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ANALYSIS OF DATA FROM PLYNLIMON RAINGAUGE NETWORKS

APRIL 1971 - MARCH 1973

by

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ABSTRACT

The characteristics of monthly rainfall data, recorded by the Plynlimon networks during the period April 1971 - March 1973, are described. Differences between the rainfall of the Wye and Severn catchments are shown to be small, although it is suggested that they may be greater when snow falls. The variability of the data is also considered, Severn rainfall being rather more variable than Wye rainfall and the precision of a mean areal monthly rainfall estimate is calculated. An investigation into domain theory confirms the assumed relationship between rainfall and altitude; the relationships between rainfall and slope and between rainfall and aspect are less clear. Domain theory is used to estimate the rainfall in gaugeless domains, thereby allowing the entire catchment area to be taken into account when calculating mean areal rainfall. Hourly rainfall from the same two-year period is also described, this time in terms of the correlation observed in the data and variability of the data over the catchments. The number of gauges required to estimate mean areal hourly rainfall to specified depth and duration for storms during the period are given.

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INTRODUCTION : OBJECTIVES

The Plynlimon study, begun in 1967, was set up to investigate the effects of differing land uses on the water yield of the two small adjacent upland catchments situated on the eastern (leeward) slopes of the Plynlimon massif in central Wales. The two catchments are the atypically forested headwaters of the River Severn (870 hectares) and the rough grassland headwaters of the River Wye (1055 hectares). Altitude ranges from 320 to 740 metres; the aspect is predominantly NE and SE in the Wye, E and SE in the Severn, with the western watershed of both catchments about 18½ km from Cardigan Bay. The prevailing rain-bearing wind is south-westerly.

Fulfilment of the objectives of the Plynlimon project necessitated close study of the water balance of each catchment. Networks of raingauges were set up to estimate the precipitation input to each catchment, and networks of access tubes were established to estimate the moisture content of the soil using the Wallingford probe. Streamflow is measured continuously in both catchments and also on three minor catchments within each (the Afon Cyff, Nant Iago and Afon Gwy on the Wye, and the Afon Hore, Nant Tanllwyth and Afon Hafren on the Severn); see Figure 1. A meteorological observation site was set up at the confluence of the Nant Tanllwyth and Afon Hafren (Moel Cynnedd, commonly known as the "Tanllwyth met. site") to provide climatic data from which potential evapotranspiration is estimated. This site was subsequently augmented by automatic weather stations (AWS) distributed throughout the catchments; these record on magnetic tape the values of climatic variables, at five-minute intervals.

In addition to the instruments of this basic hydrometric network, many others are used in the intensive study of the processes whereby rainfall is converted to streamflow. At present, studies of interception by the forest are in progress and are about to be greatly extended, whilst detailed studies of infiltration and of water movement in ephemeral first-order basins and along small subsurface channels ("pipeflow")are progressing on the upper slopes of the Wye. Further studies, on the distribution of snowfall and characteristics of snowmelt, and on the distribution and intensity of rainfall, will be undertaken beginning in winter 1973-74.

As part of the study of distribution and intensity of rainfall, mentioned in the last paragraph, the raingauge network is to be extended to include an appreciable number (30 or more) of automatic recording (tipping-bucket) Rimco gauges. The purpose of this report is to summarize the analyses made on the rainfall data collected from the original network, particularly for the two complete years April 1971 to March 1973; although some gauges were operating before the former date, the networks were incomplete and the rainfall records discontinuous.

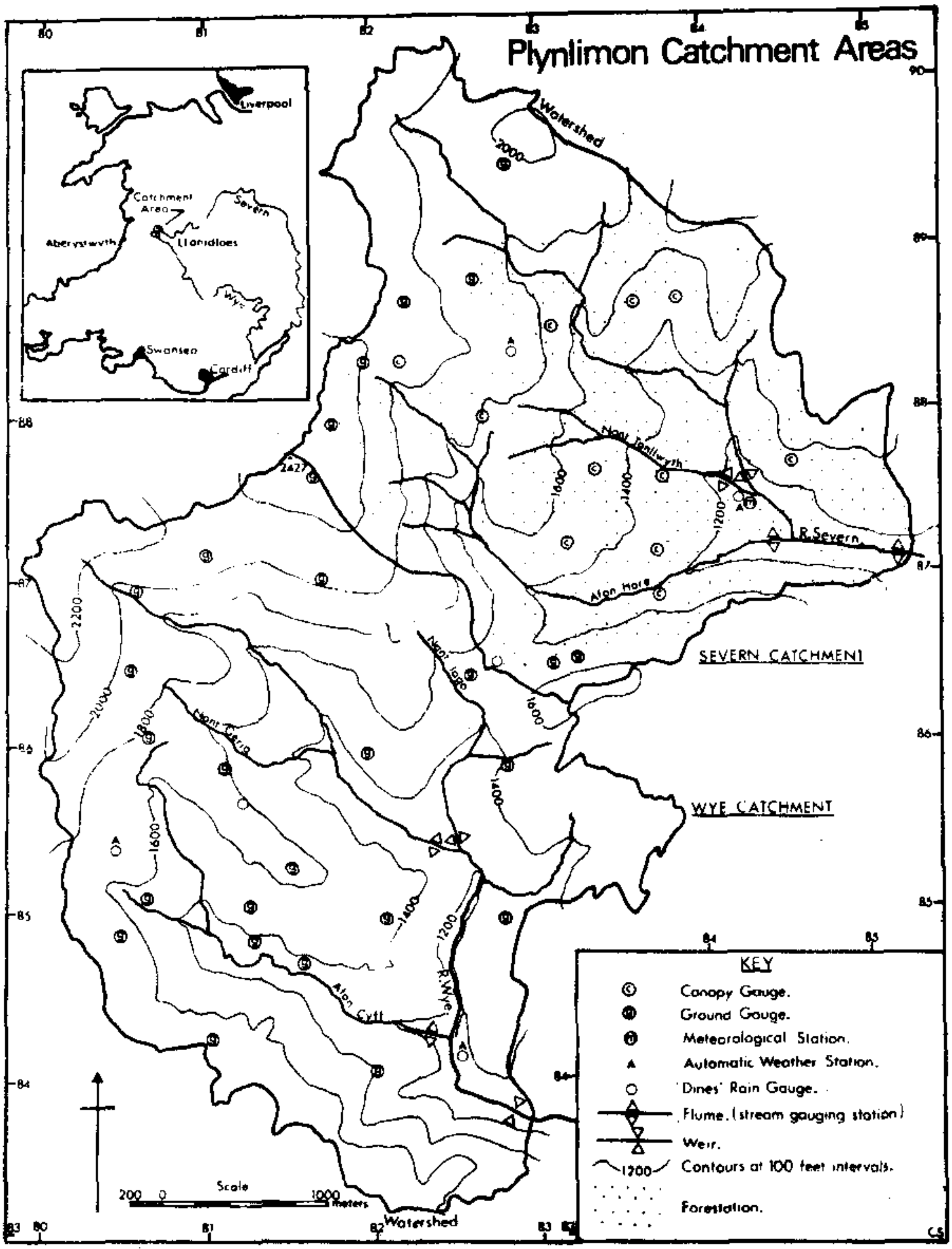

Figure 1. Plynlimon catchment areas

1.1 Formation of domains

The raingauge network was established by siting gauges within strata or "domains". A domain map was constructed from the 1:5000 Hunting Surveys map of each catchment; for the Severn, domains were delineated according to whether their altitudes, slopes and aspects fell within the following classes.

Altitude:	320 to 424 metres	(code : A)
	425 to 529 metres	(code : B)
	530 to 634 metres	(code : C)
	635 to 740 metres	(code : D)
Slope:	0 to 9 degrees	(code : 1)
	10 to 19 degrees	(code : 2)
	20 degrees or more	(code : 3).
Aspect:	Facing NE	(code : W)
	Facing SE	(code : X)
	Facing SW	(code : Y)
	Facing NW	(code : Z)

For the Wye catchment, the altitude, slope and aspect classes were as follows.

Altitude:	340 to 439 metres	(code : A)
	440 to 539 metres	(code : B)
	540 to 639 metres	(code : C)
	640 to 740 metres	(code : D)
Slope:	0 to 9 degrees	(code : 1)
	10 to 19 degrees	(code : 2)
	20 degrees or more	(code : 3)
Aspect:	Facing NE	(code : W)
	Facing SE	(code : X)
	Facing SW	(code : Y)
	Facing NW	(code : Z).

Each altitude class has an interval of 99 metres on the Wye and 104 metres on the Severn. Slope was determined from calculation of the vertical separation in each square of a ¼ km grid drawn over the map, followed by the construction of isopleths at 10^{o} intervals to show slope distribution. Aspect was mapped by delineating areas of approximately uniform direction of ground slope using the minor watersheds and stream courses as boundaries. A full account of domain delineation is given in Appendix 1. The domain map (Figure 2) consists of 38 different altitude-slope-aspect combinations out of a possible 4 x 3 x 4 = 48. The total area of each domain is, of course, not

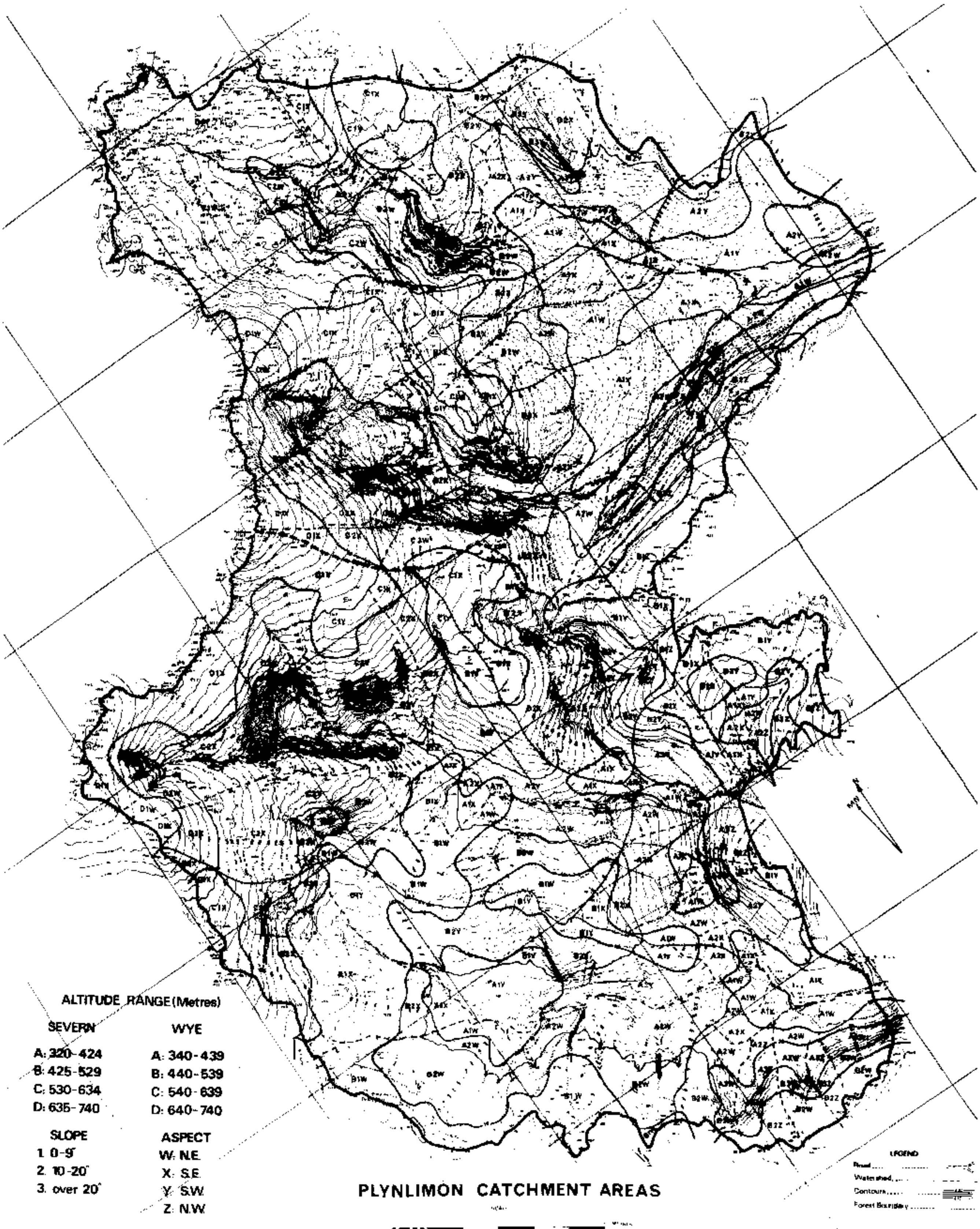

Figure 2. Plynlimon domain map

necessarily prescribed within a single boundary, and in general it is distributed as a series of domain "elements" over the catchment. Figure 2 shows that there are altogether about 275 such domain elements.

Rainguages were installed one in each domain that contributed more than 2% of the total area of each catchment. Other domains were gauged where there would otherwise have been obvious gaps in the network; for example, where a sub-catchment would have contained no gauge, or a particular class of altitude, slope or aspect would have contained none. (The lack of a raingauge in the steepest slope class in the Wye was not at first noticed, but was corrected in November 1973). Gauges were sited in each domain at random; in technical terms, therefore the sampling scheme is one of stratified random sampling. In a good stratified random sampling procedure, strata should be chosen so that differences between strata are large, and variation within each stratum is small.

