57 | .114 | .091 | 1.42 | .082 | 7 * |
| * 2051 346 2.2 45 * | 346 | 2.2 | 45 | * | 11 | 99 | 25 | * 2.54 | .182 | .155 | 1.43 | .105 | 8 * |

| | | | | | | | | | |
|---------------------|----|-----|----|--------|------|------|------|------|------|
| * 2051 245 2.2 45 * | 11 | 75 | 19 | * 2.50 | .192 | .177 | 1.70 | .105 | 8 * |
| * 2061 287 1.7 16 * | 1 | 16 | 3 | * 1.18 | .093 | .086 | 1.08 | .072 | 9 * |
| * 2051 287 1.7 15 * | 2 | 15 | 21 | * 1.12 | .112 | .091 | 1.14 | .067 | 7 * |
| * 2051 287 1.7 16 * | 7 | 32 | 6 | * 1.56 | .117 | .104 | 1.53 | .084 | 10 * |
| * 2051 287 1.7 15 * | 5 | 45 | 5 | * 2.52 | .197 | .157 | 1.99 | .127 | 14 * |
| * 2051 287 1.7 16 * | 1 | 10 | 4 | * 1.76 | .136 | .115 | 1.70 | .106 | 11 * |
| * 2051 287 1.7 15 * | 1 | 12 | 7 | * 2.35 | .225 | .184 | 2.56 | .193 | 14 * |
| * 2061 287 1.7 16 * | 15 | 37 | 23 | * 2.03 | .147 | .127 | 1.93 | .082 | 7 * |
| * 2051 287 1.7 15 * | 0 | 37 | 3 | * 2.34 | .185 | .154 | 2.27 | .149 | 12 * |
| * 2047 287 1.7 16 * | 2 | 21 | 11 | * 3.55 | .269 | .260 | 2.15 | .170 | 14 * |
| * 2051 287 1.7 15 * | 1 | 9 | 12 | * 3.44 | .245 | .237 | 2.11 | .171 | 14 * |
| * 2051 287 1.7 16 * | 2 | 12 | 16 | * 3.31 | .235 | .210 | 1.71 | .154 | 12 * |
| * 2052 297 .9 51 * | 2 | 15 | 20 | * 3.75 | .207 | .179 | 1.77 | .137 | 11 * |
| * 2062 297 .9 51 * | 2 | 30 | 5 | * 3.55 | .254 | .240 | 2.15 | .171 | 19 * |
| * 2052 297 .9 51 * | 1 | 47 | 5 | * 5.20 | .345 | .330 | 2.08 | .252 | 20 * |
| * 2052 297 .9 50 * | 2 | 21 | 11 | * 3.50 | .217 | .189 | 2.17 | .159 | 12 * |
| * 2062 297 .9 50 * | 1 | 14 | 3 | * 2.45 | .214 | .196 | 2.14 | .176 | 16 * |
| * 2052 297 .9 50 * | 1 | 12 | 7 | * 3.42 | .205 | .170 | 2.21 | .168 | 12 * |
| * 2062 297 .9 50 * | 15 | 30 | 23 | * 2.25 | .171 | .145 | 1.74 | .151 | 6 * |
| * 2052 297 .9 51 * | 0 | 37 | 8 | * 2.72 | .253 | .222 | 2.15 | .171 | 12 * |
| * 2052 297 .9 50 * | 0 | 21 | 11 | * 2.20 | .171 | .157 | 1.70 | .116 | 12 * |
| * 2052 297 .9 51 * | 1 | 5 | 12 | * 2.35 | .150 | .123 | 1.58 | .109 | 9 * |
| * 2062 297 .9 50 * | 2 | 12 | 16 | * 2.30 | .154 | .137 | 1.55 | .102 | 9 * |
| * 4175 291 2.1 9 * | 1 | 12 | 5 | * 1.43 | .145 | .131 | 1.35 | .099 | 11 * |
| * 4175 291 2.1 9 * | 5 | 64 | 13 | * 1.25 | .114 | .100 | 1.26 | .064 | 6 * |
| * 4175 291 2.1 9 * | 5 | 95 | 14 | * 1.03 | .095 | .085 | 1.05 | .050 | 5 * |