1.2 The raingauge network

Period gauges

Installation of the raingauges began in November 1969. As mentioned above, records are incomplete before April 1971 due to the consistent failure of gauges designed to record daily totals over a 32-day period; these 32-day gauges were eventually all replaced by gauges that accumulate the total rainfall over a period (period gauges). The completed network, established by April 1971, consists of 38 monthly-read period gauges, 18 on the Severn and 20 on the Wye (see Figure 1 and Table 1). Those in the grassland Wye and that part of the Upper Severn that is unforested (7 gauge sites) are set in pits with their orifices at ground level in the same plane as the slope, and surrounded by a plastic grid 1.33 metres square to prevent splash. The gauges in the Severn have their funnels mounted on masts at mean canopy level or above; their funnels are all horizontal. Details on the installation of the two types of gauges are given in Appendix 1.

Rainfall recorders

There are also six Dines recording gauges, each accompanied by an additional ground-level period gauge, called a check gauge (at the Cefn Brwyn site the check gauge is in a turf wall; at Moel Cynnedd the check gauge is a standard gauge read daily). Three Dines recorders are on the Wye (Eisteddfa Gurig, Cefn Brwyn, Esgair y Maen) and three on the Severn (Carreg Wen, Moel Cynnedd, and on the watershed between the two major catchments). All are at standard height (53 cms) above ground, and visited weekly when the clocks are wound and the charts time-marked; charts are changed at three-weekly intervals except at Moel Cynnedd where the chart is changed daily. Three Dines (Carreg Wen,

Table 1. Site details of Plynlimon period raingauges
Wye catchment at January 1971 (all at ground level

Sub Catchment	Domain	Gauge No	Grid Ref	Altitude	Slope	Aspect
				Metres	Degrees	Degrees
Afon Cyff	B2W	1	28047 E 28483 N	495	13	063
Afon Cyff	BIW	2	28105 E 28422 N	488	4	012
Afon Cyff	BIX	3	28065 E 28510 N	469	11	126
Afon Cyff	AIY	4	28138 E 28484 N	431	6	206
Afon Cyff	B2Y	5	28134 E 28511 N	476	15	206
Afon Cyff	A2Y	6	28167 E 28462 N	413	19	194
Lower Wye	A2Z	7	28286 E 28492 N	421	13	330
Affon Cyff	C2X	8	28047 E 28607 N	566	13	158
Upper Wye	BIW	9	28105 E 28584 N	511	8	080
Upper Wye	B2W	10	28143 E 28539 N	511	7	033
Nant Iago	B2Z	11	28275 E 28582 N	453	15	306
Upper Wye	C2X	12	28056 E 28648 N	608	11	126
Upper Wye	B2Y	13	28195 E 28593 N	455	16	239
Nant Iago	B2X	14	28240 E 28633 N	510	13	111
Upper Wye	C2W	15	28050 E 28689 N	612	17	095
Upper Wye	C2Y	16	28165 E 28685 N	600	13	186
Upper Wye	DIX	17	28087 E 28725 N	664	7	167
Upper Wye	DIY	18	28164 E 28751 N	700	5	200
Lower Wye	A2X	19	28216 E 28489 N	427	13	120
Afon Cyff	A2W	20	28202 E 28405 N	413	9	033

Table 1 Contd. Site details of Plynlimon period rain-gauges. Severn catchment at January 1971.

Sub Catchment	Domain	Gauge No	Grid Ref	Altitude	Slope	Aspect	Gauge Type*
				Metres	Degrees	Degrees	
Afon Hore	DIX	1	28174 E 28794 N	712	9	095	G
Upper Severn	C2W	2	28308 E 28852 N	534	18	086	C
Afon Hore	D2X	3	28319 E 28721 N	457	16	151	C
Afon Hore	B2Z	4	28309 E 28650 N	475	15	352	G
Afon Hore	BIZ	5	28324 E 28649 N	486	9	352	G
Nant Tanllwyth	AIW	6	28377 E 28762 N	387	6	020	C
Afon Hore	D2X	7	28196 E 28834 N	668	15	095	G
Upper Severn	DIW	8	28221 E 28868 N	643	7	052	G
Upper Severn	CIW	9	28262 E 28881 N	594	7	076	G
Upper Severn	CIY	10	28282 E 28951 N	593	5	211	G
Upper Severn	B2X	11	28357 E 28866 N	458	19	125	C
Nant Tanllwyth	B2W	12	28336 E 28764 N	456	12	084	C
Upper Severn	A2Y	13	28383 E 28868 N	417	12	258	C
Afon Hore	CIX	14	28267 E 28797 N	541	10	121	C
Afon Hore	AIX	15	28371 E 28714 N	388	9	151	C
Afon Hore	C2X	16	28218 E 28833 N	614	11	130	C
Lower Severn	AIY	17	28453 E 28766 N	364	10	200	C
Afon Hore	A3Z	18	28372 E 28687 N	403	23	330	C

*G denotes ground level gauge C denotes canopy level gauge

Cefn Brwyn and Moel Cynnedd) have been recording since February 1968, whilst the remainder were installed in 1971. The check gauges (except for the daily gauge at Moel Cynnedd) are all read weekly and also on the first of each month with the period gauges, to give a monthly t total.

Automatic weather stations (AWS)

In late 1971, AWS were sited within each main catchment. In the Severn, a pair was installed both at Moel Cynnedd and at Carreg Wen (one from Carreg Wen was moved in April 1972 to another catchment); in the Wye, two were sited at Eisteddfa Gurig, and two at Cefn Brwyn. Each AWS has one Rimco tipping-bucket gauge, the number of 0.5 mm tips being scanned at five-minute intervals. The count is written on a magnetic data-logger tape (Microdata loggers in the case of Moel Cynnedd, Carreg Wen and Eisteddfa Gurig; Epsylon loggers at Cefn Brwyn). Logger tapes are changed at two or three-weekly intervals; in addition to rainfall, other meteorological variables are also recorded.

1.3 Processing of rainfall data

Full details of the computer methods used to scrutinize rainfall records for the detection of gross errors ("quality control") and for the computation of areal estimates of rainfall are covered by another Institute Report, Plinston and Hill (1974). Essentially, the amount recorded by each period gauge is divided into hourly increments proportional to the hourly catches recorded by the nearest operational Dines recording gauge. Allowance is made for the fact that, over a month, the Dines total catch is less than that recorded by the check gauge near to it; the purpose of this check gauge is to give the monthly total that is used to adjust the Dines hourly catches upwards in proportion to the ratio: (month's catch of check gauge)/(month's catch from Dines rainfall recorder). When hourly catches from Dines recorders and monthly totals from network period gauges have been checked by the quality control program, these data are stored on a magnetic tape (the "copy tape"); the Thiessen estimates of mean areal rainfall in each hour are stored separately on another magnetic tape, the "process tape". A duplicate is kept of the copy tape, and for technical reasons three copies of the process tape are maintained.

Measurements from the AWS Rimco gauges are processed separately. Rainfall data are subjected to the same quality control tests as those from the period gauge/Dines recorder network; five-minute rainfall totals are stored on DEC tape on the Institute's PDP8 computer, whilst hourly and daily totals are computed for each AWS and listed by line-printer.

2. GENERAL CHARACTERISTICS OF MONTHLY RAINFALL ON THE WYE AND SEVERN CATCHMENTS

2.1 (Arithmetic) mean monthly rainfalls

For the period April 1971 to March 1973, the arithmetic mean monthly rainfalls (using all period gauges in the network) were as shown in Table 2.

Table 2. Mean monthly rainfall (mm): Wye and Severn catchment*, April 1971 to March 1973

	1971-72		1972-73		Mean:	
	Wye:	Severn:	Wye:	Severn:	Wye:	Severn:
April	70.4	71.4	273.2	303.0	171.8	187.2
May	75.3	75.0	133.5	147.0	104.4	111.0
June	198.0	192.5	191.9	198.3	195.0	195.4
July	76.9	71.1	142.1	141.0	109.5	106.0
Aug	213.5	217.9	118.4	124.4	166.0	171.2
Sept	100.7	89.4	58.9	62.1	79.8	75.8
Oct	209.3	212.8	74.4	80.9	141.8	146.8
Nov	313.9	311.8	299.0	331.4	306.4	321.6
Dec	125.3	128.2	232.6	257.5	179.0	192.9
Jan	229.1	228.6	203.5	177.4	216.3	203.0
Feb	151.7	146.6	328.6	287.7	240.2	217.2
Mar	232.2	215.5	132.9	137.9	182.6	176.7
Total:	1996.3	1960.8	2189.0	2248.6	2092.8	2104.8

*Means for the Severn include data from check gauges at Moel Cynnedd and Carreg Wen, and are therefore means of 20 gauges. Means for the Wye include data from the check gauge at Cefn Brwyn, and are therefore means of 21 gauges. The original monthly data is given in full in Appendix 2.

For completeness, Table 3 shows arithmetic mean monthly rainfall over the four years 1969 to 1972, calculated from whichever gauges were present on the catchments in each year (see section 1.2 (period gauges)). Figures in brackets denote the total number of readings from which each mean was calculated.

Table 3. Mean monthly rainfall (mm) for Wye and Severn catchments, 1969 to 1972

	Wye:	Severn:
Jan	237.6(65)	228.3(64)
Feb	227.4(65)	214.5(63)
Mar	204.7(71)	181.6(66)
Apr	221.8(71)	230.6(71)
May	107.1(76)	113.6(72)
June	162.7(77)	164.8(72)
July	133.7(80)	125.8(75)
Aug	166.0(77)	171.6(74)
Sept	120.8(77)	121.1(74)
Oct	181.1(73)	195.5(73)
Nov	324.9(72)	343.0(67)
Dec	200.5(78)	199.0(76)
Totals	2288.3	2289.4

2.2 Precipitation during months when snow falls

A not insignificant part of the precipitation in winter months occurs as snowfall, and it has been suggested that precipitation on the forested Severn catchment, recorded as it is by canopy level gauges believed to be subject to high turbulence, may be underestimated. For the period April 1971 to March 1973 the differences (mean monthly rainfall on Wye minus mean monthly rainfall on Severn), which it is convenient to denote by W-S, are shown in Table 4, together with (i) the number of days on which snow was lying at Moel Cynnedd at 9 am GMT, and (ii) the number of days on which snow fell.

Table 4. Differences W-S (see text) with (i) number of days on which snow was lying at Moel Cynnedd; (ii) number of days on which snow fell

	1971-72			1972-73		
	(i)	(ii)	W-S	(i)	(ii)	W-S
April	0	0	- 1.0	0	0	- 29.8
May	0	0	+ 0.3	0	0	- 13.5
June	0	0	+ 5.5	0	0	- 6.4
July	0	0	+ 5.8	0	0	+ 1.1
Aug	0	0	- 4.4	0	0	- 6.0
Sept	0	0	+11.3	0	0	- 3.2
Oct	0	0	- 3.5	0	0	- 6.5
Nov	1	7	+ 2.1	0	1	- 32.4
Dec	0	3	- 2.9	0	1	- 24.9
Jan	3	8	+ 0.5	4	4	+ 26.1
Feb	2	6	+ 5.1	8	9	+ 40.9
Mar	3	3	+16.7	0	1	- 5.0

Although the month in which W-S was greatest was also the month having the largest number of days on which snow fell (February 1973, when W-S = + 40.9, snow falling on 9 days), the data given above suggest no strong evidence that winter precipitation on the Severn is underestimated. The mean value of W-S for the 14 months free of snow was - 3.59, whilst its mean value over the 10 months during which snow fell was + 2.62 mm. The absolute value of the difference between these differences is 6.21 $\pm$ 6.57 mm, suggesting that, for these two years, the mean value of W-S over snow-free months did not differ significantly from its value over months when snow was observed.

It would be most unwise, however, to draw firm conclusions from only two years of data. D T Plinston (internal communication) examined precipitation during the years 1969 and 1970, for which snowfall formed a more significant part of winter precipitation on both catchments than for the years 1971/72 and 1972/73, and reported a large difference in winter-month precipitation for the two catchments. During the 9 months of the calendar years 1969 and 1970 in which snow fell, the mean value (over 9 months) of the difference W-S was 14.9 $\pm$ 6.35 mm, whilst during the remaining (snow-free) months the mean difference was - 1.3 $\pm$ 3.07 mm. The magnitude of the former difference (41.9 mm) could not be explained by bias due to the incomplete network of gauges in the catchments during those years. This result makes all the more surprising the remarkably close agreement to within a millimetre or so of the mean annual precipitation for the entire period 1969-72 shown in Table 3. The magnitude of the difference may indeed be either the result of underestimation of Severn winter-month precipitation by the canopy gauges of the Severn, which have very little storage capacity, or the result of overestimation of the Wye snowfall; one may speculate further that the observed result is a true one, such that precipitation falling as snow is greater on the Wye than on the Severn. If snow bearing winds are predominantly easterly/north-easterly, it is not inconceivable that some snow which in still air would have fallen on the tree-canopy of the Severn may be redistributed westwards/south-westwards on to the Wye.

2.3 Random variation in measured monthly rainfall

Variation amongst monthly totals of rainfall includes components of variation due to altitude differences and other factors. To judge the adequacy of the raingauge network, it is sometimes necessary to derive an estimate of that "random" or "residual" variation that cannot be simply explained in terms of the principal factors known to influence rainfall distribution.

The method by which random variation is estimated depends greatly on what can be assumed about these factors. In technical jargon, the estimation procedure depends upon what linear statistical model is chosen to approximate the true (unknown) rainfall surface (the rainfall surface is that describing rainfall depth at each point of the horizontal projection of the catchment surface). If, for example, individual gauge readings are assumed to differ from the areal mean by random variation only, this is equivalent to assuming that the measured rainfall y_j recorded by gauge j is described by the linear statistical model

$$y_j = \mu + \varepsilon_j \qquad \ldots \qquad (2.3.1)$$

where μ is the areal mean, and ε_j the random "residual". In this case, random variation is estimated by

$$\Sigma(y_j - \bar{y})^2/(n-1)$$

where n is the number of gauges in the network. If, on the other hand, the rainfall has been measured that was collected by each of G gauges in each of P periods, the catch by gauge i in period j may be represented by the linear statistical model

$$y_{ij} = \mu + g_i + p_j + \varepsilon_{ij} \qquad \ldots \qquad (2.3.2)$$

where μ represents a mean precipitation over all periods and gauges, and g_i, p_j represent disturbances associated with gauge i and period j respectively. The random variation, if such a model is assumed, is estimated by the quantity

$$\sum_i \sum_j (y_{ij} - \hat{\mu} - \hat{g}_i - \hat{p}_j)^2/(P - 1)\ (G - 1) \qquad \ldots \qquad (2.3.3)$$

where $\hat{\mu}$, $\hat{g}_i$ and $\hat{p}_j$ are the least squares estimates of μ, g_i and p_j respectively. This quantity estimates the variance of the residuals ε_{ij}, and has been used in earlier work by the Institute on the random errors of rainfall measurement, Sutcliffe (1966). Its calculation is most readily obtained from an analysis of variance table, as shown in Table 5.

The residual variance, s^2 (= (C-B-A)/(P-1)(G-1), in the notation of Table 5) was computed for each month in turn. For the Wye, G = 21, if the check gauge at Cefn Brwyn is included in the network of 20 period gauges, and P = 2, since complete records were available for 2 Januarys, 2 Februarys, etc; for the Severn, G = 20, if the check gauges at Moel Cynnedd and Carreg Wen are included in the network of 18 period gauges, and P = 2. The values of the residual variance, and the coefficients of variation calculated from them and the monthly means, are given in Table 6.

This table shows that in 11 months out of 12, the random variation in monthly rainfall in the Wye is less than that in the Severn. This illustrates the greater local variability in the Severn monthly rainfall, probably due to greater turbulence at the canopy level where the period gauges are sited.

Table 5. Analysis of variance for data from G gauges in each of P periods: formulae for calculation of residual variance

Source of variation	d.f.	S.S.	MS
Periods*	(P-1)	$\Sigma P_j^2/G - T^2/PG(=A)$	
Gauges*	(G-1)	$\Sigma G_i^2/P - T^2/PG(=B)$	
Residual	(P-1) (G-1)	C-B-A (obtained by subtraction)	(C-B-A)/(P-1)(G-1)
Total	(PG-1)	$\Sigma y_{ij}^2 - T^2/PG(=C)$	

*P_j and G_i (see the Sum of Squares column) are, respectively, the total for period j over all gauges, and the total for gauge i over all periods. T is the total over all periods and gauges.

Table 6. Residual variances (s^2) of mean monthly rainfall and coefficients of variation of monthly rainfall for the Wye and Severn catchments

	s^2		CV(percentage)	
	Wye:	Severn:	Wye:	Severn:
Jan	286.60	529.38	7.8	11.3
Feb	762.93	777.69	11.5	12.8
Mar	108.93	453.27	5.7	12.0
April	469.01	791.74	12.6	15.0
May	73.37	74.87	8.2	7.8
June	141.19	229.14	6.1	7.7
July	82.20	138.63	8.3	11.1
Aug	220.06	93.72	8.9	5.6
Sept	29.87	237.36	6.8	20.3
Oct	21.61	186.37	3.3	9.3
Nov	105.73	331.67	3.3	5.7
Dec	209.93	212.70	8.1	7.6

Similarly, the coefficient of variation of monthly rainfall for the Wye is less than that of the Severn in 9 out of 12 months. On the (null) hypothesis that random errors in monthly rainfall are comparable for both Wye and Severn, the probability that s^2 for the Wye is less than s^2 for the Severn in 11 months is 0.003, suggesting that the hypothesis of comparable random errors be rejected. With the same null hypothesis, the probability that CV for the Wye is less than CV for the Severn in 9 out of 12 months is approximately 0.05, which again casts doubt on the validity of the null hypothesis.

The conclusions of sections 2.2 and 2.3, based on two complete years of monthly rainfall data, are the following:

There was no evidence that the differences between monthly precipitation on the Wye and monthly precipitation on the Severn varied according to whether or not some of the months' precipitation fell as snow. For the particular linear statistical model used to estimate random errors in monthly precipitation, there was evidence that the random errors were greater for the Severn than for the Wye, due possibly to the greater turbulence at canopy level in the Severn where the majority of period gauges for that catchment are sited.

3. PRECISION OF AREAL ESTIMATES OF MONTHLY RAINFALL FOR VARYING NETWORK SIZES

In the last section, estimates of the residual variance in monthly rainfall were given for the particular statistical model used, and it was pointed out that different models yield different estimates of residual variance. It is possible to use these estimates of residual variance to assess the adequacy of the raingauge network - for example, by considering what number of gauges in the network would reduce the standard error of the arithmetic mean to some acceptable level.

Two points must be made about such a procedure:

First, specification of the acceptable level of precision is invariably subjective, because it is generally not possible to specify in quantitative terms the losses resulting from imprecise estimation. The most that can be said, in many cases, is that an increase in the network density beyond a certain level gives a diminishing return in terms of precision of the estimate of mean areal rainfall. Second, since the estimate of residual variance differs for different statistical models, the number of gauges required to give a mean areal estimate of the desired precision - even if this could be stated - may vary according to which model is taken to describe the rainfall surface.

One method of examining the variability in mean areal rainfall, that makes no assumption about the type of statistical model required to describe the rainfall surface, is to select subsets of gauges from the complete network, and examine how the Thiessen estimates (and others) vary from subset to subset. The "scatter" between estimates derived from dividing the total network of N gauges into subsets of size p may be expected to decrease as p increases; if the reduction in the scatter is appreciable as the subset size is increased to p^*, but is "small" for larger numbers of gauges in the subset, then p^* is, in this rather vague sense, an "optimum" number of gauges. The technique assumes that records are available from a plentiful number of gauges, so that an appreciable number of subsets can be constructed.

3.1 Variability of Thiessen estimates of mean areal monthly rainfall

For the Wye, Thiessen estimates were computed for various subsets from the network consisting of the 20 period gauges together with the check gauge from Cefn Brwyn (21 gauges altogether). Independent subsets (ie subsets with no gauges in common) of size 10, 7, 5, 4, 3, 2 and 1 gauges were constructed, and the Thiessen estimates computed for each, using (i) the mean (over two years) January rainfall, (ii) the mean (two-year) July rainfall. The scatter amongst the Thiessen estimates, computed from all subsets of a given size, was plotted as a function of the number of gauges in the subset, with the result shown in Figure 3(a).

Inspection of Figure 3(a) suggests that the scatter amongst Thiessen estimates decreases markedly as the number of gauges increases to five, beyond which the curve of variance "flattens out"; if the sole purpose

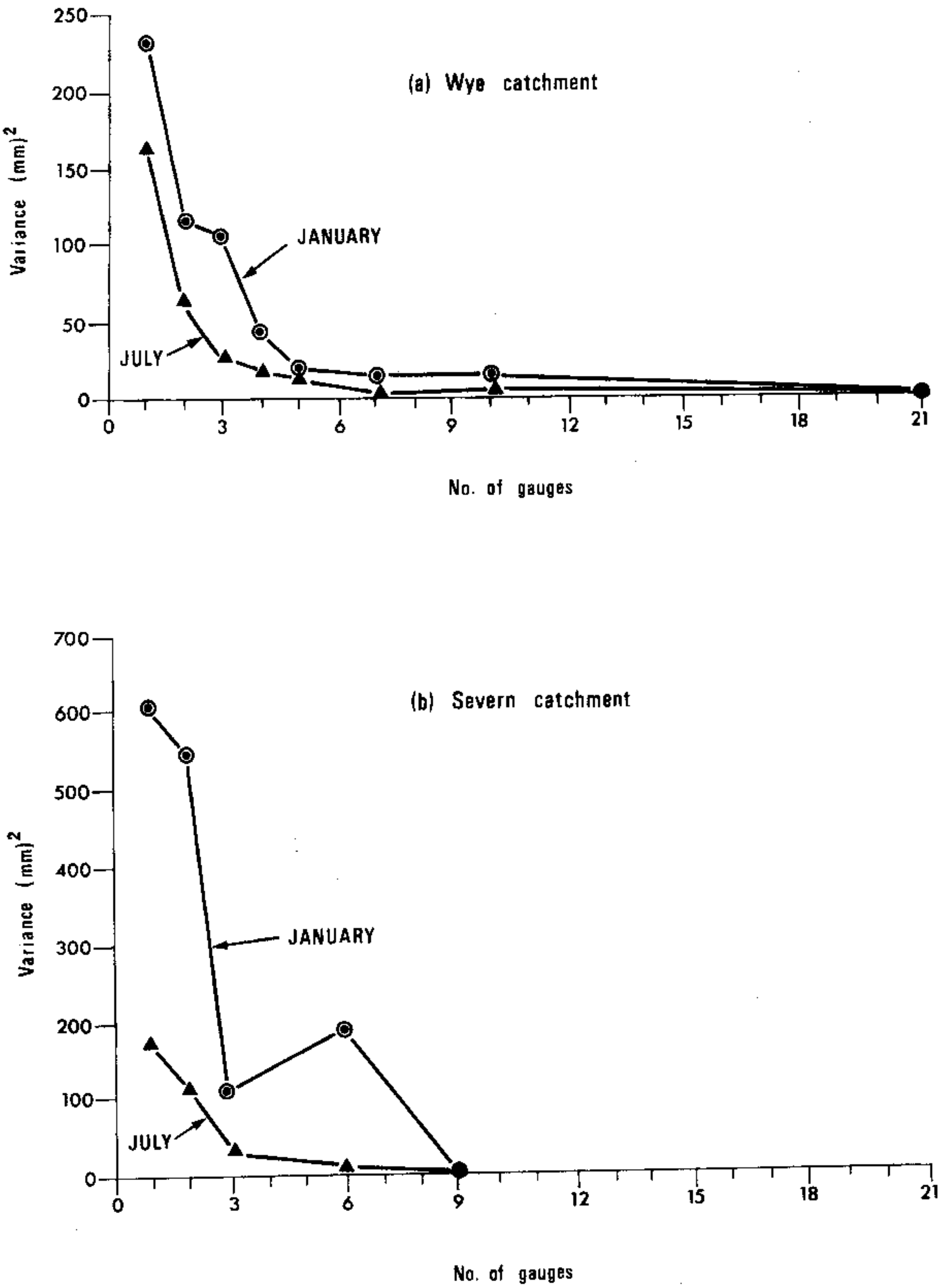

Figure 3. Variance of mean monthly rainfall estimates as a function of network size.

of the Wye network were to provide satisfactory estimates of mean monthly rainfall, therefore, Figure 3(a) suggests that roughly five period gauges would be adequate for this purpose.

A similar exercise was undertaken using data from the 18 gauges giving monthly totals on the Severn. Figure 3(b) shows a plot of variance of Thiessen estimates as a function of the number of gauges; the pattern is less regular than for the Wye, but suggests that no more than nine period gauges may be required to provide estimates of monthly rainfall in the Severn.

3.2 Variability between estimates of mean areal monthly rainfall, using different estimation methods

For completeness, Table 7 shows the mean areal estimate of monthly rainfall for the period April 1971 - March 1973, as calculated using four different methods: arithmetic mean, Thiessen polygons, isohyetal method, and, in the case of the Severn catchment, domain theory. No consistent difference was found between the various methods.

Table 7. Mean areal estimates of rainfall

Severn catchment

	Arithmetic mean (1):	Thiessen estimate:	Isohyetal estimate:	Domain theory estimate (2):
April 71	71.2	70.7	69.9	70.2
May	75.2	74.5	74.2	74.6
June	186.8	190.8	188.1	200.4
July	71.1	70.1	69.0	70.0
Aug	217.8	216.0	210.0	217.0
Sept	89.5	87.7	86.8	88.3
Oct	210.8	213.1	208.2	209.6
Nov	307.9	307.7	313.9	298.8
Dec	127.9	131.1	128.3	128.4
Jan 72	227.6	233.4	230.8	228.6
Feb	145.9	140.7	148.9	136.5
Mar	213.2	213.6	216.4	202.7
April	301.2	304.0	305.6	299.3
May	145.9	147.5	144.6	146.0
June	198.1	195.0	193.4	200.2
July	141.0	137.8	138.3	140.7
Aug	123.8	122.6	122.3	124.0
Sept	62.0	62.6	62.1	62.7
Oct	81.1	81.1	82.3	80.3
Nov	328.5	327.4	325.5	321.5
Dec	260.4	255.6	253.2	257.7
Jan 73	176.4	174.5	170.3	175.4
Feb	283.4	275.7	284.7	268.9
Mar	138.6	133.7	124.3	133.6

(1) These means were calculated using observations from 18 period gauges, plus the check gauge total from Moel Cynnedd. They differ slightly therefore from those of Table 2.