| * 4175 291 2.1 9 * | 3 | 71 | 10 | * 1.12 | .100 | .082 | 1.07 | .057 | 6 * |
| * 4175 291 2.1 9 * | 3 | 38 | 14 | * 1.05 | .085 | .067 | 1.05 | .050 | 5 * |
| * 4175 291 2.1 9 * | 4 | 28 | 14 | * .97 | .107 | .084 | 1.05 | .055 | 5 * |
| * 4175 291 2.1 9 * | 2 | 41 | 3 | * .82 | .111 | .083 | 1.05 | .070 | 7 * |
| * 4175 291 2.1 9 * | 2 | 40 | 4 | * .90 | .104 | .085 | 1.01 | .066 | 7 * |
| * 4175 291 2.1 9 * | 5 | 72 | 14 | * 1.43 | .140 | .141 | 1.34 | .079 | 9 * |
| * 4175 291 2.1 9 * | 1 | 31 | 7 | * .74 | .087 | .074 | 1.04 | .052 | 6 * |
| * 4175 291 2.1 9 * | 2 | 10 | 23 | * .71 | .083 | .074 | 1.02 | .042 | 4 * |
| * 4175 325 .9 27 * | 16 | 31 | 55 | * 3.50 | .270 | .235 | 1.84 | .127 | 10 * |
| * 4175 325 .9 27 * | 3 | 56 | 5 | * 5.10 | .317 | .214 | 2.17 | .243 | 25 * |
| * 4175 325 .9 27 * | 9 | 192 | 16 | * 3.28 | .219 | .196 | 2.04 | .134 | 12 * |
| * 4175 325 .9 27 * | 3 | 15 | 15 | * 5.57 | .265 | .257 | 2.49 | .193 | 15 * |
| * 4175 325 .9 27 * | 3 | 44 | 19 | * 5.00 | .350 | .324 | 4.15 | .200 | 30 * |
| * 4175 325 .9 27 * | 2 | 41 | 10 | * 5.07 | .291 | .272 | 4.37 | .257 | 30 * |
| * 4175 325 .9 27 * | 5 | 57 | 17 | * 5.10 | .594 | .578 | 4.02 | .300 | 30 * |
| * 4175 325 .9 27 * | 5 | 185 | 7 | * 3.32 | .208 | .236 | 3.17 | .199 | 19 * |
| * 4175 325 .9 27 * | 3 | 54 | 13 | * 6.10 | .511 | .492 | 3.12 | .220 | 22 * |
| * 4175 325 .9 27 * | 2 | 52 | 5 | * 4.54 | .396 | .391 | 3.69 | .235 | 24 * |
| * 4175 325 .9 27 * | 4 | 37 | 17 | * 6.00 | .475 | .456 | 3.00 | .200 | 24 * |
| * 4375 305 3.1 11 * | 15 | 31 | 55 | * 1.53 | .137 | .115 | 1.25 | .061 | 5 * |
| * 4375 305 3.1 11 * | 3 | 54 | 0 | * 1.35 | .111 | .109 | 1.17 | .075 | 8 * |
| * 4375 305 3.1 11 * | 9 | 192 | 15 | * 1.47 | .124 | .107 | 1.26 | .065 | 7 * |
| * 4375 305 3.1 11 * | 3 | 14 | 16 | * 1.73 | .137 | .122 | 1.11 | .079 | 7 * |
| * 4375 305 3.1 11 * | 3 | 54 | 13 | * 2.07 | .175 | .157 | 1.42 | .102 | 9 * |
| * 4375 305 3.1 11 * | 2 | 41 | 10 | * 1.42 | .114 | .113 | 1.17 | .073 | 7 * |
| * 4375 305 3.1 11 * | 5 | 52 | 17 | * 1.45 | .143 | .115 | 1.26 | .074 | 7 * |
| * 4375 305 3.1 11 * | 3 | 123 | 7 | * 2.42 | .207 | .185 | 1.72 | .134 | 12 * |
| * 4375 305 3.1 11 * | 3 | 55 | 10 | * 2.25 | .183 | .159 | 1.45 | .114 | 11 * |

| | | | | | | | | | | | | | | | |