(2) For explanation of calculation of domain theory estimates, see section 4.

Table 7. Contd. Mean areal estimates of rainfall

Wye catchment

	Arithmetic mean (3):	Thiessen estimate	Isohyetal estimate	Domain theory estimate (4):
April 71	66.9	70.5	0.2	
May	73.9	75.4	77.1	
June	198.7	197.2	196.9	
July	77.7	75.5	75.4	
Aug	214.1	211.5	210.6	
Sept	101.1	99.2	98.7	
Oct	209.7	208.7	211.6	
Nov	309.3	313.4	315.6	
Dec	125.6	125.2	120.0	
Jan 72	229.0	226.9	226.5	
Feb	153.3	148.4	148.4	
Mar	233.6	230.7	231.7	
April	273.4	274.5	268.9	
May	134.1	133.5	134.6	
June	192.8	191.6	193.3	
July	142.9	141.4	142.2	
Aug	119.1	118.4	117.7	
Sept	59.3	58.7	58.5	
Oct	74.4	74.3	72.6	
Nov	299.1	300.6	303.1	
Dec	233.0	232.1	236.2	
Jan 73	198.8	204.3	200.1	
Feb	329.7	328.4	323.1	
Mar	132.9	130.7	131.2	

(3) These means were calculated using observations from 20 period gauges only, and do not include these from Cefn Brwyn. They differ slightly, therefore, from those of Table 2.

(4) Domain theory estimates cannot be calculated for the Wye throughout this period, because the third slope class contained no gauge.

4. INVESTIGATION OF DOMAIN THEORY: ITS USE FOR ESTIMATING MEAN AREAL RAINFALL

Since there are altogether 4 x 3 x 4 = 48 combinations of altitude, slope and aspect categories for each catchment, and fewer than that number of gauges in each catchment, several domains will not contain a gauge. Estimation of the mean areal rainfall in any month may then follow one of several procedures, amongst which are the following:-

(a) calculate the unweighted (arithmetic) mean of the gauges, ignoring that part of the catchment made up of domains without gauges in them.

(b) calculate the weighted mean of the gauge catches, using, as the weight for the catch of each gauge, the total area of the domain in which that gauge is sited. (This area will usually be fragmented, the fragments being distributed throughout the catchment; because not all domains have gauges, the total area of the domains will be less than the total catchment area). A drawback of this method is that not all of the catchment area enters the calculation of the weighted mean.

(c) calculate the mean areal rainfall as a weighted mean in which the rainfall in domains without gauges is estimated; the weight associated with the estimated gauge catch in domain IJK is W_{IJK}, the total area of this domain in the catchment. Provided that each altitude category, each slope category and each aspect category contains at least one gauge, the catch of each domain can be estimated; the total of the weights W_{IJK} will then equal the total catchment area.

For each month of the period April 1971 to March 1973 inclusive, mean areal rainfall was estimated by methods (a) and (c) above and is displayed in Table 7 together with the computed Thiessen estimates and isohyetal estimates for comparison. Details of the estimation procedure are described at length in section 4.1.

4.1 Estimation of monthly rainfall for domains without gauges

For a particular month, let y_{ijk} be the catch of a gauge sited in the i^{th} altitude category (i = A, B, C, D), the j^{th} slope category (j = 1,2,3) and the k^{th} aspect category (k = W, X, Y, Z). This catch may then be written

$$y_{ijk} = \mu + a_i + s_j + l_k + \varepsilon_{ijk} \quad \ldots \quad (4.1.1)$$

where a_i is an increment (which may be negative) to be added to a mean value μ for all domains with the i^{th} altitude category; s_j is the increment to be added for domains in the j^{th} slope category; and l_k is the increment to be added for domains in the k^{th} aspect category. The quantity ε_{ijk} is a residual, to be regarded as a random variable with zero mean and varance σ_ε^2, representing the depature of an observed gauge reading y_{ijk} from the "fitted" gauge reading $\mu + a_i + s_j + l_k$. The constants (parameters) μ, a_i, s_j and l_k are to be estimated from the monthly totals recorded by the network of storage gauges; they are estimated by minimizing the sum of squares SS, regarded as a function of a_i, s_j and l_k, such that

$$SS = \sum_{\substack{i=A,B,C,D \\ j=1,2,3 \\ k=W,X,Y,Z}} (y_{ijk} - \mu - a_i - s_j - l_k)^2 \quad \ldots \quad (4.1.2)$$

For the Severn data, let the totals recorded by the network of gauges in a particular month be T_1, T_2 .... T_{18}, T_{30}. Then T_1, T_2, T_3 .... are estimates of the quantities given by the following equations:-

$$\begin{aligned}
\mu + a_D + s_1 + l_X &= T_1 \\
\mu + a_C + s_2 + l_W &= T_2 \\
\mu + a_B + s_2 + l_X &= T_3 \\
\mu + a_B + s_2 + l_Z &= T_4 \\
\mu + a_B + s_1 + l_Z &= T_5 \\
\mu + a_A + s_1 + l_W &= T_6 \\
\mu + a_D + s_2 + l_X &= T_7 \\
\mu + a_D + s_1 + l_W &= T_8 \\
\mu + a_C + s_1 + l_W &= T_9 \\
\mu + a_C + s_1 + l_Y &= T_{10} \\
\mu + a_B + s_2 + l_X &= T_{11} \\
\mu + a_B + s_2 + l_W &= T_{12} \\
\mu + a_A + s_2 + l_Y &= T_{13} \\
\mu + a_C + s_1 + l_X &= T_{14} \\
\mu + a_A + s_1 + l_X &= T_{15} \\
\mu + a_C + s_2 + l_X &= T_{16} \\
\mu + a_A + s_1 + l_Y &= T_{17} \\
\mu + a_A + s_3 + l_Z &= T_{18} \\
\mu + a_A + s_1 + l_X &= T_{30} \qquad (4.1.3)
\end{aligned}$$

Adding all 19 equations gives

$$19\mu + (6a_A + 5a_B + 5a_C + 3a_D) + (10s_1 + 8s_2 + s_3) + (5l_W + 8l_X + 3l_Y + 3l_Z) = G \quad (4.1.4)$$

where G is the sum of the catches from all gauges. Equation (4.1.4) shows that it is convenient to impose certain constraints on the a_i, s_j and l_k parameters; if the constraints are

$$\begin{aligned} 6a_A + 5a_B + 5a_C + 3a_D &= 0 \\ 10s_1 + 8s_2 + s_3 &= 0 \\ 5l_W + 8l_X + 3l_Y + 3l_Z &= 0 \end{aligned} \quad (4.1.5)$$

then the parameter μ is easily estimated as G/19, the arithmetic mean of all gauge catches in the network. Manipulation of the equations (4.1.3) above then shows that the parameters a_i, s_j and l_k must be estimated by solving the equations, which, written in matrix notation, are

$$\underline{A}\,\underline{t} = \underline{b} \quad (4.1.6)$$

where $\underline{A}$ is an 11 x 11 matrix given by

$$\underline{A} = \begin{bmatrix} 6 & 0 & 0 & 0 & 4 & 1 & 1 & 1 & 2 & 2 & 1 \\ 0 & 5 & 0 & 0 & 1 & 4 & 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 5 & 0 & 3 & 2 & 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 3 & 2 & 1 & 0 & 1 & 2 & 0 & 0 \\ 4 & 1 & 3 & 2 & 10 & 0 & 0 & 3 & 4 & 2 & 1 \\ 1 & 4 & 2 & 1 & 0 & 8 & 0 & 2 & 4 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 1 & 3 & 2 & 0 & 5 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 4 & 4 & 0 & 0 & 8 & 0 & 0 \\ 2 & 0 & 1 & 0 & 2 & 1 & 0 & 0 & 0 & 3 & 0 \\ 1 & 2 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 3 \end{bmatrix}$$

and $\underline{t}$ and $\underline{b}$ are given by

$$\underline{t} = \begin{bmatrix} a_A \\ a_B \\ a_C \\ a_D \\ s_1 \\ s_2 \\ s_3 \\ l_W \\ l_X \\ l_Y \\ l_Z \end{bmatrix}$$

$$\underline{b} = \begin{bmatrix} T_6 + T_{13} + T_{15} + T_{17} + T_{18} + T_{30} - 6G/19 \\ T_3 + T_4 + T_5 + T_{11} + T_{12} - 5G/19 \\ T_2 + T_9 + T_{10} + T_{14} + T_{16} - 5G/19 \\ T_1 + T_7 + T_8 = 3G/19 \\ T_1 + T_5 + T_6 + T_8 + T_9 + T_{10} + T_{14} + T_{15} + T_{17} + T_{30} - 10G/19 \\ T_2 + T_3 + T_4 + T_7 + T_{11} + T_{12} + T_{13} + T_{16} - 8G/19 \\ T_{18} - G/19 \\ T_2 + T_6 + T_8 + T_9 + T_{12} - 5G/19 \\ T_1 + T_3 + T_7 + T_{11} + T_{14} + T_{15} + T_{16} + T_{30} - 8G/19 \\ T_{10} + T_{13} + T_{17} - 3G/19 \\ T_4 + T_5 + T_{18} - 3G/19 \end{bmatrix}$$

A complication arises because the matrix is a singular as a consequence of the constraints given by equations (4.1.5). The device used to overcome this difficulty is to add three extra rows and three extra columns to the matrix $\underline{A}$, and three extra elements to the vectors $\underline{t}$ and $\underline{b}$, as shown in the following equations (dotted lines indicate the partititioning of the matrix):-

$$\left[\begin{array}{ccccccccccc|ccc}
 & & & & & & & & & & & 6 & 0 & 0 \\
 & & & & & & & & & & & 5 & 0 & 0 \\
 & & & & & & & & & & & 5 & 0 & 0 \\
 & & & & & & & & & & & 3 & 0 & 0 \\
 & & & & & & & & & & & 0 & 10 & 0 \\
 & & & & & \underline{A} & & & & & & 0 & 8 & 0 \\
 & & & & & & & & & & & 0 & 1 & 0 \\
 & & & & & & & & & & & 0 & 0 & 8 \\
 & & & & & & & & & & & 0 & 0 & 5 \\
 & & & & & & & & & & & 0 & 0 & 3 \\
 & & & & & & & & & & & 0 & 0 & 3 \\
\hline
6 & 5 & 5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 10 & 8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 5 & 3 & 3 & 0 & 0 & 0
\end{array}\right]
\left[\begin{array}{c}
a_A \\ a_B \\ a_C \\ a_D \\ s_1 \\ s_2 \\ s_3 \\ l_W \\ l_X \\ l_Y \\ l_Z \\ \hline \lambda_1 \\ \lambda_2 \\ \lambda_3
\end{array}\right]
=
\left[\begin{array}{c}
\\ \\ \\ \\ \\ \underline{b} \\ \\ \\ \\ \\ \\ \hline 0 \\ 0 \\ 0
\end{array}\right]
\qquad (4.1.7)$$

Denote the square matrix on the left-hand side by $\underline{A}^*$, and write the equations (4.1.7) as

$$\underline{A}^* \ \underline{t}^* = \underline{b}^* ; \qquad (4.1.8)$$

the matrix $\underline{A}^*$ is then non-singular, so that equations (4.1.7) can be solved to give values of a_A, a_B, a_C, a_D; s_1, s_2, s_3; and l_W, l_X, l_Y, l_Z, that satisfy the constraints (4.1.5). (The above procedure may be shown to be equal to minimization of the function SS + λ_1 C_1 + λ_2 C_2 + λ_3 C_3, where C_1, C_2, C_3 are the constraints of equations (4.1.5) and λ_1, λ_2, λ_3 are Lagrangian multipliers). The matrix $\underline{A}^{*-1}$, required for the solution

$$\underline{t}^* = \underline{A}^{*-1} \ \underline{b}^* \qquad (4.1.9)$$

and which is the same for each month in which all gauges in the network gave readings, was computed on the Institute's PDP8 computer together with the parameter estimates $\underline{t}^*$ for each month from April 1971 to March 1973 inclusive. For each month, σ^2 was also estimated, the estimate (as given by the Gauss-Markov theorem of least squares) being for the Severn catchment:-

$$\sum_{i,j,k} (y_{ijk} - \hat{\mu} - \hat{a}_i - \hat{s}_j - \hat{l}_k)^2/10 \qquad (4.1.10)$$

where the "hats" over parameters indicate that their estimated values are used to compute the expression (4.1.10).

Given that the constants a_A, a_B, a_C, a_D; s_1, s_2, s_3 and l_W, l_X, l_Y, l_Z have been estimated for a particular month, suppose that the domain C2Z has no gauge sited in it. Then the estimate of the monthly rainfall on this domain is

$$\hat{\mu} + \hat{a}_C + \hat{s}_2 + \hat{l}_Z \qquad (4.1.11)$$

and if the area of this domain is W_{C2Z} (more generally, if the area of domain ijk is W_{ijk}) the estimate of mean areal rainfall for the whole catchment is

$$\hat{\mu} + \sum_{i,j,k} W_{ijk}\,(\hat{a}_i + \hat{s}_j + \hat{l}_k) / \sum_{i,j,k} W_{ijk}. \qquad (4.1.12)$$

The fitted parameters $\hat{a}_i$, $\hat{s}_j$, and $\hat{l}_k$ are themselves of interest. If rainfall increases with altitude, this should be demonstrated by values of $\hat{a}_A$, $\hat{a}_B$, $\hat{a}_C$ and $\hat{a}_D$ that form an increasing sequence (subject, of course, to the constrain $6a_A + 5a_B + 5a_C + 3a_D = 0$). Similarly, if rain comes more frequently from the SW direction than from any other, as suggested in section 1, this should be demonstrated by the value of l_Y, which should be greater than l_W, l_X and l_Z (also subject to the constraint $5l_W + 8l_X + 3l_Y + 3l_Z = 0$). Because the parameter estimates are of interest, their values are given in section 4.2; first, however, we give a summary of the above calculations for estimating a_i, s_j and l_k for the Wye catchment.

For the Wye, equations (4.1.3) become

$$\mu + a_B + s_2 + l_W = T_1$$
$$\mu + a_B + s_1 + l_W = T_2$$
$$\mu + a_B + s_1 + l_X = T_3$$
$$\mu + a_A + s_1 + l_Y = T_4$$
$$\mu + a_B + s_2 + l_Y = T_5$$
$$\mu + a_A + s_2 + l_Y = T_6$$
$$\mu + a_A + s_2 + l_Z = T_7$$
$$\mu + a_C + s_2 + l_X = T_8$$
$$\mu + a_B + s_1 + l_W = T_9$$
$$\mu + a_B + s_2 + l_W = T_{10}$$
$$\mu + a_B + s_2 + l_Z = T_{11}$$

$$\mu + a_C + s_2 + l_X = T_{12}$$
$$\mu + a_B + s_2 + l_Y = T_{13}$$
$$\mu + a_B + s_2 + l_X = T_{14}$$
$$\mu + a_C + s_2 + l_W = T_{15}$$
$$\mu + a_C + s_2 + l_Y = T_{16}$$
$$\mu + a_D + s_1 + l_X = T_{17}$$
$$\mu + a_C + s_1 + l_Y = T_{18}$$
$$\mu + a_A + s_2 + l_X = T_{19}$$
$$\mu + a_A + s_2 + l_W = T_{20} \qquad (4.1.13)$$

Addition of these equations gives

$$20\mu + (5a_A + 9a_B + 4a_C + 2a_D) + (6s_1 + 14s_2) + (6l_W + 6l_X + 6l_Y + 2l_Z) = G \qquad (4.1.14)$$

suggesting the constraints

$$5a_A + 9a_B + 4a_C + 2a_D = 0$$
$$6s_1 + 14s_2 = 0$$
$$6l_W + 6l_X + 6l_Y + 2l_Z = 0 \qquad (4.1.15)$$

Then $\hat{\mu} = G/20$. Note that the constraint on the slope parameters contains no term in s_3, since no gauge on the Wye (until November 1973) was sited in this slope category. The equation (4.1.7) then becomes

$$\underline{A}^* \underline{t}^* = \underline{b}^* \qquad (4.1.16)$$

where $\underline{A}^*$ is given by

$$\underline{A}^* = \begin{bmatrix} 5 & 0 & 0 & 0 & 1 & 4 & 1 & 1 & 2 & 1 & 5 & 0 & 0 \\ 0 & 9 & 0 & 0 & 3 & 6 & 4 & 2 & 2 & 1 & 9 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 4 & 1 & 2 & 1 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 & 0 & 0 & 1 & 1 & 0 & 2 & 0 & 0 \\ 1 & 3 & 0 & 2 & 6 & 0 & 2 & 2 & 2 & 0 & 0 & 6 & 0 \\ 4 & 6 & 4 & 0 & 0 & 14 & 4 & 4 & 4 & 2 & 0 & 14 & 0 \\ 1 & 4 & 1 & 0 & 2 & 4 & 6 & 0 & 0 & 0 & 0 & 0 & 6 \\ 1 & 2 & 2 & 1 & 2 & 4 & 0 & 6 & 0 & 0 & 0 & 0 & 6 \\ 2 & 2 & 1 & 1 & 2 & 4 & 0 & 0 & 6 & 0 & 0 & 0 & 6 \\ 1 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 2 \\ 5 & 9 & 4 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 6 & 6 & 2 & 0 & 0 & 0 \end{bmatrix}$$

and $\underline{t}^*$ and $\underline{b}^*$ are given by

$$\underline{t}^* = \begin{bmatrix} a_A \\ a_B \\ a_C \\ a_D \\ s_1 \\ s_2 \\ l_W \\ l_X \\ l_Y \\ l_Z \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

$$\underline{b}^* = \begin{bmatrix} T_4 + T_6 + T_7 + T_{19} + T_{20} - 5G/20 \\ T_1 + T_2 + T_3 + T_5 + T_9 + T_{10} + T_{11} + T_{13} + T_{14} - 9G/20 \\ T_8 + T_{12} + T_{15} + T_{16} - 4G/20 \\ T_{17} + T_{18} - 2G/20 \\ T_2 + T_3 + T_{14} + T_9 + T_{17} + T_{18} - 6G/20 \\ T_1 + T_5 + T_6 + T_7 + T_8 + T_{10} + T_{11} + T_{12} + T_{13} + T_{14} + T_{15} + T_{16} \\ \qquad + T_{19} + T_{20} - 14G/20 \\ T_1 + T_2 + T_9 + T_{10} + T_{15} + T_{20} - 6G/20 \\ T_3 + T_8 + T_{14} + T_{17} + T_{19} + T_{12} - 6G/20 \\ T_4 + T_5 + T_6 + T_{13} + T_{16} + T_{18} - 6G/20 \\ T_7 + T_{11} - 2G/20 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The estimate of σ_ε^2 for the Wye catchment, to be calculated for each month, is

$$\sum_{i,j,k} (y_{ijk} - \hat{\mu} - \hat{a}_i - \hat{s}_j - \hat{l}_k)^2/12. \qquad (4.1.17)$$

4.2 Computed values of altitude, aspect and slope parameters for the Wye and Severn catchments

The estimated quantities a_A, a_B, a_C, a_D are shown in Table 8; the values $\hat{s}_1$, $\hat{s}_2$ and $\hat{s}_3$ (where relevant) are shown in Table 9 and the values $\hat{l}_W$, $\hat{l}_X$, $\hat{l}_Z$ are shown in Table 10.

Table 8. Estimates of the altitude parameters a_A, a_B, a_C, a_D and μ for the Wye and Severn catchments (units: millimetres)

(Values shown with asterisks are greater than twice their standard error in absolute magnitude)

		a_A:	a_B:	a_C:	a_D:	μ
April 71	Wye:	-2.14±2.62	+2.02±1.77	-2.76±3.57	+1.76±5.68	69.96
	Severn:	-4.14±2.02*	+2.46±2.64	-1.06±1.86	+5.94±2.58*	71.04
May 71	W:	-3.14±1.45*	-3.45±0.98*	+3.69±1.97	16.00±3.14*	75.42
	S:	-7.28±2.20*	-0.04±2.87	+4.86±2.03*	+ 6.54±2.81*	74.66
June 71	W:	-11.12±7.01	+1.55±4.74	+13.33±9.54	- 5.85±15.18	198.71
	S:	-29.79±7.97*	+4.73±10.40	+20.59±7.35*	+17.38±10.18	190.93
July 71	W:	- 7.11±1.79*	-3.19±1.21*	+13.43±2.43*	+ 5.29±3.87	77.69
	S:	- 8.71±2.36*	-1.16±3.08	+ 6.04±2.18*	+ 9.28±3.01*	70.64
Aug 71	W:	-10.24±3.08*	-3.21±2.08	+10.72±4.19*	+18.61±6.67*	214.11
	S:	-19.83±5.40*	+1.83±7.05	+14.71±4.98*	+12.11±6.90	217.08
Sept 71	W:	- 5.11±3.12	-0.80±2.11	+ 7.22±4.26	+ 1.90±6.77	101.11
	S:	-15.96±2.76*	+0.69±3.60	+10.73±2.55*	+12.88±3.53*	88.95
Oct 71	W:	-10.38±2.34*	-6.42±1.58*	+19.56±3.18*	+15.74±5.06*	209.62
	S:	-18.48±9.09*	+13.29±11.86	+ 1.80±8.38	+11.79±11.61	211.27
Nov 71	W:	- 8.86±8.34	- 3.09±5.63	+ 6.70±11.35	+22.69±18.05	313.78
	S:	-31.18±14.24*	+ 4.24±18.58	+ 2.80±13.13	+50.63±18.18*	307.21
Dec 71	W:	- 2.01±1.72	+ 1.29±1.16	- 1.22±2.34	+ 1.63±3.73	125.64
	S:	-14.21±8.60	+11.66±11.23	+ 0.39±7.98	+ 8.35±10.99	127.39
Jan 72	W:	-15.88±4.40*	- 4.47±2.98	±23.50±6.00*	+12.81±9.54	229.04
	S:	-22.87±9.54*	+ 4.63±12.44	+ 4.28±8.79	+30.91±12.18*	227.27
Feb 72	W:	-23.52±6.30*	-16.90±4.26*	+16.68±8.57	+101.49±13.63*	153.33
	S:	-23.01±7.66*	-19.48±10.00	-12.36±7.06	+ 99.08± 9.78*	144.95
Mar 72	W:	-25.20±7.50*	- 1.99± 5.08	+ 9.72±10.23	+ 52.52±16.26*	233.57
	S:	-30.17±7.70*	- 9.41±10.05	- 6.94± 7.10	+ 87.59± 9.84*	212.44

April 72	W:	- 8.86±12.84	+ 2.39± 8.68	- 9.57±17.48	+30.53±27.81	273.36
	S:	-24.39±15.54	+50.59±20.28*	-23.16±14.33	+ 3.04±19.85	301.26
May 72	W:	-14.03± 4.15*	- 3.77±2.80	+14.11± 5.65*	+23.85± 9.00*	134.16
	S:	- 7.19± 6.35	+ 3.46±8.28	+ 2.95± 5.85	+ 3.69± 8.10	145.79
June 72	W:	-16.75±4.98*	- 3.82±3.37	+19.41± 6.78*	+20.23±10.79	192.78
	S:	-30.58±9.62*	+13.50±12.55	+10.92± 8.87	+20.45±12.28	196.85
July 72	W:	- 9.97± 3.81*	- 3.69± 2.58	+15.05± 5.19*	+11.43± 8.26	142.95
	S:	-22.31± 5.07*	+ 3.10± 6.61	+12.78± 4.68*	+18.15± 6.47*	140.17
Aug 72	W:	- 9.44± 2.02*	- 1.07± 1.37	+ 8.12± 2.75*	+12.19± 4.38*	119.06
	S:	-10.73± 3.96*	+ 2.18± 5.16	+ 7.68± 3.65*	+ 5.02± 5.05	123.56
Sept 72	W:	- 2.90± 1.13*	- 1.15± 0.76	+ 3.80± 1.54*	+ 4.83± 2.45	59.26
	S:	- 2.76± 1.15*	+ 0.76± 1.51	+ 1.49± 1.06	+ 1.76± 1.47	61.99
Oct 72	W:	-10.70± 1.57*	- 3.50± 1.06*	+11.79± 2.13*	+18.91± 3.40*	74.82
	S:	-11.23± 2.94*	- 1.34± 3.84	+ 7.30± 2.71*	+12.53± 3.75*	80.34
Nov 72	W:	- 9.99± 8.69	- 1.30± 5.87	+ 1.25±11.83	+28.30±18.81	299.06
	S:	-28.77±10.92*	-11.40±14.24	+20.62±10.06*	+42.20±13.94*	328.23
Dec 72	W:	- 6.69± 8.80	- 6.43± 5.95	+ 9.48±11.99	+26.69±19.06	233.04
	S:	-13.96±11.98	+12.35±15.62	+ 7.51±11.04	- 5.18±15.29	255.66
Jan 73	W:	- 7.24± 7.45	+ 4.90± 5.04	+ 6.01±10.14	-15.97±16.13	203.86
	S:	-16.00±13.67	- 7.18±17.84	+21.95±12.61	+ 7.40±17.46	175.45
Feb 73	W:	-13.36±11.92	- 6.71± 8.06	-11.47±16.23	+86.55±25.81*	329.67
	S:	-59.13±13.86*	+ 4.76±18.08	+ 6.70±12.78	+99.16±17.70*	282.00
Mar 73	W:	- 9.48± 6.30	- 5.78± 4.26	+ 1.91± 8.58	+45.90±13.64*	132.92
	S:	-27.15± 8.00*	+ 1.50±10.43	- 1.36± 7.37	+54.08±10.21*	137.21

*Values shown with asterisks are greater than twice their standard error in absolute magnitude.