|--------|-----|-----|----|---|---|-----|----|---|------|------|------|------|------|----|---|
| * 4376 | 306 | 3.1 | 11 | * | 2 | 62 | 5 | * | 2.09 | .190 | .179 | 1.75 | .135 | 13 | * |
| * 4375 | 305 | 3.1 | 11 | * | 6 | 97 | 17 | * | 2.07 | .184 | .152 | 1.76 | .107 | 9 | * |
| * 4476 | 277 | 1.6 | 10 | * | 3 | 56 | 6 | * | 2.07 | .153 | .131 | 1.60 | .107 | 9 | * |
| * 4475 | 277 | 1.5 | 10 | * | 9 | 192 | 15 | * | 1.53 | .131 | .104 | .84 | .072 | 7 | * |
| * 4476 | 277 | 1.6 | 10 | * | 3 | 15 | 16 | * | 2.27 | .157 | .136 | 1.45 | .096 | 9 | * |
| * 4475 | 277 | 1.5 | 10 | * | 3 | 91 | 13 | * | 1.93 | .159 | .137 | 1.45 | .092 | 8 | * |
| * 4476 | 277 | 1.6 | 10 | * | 2 | 41 | 10 | * | 1.71 | .129 | .119 | 1.37 | .082 | 8 | * |
| * 4475 | 277 | 1.5 | 10 | * | 5 | 52 | 17 | * | 1.41 | .137 | .120 | 1.11 | .070 | 7 | * |
| * 4476 | 277 | 1.6 | 10 | * | 3 | 133 | 7 | * | 1.98 | .172 | .153 | 1.69 | .108 | 11 | * |
| * 4475 | 277 | 1.5 | 10 | * | 3 | 55 | 10 | * | 1.78 | .169 | .144 | 1.37 | .095 | 9 | * |
| * 4476 | 277 | 1.6 | 10 | * | 2 | 62 | 5 | * | 1.88 | .175 | .159 | 1.76 | .119 | 13 | * |
| * 4375 | 277 | 1.5 | 10 | * | 6 | 97 | 17 | * | 1.09 | .103 | .032 | .86 | .053 | 5 | * |
| * 4177 | 335 | 2.3 | 32 | * | 2 | 10 | 18 | * | 1.87 | .138 | .145 | 1.11 | .090 | 9 | * |
| * 4177 | 335 | 2.3 | 32 | * | 2 | 37 | 5 | * | 1.75 | .152 | .140 | 1.24 | .103 | 10 | * |
| * 4177 | 335 | 2.3 | 32 | * | 5 | 35 | 32 | * | 1.83 | .171 | .146 | 1.06 | .092 | 6 | * |
| * 4177 | 335 | 2.3 | 32 | * | 2 | 92 | 5 | * | 1.39 | .189 | .158 | 1.85 | .130 | 12 | * |
| * 4177 | 335 | 2.3 | 32 | * | 2 | 43 | 9 | * | .90 | .111 | .102 | .92 | .063 | 6 | * |
| * 4177 | 335 | 2.3 | 32 | * | 2 | 10 | 15 | * | .97 | .043 | .039 | .73 | .039 | 5 | * |
| * 4177 | 335 | 2.3 | 32 | * | 2 | 31 | 27 | * | 1.51 | .141 | .119 | .89 | .070 | 7 | * |
| * 4177 | 335 | 2.3 | 32 | * | 2 | 27 | 7 | * | 1.42 | .122 | .106 | 1.14 | .078 | 8 | * |
| * 4177 | 335 | 2.3 | 32 | * | 9 | 150 | 22 | * | 2.23 | .167 | .144 | 1.20 | .097 | 8 | * |
| * 4177 | 335 | 2.3 | 32 | * | 2 | 22 | 10 | * | 2.64 | .219 | .198 | 1.94 | .130 | 8 | * |
| * 4177 | 335 | 2.3 | 32 | * | 3 | 30 | 29 | * | 2.32 | .211 | .192 | 1.31 | .123 | 9 | * |
| * 4177 | 335 | 2.3 | 32 | * | 2 | 20 | 8 | * | 1.76 | .146 | .129 | 1.34 | .095 | 9 | * |
| * 4177 | 335 | 2.3 | 32 | * | 2 | 42 | 3 | * | 1.32 | .159 | .137 | 1.05 | .119 | 9 | * |