Means of altitude parameters over two years:-

		a_A:	a_B:	a_C:	a_D:	μ
April	W:	5.5±6.55	+ 2.20± 4.43	- 6.16± 8.92	+16.14±14.19	171.66
	S:	-14.26±7.84	+26.52±10.23*	-12.11± 7.23	4.49±10.01	186.15
May	W:	- 8.58±2.2*	- 3.61± 1.48*	+ 8.90± 3.34*	+19.92± 4.77*	104.79
	S:	- 7.23±3.36*	+ 1.71± 4.38	+ 3.90± 3.10	+ 5.12± 4.29	110.22
June	W:	-13.93±4.30*	- 1.14± 2.91	+16.37± 5.85*	+ 7.19± 9.31	166.44
	S:	-30.18±6.25*	+ 9.12± 8.15	+15.76± 5.76*	+18.92± 7.98*	168.36
July	W:	- 8.54±2.10*	- 3.44± 1.42*	+14.24± 2.87*	+ 8.36± 4.56	110.32
	S:	-15.51±2.80*	+ 0.97± 3.65	+ 9.41± 2.58*	+13.72± 3.57*	105.40
Aug	W:	- 9.84±1.84*	- 2.14± 1.25	+ 9.42± 2.51*	+15.40± 3.99*	166.58
	S:	- 9.92±3.35*	+ 2.00± 4.37	+11.19± 3.09*	+ 8.56± 4.28*	170.32
Sept	W:	- 4.00±1.66*	- 0.98± 1.12	+ 5.51± 2.26*	+ 3.36± 3.60	80.18
	S:	- 9.36±1.49*	+ 0.72± 1.95	+ 6.11+ 1.38*	+ 7.32± 1.91*	75.47
Oct	W:	-10.54±1.41*	- 4.96± 0.95*	+15.68± 1.91*	+17.32± 3.05*	142.22
	S:	-14.86±4.61*	+ 5.98± 6.23	+ 4.55± 4.14	+12.16± 6.10	145.80
Nov	W:	- 9.42±6.02	- 2.19± 4.07	+ 3.98± 8.20	+25.50±13.03	306.42
	S:	-29.98±8.97*	- 3.58+11.7	+11.71± 8.27	+46.42±11.45*	317.72
Dec	W:	- 4.35±4.48	- 2.57± 3.03	+ 4.13± 6.11	+14.16± 9.71	179.34
	S:	-14.08±7.37	12.00± 9.62	+ 3.95± 6.80	+ 1.58± 9.42	191.52
Jan	W:	-15.94±4.33*	- 5.82± 2.93	+22.72± 5.89*	+10.10± 9.37	216.45
	S:	-19.44±8.33*	- 1.28±10.87	+13.12± 7.69	+19.16±10.64	201.36
Feb	W:	-18.44±6.74*	-11.80± 4.56	+ 2.60± 9.18	+94.02±14.59*	241.50
	S:	-41.07±7.92*	- 7.36±10.33	- 2.83± 7.30	+99.12±10.11*	213.48
Mar	W:	-17.34±4.90*	- 3.89± 3.31	+ 5.82± 6.68	+49.21±10.61*	183.24
	S:	-28.66±5.55*	- 3.95± 7.24	- 4.15± 5.12	+70.33± 7.09*	174.82
Mean:	W:	-10.54±1.76*	- 3.36± 1.19*	+ 8.60± 2.40*	+23.39± 3.81*	172.43
	S:	-19.55±2.51*	+ 3.57± 3.27*	+ 5.05± 2.31	+25.58± 3.20*	171.72

Table 9. Estimates of the slope parameters s_1, s_2, s_3 and μ for the Wye and Severn catchments (units: millimetres)

(Values shown with asterisks are greater than twice their standard error in absolute magnitude)

		s_1:	s_2:	s_3:	μ
April 71	Wye:	0.13±2.98	-0.06±1.29	-	69.96
	Severn:	0.74±1.10	-0.93±1.59	0.00±6.61	71.04
May 71	W:	-0.47±1.65	+0.20±0.70	-	75.42
	S:	1.19±1.20	-1.83±1.73	2.77±7.19	74.66
June 71	W:	10.31±7.97	-4.42±3.41	-	198.71
	S:	28.58±4.36*	-3.46±6.27	-25.13±26.09	190.93
July 71	W:	6.18±2.03*	-2.65±0.87*	-	77.69
	S:	0.82±1.29	-2.17±1.85	9.18± 7.72	70.64
Aug 71	W:	5.80±3.50	-2.48±1.50	-	214.11
	S:	3.24±2.95	-6.74±4.25	21.51±17.68	217.08
Sept 71	W:	5.71±3.55	-2.45±1.52	-	101.11
	S:	2.02±1.51	-3.94±2.17	11.38± 9.04	88.95
Oct 71	W:	6.54±2.66*	-2.80±1.14*	-	209.62
	S:	8.23±4.97	-13.20±7.14	23.33±29.74	211.27
Nov 71	W:	14.58±9.47	- 6.25±4.06	-	313.78
	S:	8.85±7.78	- 9.71±11.19	-10.86±46.59	307.21
Dec 71	W:	4.68±1.96*	- 2.00± 0.84*	-	125.64
	S:	4.11±4.70	- 6.80± 6.76	13.32±28.15	127.39
Jan 72	W:	1.64±5.00	- 0.70± 2.14	-	229.04
	S:	8.30±5.21	-14.57± 7.49	33.52±31.20	227.27
Feb 72	W:	0.27±7.16	- 0.12± 3.07	-	153.33
	S:	8.40±4.19	- 8.70± 6.02	-14.41±25.06	144.95
Mar 72	W:	8.65±8.53	- 3.70± 3.66	-	233.57
	S:	5.11±4.21	- 4.44± 6.06	-15.65±25.21	212.44

April 72	W:	12.01+14.59	- 5.15+ 6.25	-	273.36
	S:	20.38+ 8.49*	-24.28+12.22	- 9.57+50.86	301.26
May 72	W:	3.84+ 4.71	- 1.65+ 2.02	-	134.16
	S:	0.93+ 3.47	- 2.48+ 4.99	10.58+20.76	145.79
June 72	W:	8.17+ 5.66	- 3.50+ 2.43	-	192.78
	S:	- 2.09+ 5.26	- 2.87+ 7.56	43.90+31.47	196.85
July 72	W:	8.98+ 4.33*	- 3.85+ 1.86*	-	142.95
	S:	0.45+ 2.77	- 3.58+ 3.98	24.15+16.59	140.17
Aug 72	W:	3.29+ 2.30	- 1.41+ 0.98	-	119.06
	S:	- 0.19+ 2.16	- 0.98+ 3.11	9.69+12.94	123.56
Sept 72	W:	2.38+ 1.29	- 1.02+ 0.55	-	59.26
	S:	- 0.03+ 0.63	0.18+ 0.91	- 1.16+ 3.78	61.99
Oct 72	W:	0.46+ 1.78	- 0.20+ 0.76	-	74.82
	S:	0.69+ 1.61	- 1.85+ 2.31	7.90+ 9.62	80.34
Nov 72	W:	10.17+ 9.87	- 4.36+ 4.23	-	299.06
	S:	4.82+ 5.96	- 4.47+ 8.58	-12.46+35.71	328.23
Dec 72	W:	2.41+10.00	- 1.03+ 4.29	-	233.04
	S:	1.70+ 6.54	- 4.66+ 9.41	20.33+39.18	255.66
Jan 73	W:	19.76+ 8.47*	- 8.47+ 3.63*	-	203.86
	S:	- 4.13+ 7.47	6.74+10.74	-12.57+44.73	175.45
Feb 73	W:	1.00+13.55	- 0.43+ 5.81	-	329.67
	S:	13.87+ 7.57	-17.87+10.89	4.29+45.34	282.00
Mar 73	W:	6.54+ 7.16	- 2.80+ 3.07	-	132.92
	S:	- 1.83+ 4.37	1.99+ 6.28	2.43+26.16	137.21

Means of slope parameters over two years:-

		s_1:	s_2:	s_3:	μ
April:	W:	+ 6.07± 7.45	- 2.60±3.19	-	171.66
	S:	+10.56± 4.28*	-12.60±6.16*	- 4.78±25.64	186.15
May:	W:	+ 1.68± 2.50	- 0.72±1.07	-	104.79
	S:	+ 1.06± 1.84	- 2.16±2.64	+ 6.68±10.98	110.22
June:	W:	+ 9.24± 4.89	- 3.96±2.09	-	166.44
	S:	+13.24± 4.05*	- 3.16±4.91	+ 9.38±20.44	168.36
July:	W:	+ 7.58± 2.39*	- 3.25±1.03*	-	110.32
	S:	+ 0.64± 1.53	- 2.88±2.19	+16.66± 9.15	105.40
Aug:	W:	+ 4.54± 2.09*	- 1.94±0.90*	-	166.58
	S:	+ 1.52± 1.83	- 3.86±2.63	+15.60±10.95	170.32
Sept:	W:	+ 4.04± 1.89*	- 1.74±0.81*	-	80.18
	S:	+ 0.99± 0.82	- 1.88±1.18	+ 5.11± 4.90	75.47
Oct:	W:	+ 3.50± 1.60*	- 1.50±0.69*	-	142.22
	S:	+ 4.46± 2.61	- 7.52±3.75*	+15.62±15.63	145.80
Nov:	W:	+12.38± 6.84	- 5.30±2.93	-	306.42
	S:	+ 6.83± 4.90	- 7.09±7.05	-11.66±29.35	317.72
Dec:	W:	+ 3.54± 5.10	- 1.52±2.19	-	179.34
	S:	+ 2.90± 4.03	- 5.73±5.79	+16.82±24.12	191.52
Jan:	W:	+10.70± 4.92*	- 4.58±2.11*	-	216.45
	S:	+ 2.08± 4.55	- 3.92±6.55	+10.48±27.27	201.36
Feb:	W:	+ 0.64± 7.66	- 0.28±3.29	-	241.50
	S:	+11.14± 4.33*	-13.28±6.22*	- 5.06±25.90	213.48
Mar:	W:	+ 7.59± 5.57	- 3.25±2.39	-	183.24
	S:	+ 1.64± 3.03	- 1.22±4.36	- 6.61±18.16	174.82
Mean:	W:	+ 5.96± 2.00*	- 2.55±0.86*	-	172.43
	S:	+ 4.75± 1.39*	- 5.44±1.97*	+ 5.69± 8.21	171.72

Table 10. Estimates of the aspect parameters l_W, l_X, l_Y, l_Z and μ for the Wye and Severn catchments (units: millimetres)

(Values shown with asterisks are greater than twice their standard error in absolute magnitude)

		l_W:	l_X:	l_Y:	l_Z:	μ
April 71	Wye:	0.46±2.58	- 0.32±2.21	- 0.27± 7.35	0.41±1.80	69.96
	Severn:	2.13±1.51	- 2.18±1.35	0.34± 2.86	-2.34±3.76	71.04
May 71	W:	1.34±1.42	- 0.91±1.22	0.01± 4.06	-1.33±1.00	75.42
	S:	0.47±1.64	- 0.56±1.47	0.17± 3.12	-0.46±4.09	74.66
June 71	W:	10.06±6.88	- 0.10±5.91	- 6.04±19.64	-11.75±4.82*	198.71
	S:	8.52±5.94	- 5.56±5.34	- 2.99±11.31	-10.46±14.84	190.93
July 71	W:	1.26±1.76	2.01±1.51	- 0.44± 5.01	-8.49±1.23*	77.69
	S:	3.34±1.76	- 2.20±1.58	- 1.47± 3.34	-3.71±4.39	70.64
Aug 71	W:	1.39±3.02	3.58±2.60	- 4.32± 8.63	-1.95±2.12	214.11
	S:	6.94±4.03	- 1.46±3.62	- 7.50± 7.66	-8.48±10.06	217.08
Sept 71	W:	3.22±3.07	2.15±2.64	- 3.11± 8.76	-6.82±2.15*	101.11
	S:	2.81±2.06	- 1.45±1.85	- 4.72± 3.92	-0.25±5.14	88.95
Oct 71	W:	3.46±2.30	- 2.43±1.97	- 1.37± 6.55	1.04±1.61	209.62
	S:	11.49±6.78	-11.06±6.08	5.27±12.89	-17.39±16.92	211.27
Nov 71	W:	5.15±8.18	5.17±7.03	-9.19±23.36	-3.40± 5.74	313.78
	S:	10.94±10.62	-18.83+9.53	3.08±20.19	-0.52±26.51	307.21
Dec 71	W:	0.12±1.69	3.00±1.45*	-3.72±4.82	1.77±1.18	125.64
	S:	2.26±6.42	3.14±5.76	10.96±12.20	-11.70±16.02	127.39
Jan 72	W:	-0.72±4.32	0.55±3.71	1.98±12.34	-5.42±3.03	229.04
	S:	7.55±7.11	-9.70±6.38	26.15±13.52	-29.92±17.75	227.27
Feb 72	W:	4.59±6.18	-3.30±5.31	-2.69±17.65	4.19±4.33	153.33
	S:	8.06±5.71	-14.54±5.13*	6.64±10.86	-3.19±14.26	144.95