| * 4277 | 78 | 4.1 | 6 | * | 2 | 10 | 18 | * | 2.30 | .180 | .170 | 1.44 | .111 | 10 | * |
| * 4277 | 73 | 4.1 | 5 | * | 5 | 35 | 32 | * | 1.03 | .117 | .031 | .83 | .053 | 4 | * |
| * 4277 | 73 | 4.1 | 6 | * | 2 | 92 | 5 | * | 1.67 | .163 | .148 | 1.53 | .114 | 10 | * |
| * 4277 | 73 | 4.1 | 5 | * | 2 | 43 | 9 | * | 1.09 | .124 | .104 | .84 | .071 | 9 | * |
| * 4277 | 73 | 4.1 | 6 | * | 2 | 10 | 16 | * | 1.10 | .121 | .090 | .90 | .064 | 6 | * |
| * 4277 | 73 | 4.1 | 5 | * | 5 | 31 | 27 | * | 1.14 | .109 | .097 | .75 | .052 | 5 | * |
| * 4277 | 78 | 4.1 | 6 | * | 2 | 27 | 7 | * | 1.75 | .148 | .126 | 1.42 | .093 | 10 | * |
| * 4277 | 73 | 4.1 | 5 | * | 9 | 150 | 22 | * | 1.47 | .130 | .114 | .95 | .071 | 5 | * |
| * 4277 | 78 | 4.1 | 6 | * | 2 | 22 | 10 | * | 1.32 | .155 | .142 | 1.51 | .094 | 9 | * |
| * 4277 | 73 | 4.1 | 5 | * | 3 | 30 | 29 | * | 1.44 | .134 | .120 | .91 | .070 | 6 | * |
| * 4277 | 73 | 4.1 | 6 | * | 2 | 20 | 3 | * | 1.05 | .105 | .091 | .76 | .063 | 6 | * |
| * 4277 | 73 | 4.1 | 5 | * | 2 | 42 | 3 | * | 1.44 | .098 | .032 | 1.15 | .065 | 7 | * |
| * 4377 | 413 | 2.3 | 13 | * | 2 | 10 | 13 | * | 2.51 | .169 | .145 | 1.64 | .115 | 9 | * |
| * 4377 | 413 | 2.3 | 13 | * | 2 | 37 | 5 | * | 1.15 | .111 | .102 | .98 | .110 | 9 | * |
| * 4377 | 413 | 2.3 | 13 | * | 5 | 35 | 32 | * | 1.11 | .125 | .095 | .80 | .060 | 7 | * |
| * 4377 | 413 | 2.3 | 13 | * | 2 | 92 | 5 | * | 2.74 | .260 | .231 | 1.97 | .190 | 14 | * |
| * 4377 | 413 | 2.3 | 13 | * | 2 | 42 | 9 | * | 1.21 | .137 | .127 | 1.02 | .081 | 7 | * |
| * 4377 | 413 | 2.3 | 13 | * | 2 | 10 | 15 | * | 1.59 | .154 | .125 | 1.28 | .090 | 7 | * |
| * 4377 | 413 | 2.3 | 13 | * | 5 | 31 | 27 | * | 2.28 | .200 | .177 | 1.11 | .102 | 8 | * |
| * 4377 | 413 | 2.3 | 13 | * | 2 | 27 | 7 | * | 1.30 | .149 | .131 | 1.73 | .095 | 9 | * |
| * 4377 | 413 | 2.3 | 13 | * | 9 | 150 | 22 | * | 1.91 | .157 | .136 | 1.13 | .088 | 8 | * |
| * 4377 | 413 | 2.3 | 13 | * | 2 | 22 | 10 | * | 2.70 | .197 | .130 | 2.00 | .129 | 11 | * |
| * 4377 | 413 | 2.3 | 13 | * | 3 | 30 | 29 | * | 3.95 | .264 | .231 | 1.72 | .156 | 11 | * |
| * 4377 | 413 | 2.3 | 13 | * | 2 | 20 | 3 | * | 1.37 | .159 | .142 | 1.49 | .102 | 10 | * |
| * 4377 | 413 | 2.3 | 13 | * | 2 | 42 | 8 | * | 2.10 | .141 | .121 | 1.63 | .099 | 9 | * |