Mar 72	W:	6.25 ± 7.37	4.78 ± 6.33	-10.03 ± 21.04	-3.01 ± 5.17	233.57
	S:	0.01 ± 5.74	-19.26 ± 5.16*	31.22 ± 10.93*	1.41 ± 14.34	212.44
April 72	W:	6.25 ± 12.61	4.78 ± 10.83	-10.03 ± 35.99	-3.01 ± 8.84	273.36
	S:	-2.34 ± 11.59	-12.10 ± 10.40	15.34 ± 22.04	11.06 ± 28.94	301.26
May 72	W:	-3.57 ± 4.07	5.10 ± 3.50	- 2.30 ± 11.62	2.30 ± 2.85	134.16
	S:	6.86 ± 4.73	-2.65 ± 4.25	0.59 ± 9.00	-14.44 ± 11.82	145.79
June 72	W:	2.73 ± 4.89	6.75 ± 4.20	- 7.81 ± 13.96	- 5.00 ± 3.43	192.78
	S:	10.54 ± 7.17	7.10 ± 6.44	- 8.03 ± 13.64	-31.72 ± 17.91	196.85
July 72	W:	5.82 ± 3.74	4.08 ± 3.22	- 5.89 ± 10.69	-12.02 ± 2.62*	142.95
	S:	6.06 ± 3.78	2.54 ± 3.39	- 9.22 ± 7.19	-11.02 ± 9.44	140.17
Aug 72	W:	-0.50 ± 1.98	2.48 ± 1.70	- 2.00 ± 5.66	0.04 ± 1.39	119.06
	S:	4.99 ± 2.95	0.16 ± 2.65	- 5.61 ± 5.61	- 7.92 ± 7.36	123.56
Sept 72	W:	-0.19 ± 1.11	1.55 ± 0.95	- 1.50 ± 3.17	0.43 ± 0.78	59.26
	S:	0.56 ± 0.86	0.70 ± 0.77	2.40 ± 1.64	- 5.04 ± 2.15*	61.99
Oct 72	W:	2.28 ± 1.54	-0.25 ± 1.32	- 2.40 ± 4.39	1.13 ± 1.08	74.82
	S:	1.83 ± 2.19	-0.81 ± 1.97	1.62 ± 4.17	- 5.04 ± 5.47	80.34
Nov 72	W:	6.01 ± 8.53	9.33 ± 7.32	-15.49 ± 24.34	0.44 ± 5.98	299.06
	S:	17.20 ± 8.14*	-12.38 ± 7.31	-11.55 ± 15.48	-13.34 ± 20.32	328.23
Dec 72	W:	-6.72 ± 8.64	3.76 ± 7.42	- 6.24 ± 24.67	27.60 ± 6.06*	233.04
	S:	11.26 ± 8.93	-2.78 ± 8.02	- 5.65 ± 16.98	-19.76 ± 22.29	255.66
Jan 73	W:	6.48 ± 7.32	8.54 ± 6.28	-13.92 ± 20.88	- 3.31 ± 5.13	203.86
	S:	-3.59 ± 10.19	-0.42 ± 9.15	- 8.85 ± 19.38	19.16 ± 25.45	175.45
Feb 73	W:	15.58 ± 11.70	14.00 ± 10.05	-31.44 ± 33.41	5.60 ± 8.20	329.67
	S:	10.46 ± 10.33	-16.49 ± 9.28	-12.09 ± 19.65	12.40 ± 25.80	282.00
Mar 73	W:	6.90 ± 6.19	- 0.92 ± 5.31	- 4.50 ± 17.66	- 4.44 ± 4.34	132.92
	S:	6.77 ± 5.96	2.97 ± 5.35	- 8.93 ± 11.34	-13.67 ± 14.88	137.21

Means of aspect parameters over two years:-

		l_W:	l_X:	l_Y:	l_Z:	μ
April	W:	+ 3.36± 6.44	+ 2.23±5.53	- 5.15±18.37	- 1.30± 4.51	171.66
	S:	- 0.10± 5.84	- 7.14±5.24	+ 7.84±11.11	+ 4.36±14.59	186.15
May	W:	- 1.12± 2.16	+ 2.10±1.85	- 1.14± 6.15	+ 0.48± 1.51	104.79
	S:	+ 3.66± 2.50	- 1.60±2.25	+ 0.38± 4.76	- 7.45± 6.25	110.22
Jun	W:	+ 6.40± 4.22	+ 3.32±3.63	- 6.93±12.05	- 8.38± 2.96*	166.44
	S:	+ 9.53± 4.66*	+ 0.77±4.18	- 5.51± 8.86	-21.09±11.63	168.36
July	W:	+ 3.54± 2.07	+ 3.04±1.78	- 3.16± 5.90	-10.26± 1.45*	110.32
	S:	+ 4.70± 2.08*	+ 0.17±1.87	- 5.34± 3.96	- 7.36± 5.21	105.40
Aug	W:	+ 0.44± 1.81	+ 3.03±1.55	- 3.16± 5.16	- 0.96± 1.27	166.58
	S:	+ 5.96± 2.50*	- 0.65±2.24	- 6.56± 4.75	- 8.20± 6.23	170.32
Sept	W:	+ 1.52± 1.63	+ 1.85±1.40	- 2.30± 4.66	- 3.20± 1.14*	80.18
	S:	+ 1.68± 1.12	- 0.38±1.00	- 1.16± 2.12	- 2.64± 2.79	75.47
Oct	W:	+ 2.87± 1.38*	- 1.34±1.19	- 1.88± 3.94	+ 1.08± 0.97	142.22
	S:	+ 6.66± 3.56	- 5.94±3.20	+ 3.44± 6.77	-11.22± 8.89	145.80
Nov	W:	+ 5.58± 5.91	+ 7.25±5.07	-12.34±16.87	- 1.48± 4.14	306.42
	S:	+14.07± 6.69	-15.60±6.01	- 4.24±12.72	- 6.93±16.70	317.72
Dec	W:	- 3.30± 4.40	- 3.38±3.78	- 4.98±12.57	+14.68± 3.09*	179.34
	S:	+ 6.76± 5.50	- 2.96±4.94	+ 2.66±12.05	-15.73±13.73	191.52
Jan	W:	+ 2.88± 4.25	+ 4.54±3.65	- 5.97±12.13	- 4.36± 2.98	216.45
	S:	+ 1.98± 6.21	- 5.06±5.58	+ 8.65±11.82	- 5.38±15.51	201.36
Feb	W:	+10.08± 6.62	+ 5.35±5.68	-17.06±18.89	+ 4.90± 4.64	241.50
	S:	+ 9.26± 5.90	-15.52±5.30*	- 2.72±11.23	+ 4.60±14.74	213.48
Mar	W:	+ 6.58± 4.81	+ 1.93±4.13	- 7.26±13.73	- 3.73± 3.38	183.24
	S:	+ 3.39± 4.74	- 8.14±3.72	+11.14± 7.88	- 6.13±10.33	174.82
Mean	W:	+ 3.24± 1.73	+ 3.06±1.48*	- 5.94± 4.94	- 1.04± 1.21	172.43
	S:	+ 5.63± 1.87*	- 5.17±1.68*	+ 0.72± 3.63	- 6.93± 4.67	171.72

As was to be expected, inspection of Table 8 shows that there is a strong tendency for values of the altitude parameters to increase with altitude. On average, gauges in the first altitude class on the Severn (class A: 320 to 424 metres) caught 235 mm less rainfall over a year than the mean for all gauges, whilst those in the fourth altitude range (635 to 740 metres) caught 307 mm more rainfall than the mean for all gauges. For the Wye, the pattern was similar; gauges in class A (340 to 439 metres) caught 126 mm less rainfall than the mean, whilst those in class D (640 to 740 metres) caught 281 mm more rainfall than the mean. In Table 8, values of the altitude parameters which, in absolute magnitude, are greater than twice their standard errors are marked with an asterisk; if monthly rainfall showed no significant association with altitude, roughly one in twenty of the entries would be expected to exceed twice its standard error, say 10 of the 192 entries. In fact, 81 asterisks appear, a further indication of the significant association between monthly rainfall and altitude.

Table 9 shows the values of the slope parameters with their standard errors. By contrast with the strong association between monthly rainfall and altitude, the association between monthly rainfall and slope is much less well-defined, and rarely significant statistically. Nevertheless, some kind of association between slope and monthly rainfall is apparently present, as is shown by a study of the mean values at the end of the table; in every month, on both the Wye and the Severn, the catch by gauges in the slope range 0° to 9° is greater than the arithmetic mean of all gauges for the month, whilst the converse is true for gauges in the slope range 10° to 19°. The explanation of this phenomenon is not easy to find, particularly as the pattern is similar (so far as can be judged in the absence of data from the third slope class on the Wye) for both Wye and Severn catchments. Had it been observed only for the Wye, it could perhaps have been explained by the fact that gauge orifices in that catchment are parallel to the ground surface, so that if, on average, the rainfall tends to fall vertically over a month, gauges on steeper slopes would tend to catch less; however, the canopy gauges on the Severn have their orifices horizontal, so that differential catch by gauges with varying orifice angles is unlikely to be the explanation. A second possibility may be that more of the higher altitude gauges (having above average catches) tend to be in domains with shallower slope, whereas lower altitude gauges are concentrated on steeper slopes. Table 11 shows the distribution of gauges by slope and altitude for the Wye and Severn.

Table 11. Distribution of gauge numbers with slope and altitude

Wye catchment

		Altitude class: A	B	C	D	
Slope class	1	1	3	0	2	6
	2	4	6	4	0	14
	3	0	0	0	0	0
		5	9	4	2	20

Severn catchment

		Altitude class: A	B	C	D	
Slope class	1	4	1	3	2	10
	2	1	4	2	1	8
	3	1	0	0	0	1
		6	5	5	3	19

Although it is true that on the Wye there is a preponderance of lower-altitude gauges (classes A and B) in the higher slope class (class 2), this statement is less valid for the Severn; whether the observed result is attributable to distribution of gauges is therefore doubtful.

Table 10 shows that there is no strong relation, either, between monthly rainfall and aspect classes. Of the 192 aspect parameters in Table 10, the absolute values of 11 exceed twice their standard error, a proportion (11/192) that is very close to what would be expected purely on the basis of chance. As with the slope parameters, however, there are certain consistencies of sign which suggest that an association between aspect and monthly rainfall, although small, is nevertheless present: for both the Wye and Severn, for example, monthly rainfall caught by North-East-facing gauges (class W) tend to be greater than the mean for the whole catchment, whilst in the Wye the month's catch by South-East-facing gauges tends to be less than the mean. This is an odd result because prevailing rain-bearing winds come from the South-West, sometimes veering locally up the valley of the Wye from the South-East. This could be explained by the observed tendency in the Gerig sub-catchment for precipitation to be "swept" from windward slopes and deposited in the lee.

To examine how far the slope and aspect effects, although generally small and not significant statistically, may be explained in terms of altitudinal differences between gauges, an analysis of variance was used to answer the following questions:

Given that the variation in monthly rainfall is partly explained by altitude-class differences, how much of the *remaining variation* is accounted for by slope class differences (after allowing for the altitude class differences)?

After the variation in monthly rainfall that is accounted for by altitude and slope class-differences has been allowed for, how much of the remaining variation can yet be accounted for in terms of differences between aspect classes?

The analysis of variance was constructed month by month for the period April 1971 to March 1973. The sources of variation were partitioned as shown in the following (using the 19 gauges of the Severn, ie 18 from the network, plus the check gauge from Moel Cynnedd):

Source of variation	Degrees of freedom
Variation in monthly rainfall accounted for by altitude differences	3
(ie fitting a_A, a_B, a_C, a_D)	
Variation due to slope class differences, after altitude class differences are allowed for	2
(fitting s_1, s_2, s_3, allowing for a_A, a_B a_C, a_D)	
Variation accounted for by both altitude and slope class differences	5
Variation due to aspect class differences, after altitude and slope class differences are allowed for	3
(ie fitting l_w, l_x, l_y, l_z, allowing for a_A, a_B, a_C, a_D and s_1, s_2, s_3)	
Variation due to altitude, aspect and slope class differences	8
(ie fitting a_A, a_B, a_C, a_D; s_1, s_2, s_3; l_w, l_x, l_y, l_z)	
Residual (unexplained) variation	10
Total variation	18

Table 12 shows summaries of the relevant parts of the analyses of variance of each month's data. This confirms the significance of the relation between monthly rainfall and altitude; with the effect of altitude removed, the relation between monthly rainfall and slope is seldom statistically significant. (This does not, of course, imply that none exists; it is simply that the data were such that the relation between monthly rainfall and slope was too small to be distinguished from the background of random variation). Similarly, when both slope and altitude effects have been allowed for, the relation between monthly rainfall and aspect was no greater than the random variation inherent in the data.

The residual variances, obtained from each month's analysis of variance, were used to compute the coefficients of variation (CV) month by month. These are shown in Table 13. A comparison with the CV values shown in Table 6 is of interest; those in the latter table were computed by a different method, namely, by use of the residual variance from a two-factor (periods, gauges) analysis of variance. The mean CV values for the method of Table 13 were 6.7% and 9.6% for the Wye and Severn respectively, compared with 7.5% and 10.5% using the method of Table 6. The values given by different methods are therefore not too dissimilar in this instance. Taking the CV's for the Wye and Severn to be 7% and 10% respectively, we can estimate very roughly the numbers of period gauges required to give monthly estimates of mean areal rainfall with

Table 12. Summaries of analyses of variance† of monthly rainfall, Wye and Severn catchments

Severn catchment

Mean square (April 1971 - March 1973):

	df	A	M	J	J	A	S
Altitude	3	67.48	169.28**	2727.90**	299.06**	1258.99**	787.78***
Slope|altitude	2	7.35	17.15	167.02	29.35	233.99	134.48
Aspect|slope, altitude	3	23.05	1.23	280.14	41.54	179.36	37.16
Residual	10	21.08	24.94	328.40	28.74	150.77	39.42

O	N	D	J	F	M
461.47	4011.40**	237.20	1781.98*	11774.27***	7876.90***
712.36	872.75	140.94	587.23	796.35	421.15
702.35	1109.70	167.06	1145.72	675.03	1713.77*
426.90	1047.9	382.38	469.73	303.10	306.64

A	M	J	J	A	S
3247.20	160.27	2714.85*	1673.30***	422.17*	31.70*
3954.40	13.35	107.80	101.83	4.40	26.90*
629.71	191.03	798.48	214.35	103.47	13.62
1247.90	208.06	477.89	132.76	80.83	6.88

O	N	D	J	F	M
491.02**	5628.43**	540.61	1463.66	15713.13***	5236.73***
6.80	398.80	30.20	157.71	2556.05	99.33
18.79	1136.77	458.69	199.63	981.43	257.33
44.64	615.31	740.77	965.22	991.94	330.24

†Mean squares that are marked *, ** and *** are statistically significant at the 5%, 1% and 0.1%, levels respectively.

Wye catchment

Mean square (April 1971 - March 1973):

	df	A	M	J	J	A	S
Altitude	3	33.33	219.26***	569.14	458.93	769.86***	172.76
Slope|altitude	1	4.14	0.17	987.86	324.72**	234.91	289.54*
Aspect|slope, altitude	3	0.76	5.86	322.26	51.32	64.92	70.23
Residual	12	38.19	11.65	272.66	17.74	52.64	54.20

	O	N	D	J	F	M
Altitude	924.02***	1132.63	57.48*	1411.86***	8482.63***	3745.43***
Slope|altitude	260.47*	1456.47	121.88*	29.12	0.30	583.30
Aspect|slope, altitude	35.15	261.49	46.72	26.22	78.13	305.27
Residual	30.34	385.46	16.44	107.58	220.02	312.93

	A	M	J	J	A	S
Altitude	1703.37	1226.29***	1653.86***	832.29**	416.53***	71.65**
Slope|altitude	297.59	63.30	516.42	755.05**	63.94	32.37
Aspect|slope, altitude	981.18	86.56	226.26	233.56	19.89	9.23
Residual	914.94	95.42	137.71	80.68	22.68	7.10

	O	N	D	J	F	M
Altitude	622.04***	1186.17	757.74	449.03	4879.37**	1935.85**
Slope|altitude	2.08	740.03	19.06	2635.50*	53.60	363.74
Aspect|slope, altitude	20.59	689.60	656.20	583.85	2712.20	132.26
Residual	13.64	418.57	429.97	308.01	788.48	220.25

given precision. These estimates, calculated from the naive formula

$$n = \frac{4(CV)^2}{d^2} \qquad (4.2.1)$$

(where d is the required precision) are shown in Table 14, which suggests that with the existing network of period gauges mean areal rainfall is estimated to within rather less than 5% of its true value.

Table 13. Coefficients of variation calculated from residual variance from analyses of monthly rainfall

Coefficients of variation (CV) for Wye and Severn catchments

Wye catchment

	1971-72	1972-73
Apr	8.8	11.1
May	4.5	7.3
June	8.3	6.1
July	5.4	6.3
Aug	3.4	4.0
Sept	7.3	4.5
Oct	2.6	4.9
Nov	6.2	6.8
Dec	3.2	8.9
Jan	4.5	8.6
Feb	9.7	8.5
Mar	7.6	11.2
Mean	6.0	7.4

Severn catchment

	1971-72	1972-73
Apr	6.5	11.7
May	6.7	9.9
June	9.5	11.1
July	7.6	8.2
Aug	5.6	7.3
Sept	7.0	4.2
Oct	9.8	8.3
Nov	10.5	7.6
Dec	15.4	10.6
Jan	9.5	17.7
Feb	12.0	11.1
Mar	8.2	13.2
	9.0	10.1

Table 14. Numbers of period gauges required to estimate monthly mean rainfall with given precision

	Estimate to lie within X% of the true areal mean rainfall:				
	X = 20%	X = 10%	X = 5%	X = 2.5%	X = 1%
Approx. number of gauges (Wye):	1	2	8	31	196
Approx. number of gauges (Severn):	1	4	16	64	400

4.3 Examination for possible differences, from catchment to catchment, in the relation between monthly gauge catch and altitude class

Since altitude appears to be the one "domain factor' having strong association with the varying amounts of rainfall caught by period gauges, a calculation was made to assess the evidence showing whether or not the pattern of increasing catch with altitude class differed between the Wye and Severn. In statistical terms, a test was made to show whether an "interaction" existed between catchment and altitude; this procedure is not entirely valid because the altitude classes differed slightly from catchment to catchment, and it has already been suggested that this may help to explain pattern in the fitted altitude constants of Table 8; however, differences between the altitude classes in the two catchments are not great, and were ignored for the purpose of the present calculation.

A simple example using hypothetical data may clarify the meaning of the term "interaction between catchment and altitude", a term less familiar to hydrologists than statisticians. (The term may be further abbreviated to "catchment x altitude interaction", or even C x A, or CA). Suppose that monthly catches by gauges were such that their means could be arranged in a two-way table, as follows:-

		Wye:	Severn:	Mean:
Altitude class:	A:	110	110	110
	B:	105	105	105
	C:	95	95	95
	D:	90	90	90
Mean:		100	100	

Clearly there is no difference between the mean catches in the two catchments, but (assuming that no random errors are present) there are clear differences between the catches in gauges of different altitude class. However, since the differences between catches by gauges in different catchments, but in the same altitude class, are all equal (110-110 = 0; 105-105 = 0; 95-95 = 0- 90-90 = 0) there is said to be no interaction between catchment and altitude class. If the two-way table had been:-

		Wye:	Severn:	Mean:
Altitude class:	A:	105	95	100
	B:	105	95	100
	C:	105	95	100
	D:	105	95	100
Mean:		105	95	

then there is now a clear difference between the mean catches from the two catchments, but no altitude effect; also since the difference between catches by gauges in different catchments, but in the same altitude class, are all equal (105-95 = 10; 105-95 = 10; ...) there is said to be no interaction between catchment and altitude.

If, however, the two-way table had been either of the following:-

	Wye:	Severn:	Mean:		Wye:	Severn:	Mean:
	115	85	100		90	80	85
	105	95	100		95	90	92.5
	95	105	100		105	110	107.5
	85	115	100		110	120	115
Mean:	100	100		Mean:	100	100	

then in both cases the difference between catchments varies from altitude to altitude (115-85 = 30; 105-95 = 10; 95-105 = -10; 85-115 = -30, and 90-80 = 10; 95-90 = 5; 105-110 = -5; 110-120 = -10). In the first case the interaction is said to be negative (because rainfall would be increasing with altitude in the Severn, but decreasing in the Wye), whilst in the second it is positive (because rainfall increases with altitude in both catchments, but at a faster rate in the Severn).

The possible interaction between catchment and altitude class was examined by the analysis of variance of each month's data, using a linear model of the form

$$y_{ijk} = \mu + c_i + a_j + (ca)_{ij} + \varepsilon_{ijk}$$

where y_{ijk} is the catch by gauge k in the i^{th} catchment and j^{th} altitude class, μ, c_i and a_j are constants for the overall mean, catchment and altitude effects respectively, and $(ca)_{ij}$ is a term measuring the interaction. The test of the hypothesis of no interaction is then equivalent to a test of the hypothesis that $(ca)_{ij} = 0$ for all i and j.

Table 15 shows the relevant parts of the analysis of variance for each month of the two-year period. There was never any significant catchment x altitude interaction, suggesting that we can safely assume that the increase in raingauge catch with altitude is similar for both the Wye and the Severn. Table 15 shows, however, that differences between the (arithmetic) mean catch of gauges in the two catchments differ significantly in several months.

Table 15. Analyses of variance† of monthly totals: differences between catchments, differences between altitude classes, and catchment x altitude class interaction mean classes

	df	A	M	J	J	A	S
Catchments	1	15.54	13.69	591.51	573.35***	40.30	1510.02***
Altitudes	3	84.54*	264.53***	2887.72***	706.85***	1958.86***	860.33**
Catchment x altitude	3	16.24	37.75	419.39	21.59	85.20	77.72
Residual	31	24.37	14.08	311.38	37.49	115.32	62.10
		O	N	D	J	F	M
Catchments	1	0.13	538.18	35.16	90.09	1374.29*	5129.38***
Altitudes	3	1290.40**	4835.97**	242.55	2799.39***	19677.86***	10825.76***
Catchment x altitude	3	103.94	268.87	50.37	374.61	349.20	538.36
Residual	31	275.18	722.99	163.42	345.41	307.22	461.25
		A	M	J	J	A	S
Catchments	1	8531.22*	1145.90**	115.61	110.49	170.71	61.36*
Altitudes	3	2595.09	1106.47***	4269.91***	2450.54***	812.63***	97.46***
Catchment x altitude	3	2040.44	338.16	113.86	43.26	31.70	9.63
Residual	31	1177.34	133.82	330.25	148.33	49.22	9.96
		O	N	D	J	F	M
Catchments	1	217.51**	7467.72***	4726.13**	7454.13**	23734.74***	82.03
Altitudes	3	1123.88***	5981.04***	844.57	1736.55	18644.50***	6836.67***
Catchment x altitude	3	15.89	1108.19	539.91	311.94	1416.97	341.23
Residual	31	24.00	586.85	515.86	601.48	1149.28	247.63

†Mean squares that are marked *, ** and *** are statistically significant at the 5%, 1% and 0.1%, level respectively.

4.4 Use of domain theory to estimate mean areal rainfall at ungauged sites

Wye catchment period gauges

The purpose of a raingauge network is two-fold. First, it must provide an estimate of the mean areal rainfall over the region sampled by the network; second, it must provide estimates of rainfall at points in the region that are ungauged. Section 4.1 describes how domain theory could be used to provide estimates at an ungauged point: if it is sited in altitude class I, slope class J and aspect class K, the estimated rainfall is.

$$\hat{\mu} + \hat{a}_I + \hat{s}_J + \hat{l}_K$$

where $\hat{\mu}$, $\hat{a}_I$, $\hat{s}_J$ and l_k are estimated using records from the gauges constituting the network.

In addition to the records from 20 period gauges used to estimate the parameters on the Wye, data are also available from a period gauge from Cefn Brwyn that did not form part of the domain network. Using the method described above, monthly rainfall at this site can be estimated for comparison with that actually observed there. It is also of interest to compare the estimates of monthly rainfall given by domain theory with those given by other procedures for interpolation, such as trend surface analysis. Table 16 shows observed monthly rainfall at Cefn Brwyn (y) together with estimates given by: (a) domain theory; (b) fitting linear trend surfaces; (c) fitting quadratic surfaces; (d) fitting cubic surfaces. The method of orthogonal polynomials is used in (b) - (d). The table shows that domain theory estimates are to be preferred in this instance to those given by trend surface analysis; the mean difference between true values and estimates is smaller, and they are to be preferred also for their smaller mean square error. Amongst the trend surface estimates, those derived from a quadratic trend surface had greatest accuracy, those derived from a linear trend surface had smallest mean square error; those derived from a cubic trend surface were least satisfactory on both accounts.

Severn catchment period gauges

The same calculation was repeated for the period gauge at Carreg Wen in the Upper Severn. In this case, both domain theory estimates and trend surface estimates considerably underestimate the true rainfall, with little to choose between them either in terms of accuracy or mean square error (Table 17).

Table 17. Interpolated rainfall at Carreg Wen (Severn catchment, CIX): comparison of observed monthly rainfall with estimates given by different methods. (For explanation see text).

Observed rainfall (y):		(a):	(y)-(a):	(b):	(y)-(b):	(c):	(y)-(c)	(d):	(y)-(d):
April 71	79.4	68.5	+10.9	72.0	+ 7.4	72.2	+ 7.2	73.6	+ 5.8
May	81.5	80.1	+ 1.4	76.5	+ 5.0	76.7	+ 4.8	78.7	+ 2.8
June	222.3	209.7	+12.6	199.9	+22.4	207.7	+ 14.6	220.9	+ 1.4
July	80.7	75.3	+ 5.4	73.9	+ 6.8	75.4	+ 5.3	79.2	+ 1.5
Aug	233.3	233.6	- 0.3	222.1	+11.2	227.8	+ 5.5	241.8	- 8.5
Sept	97.7	100.2	- 2.5	92.5	+ 5.2	94.9	+ 2.8	103.4	- 5.7
Oct	242.4	198.8	+43.6	214.0	+28.4	213.1	+ 29.3	228.3	+ 14.1
Nov	399.8	300.0	+99.8	313.4	+86.4	301.7	+ 98.1	332.7	+ 67.1
Dec	144.5	128.7	+15.8	127.5	+17.0	123.0	+ 21.5	131.5	+ 13.0
Jan 72	253.9	230.2	+23.7	236.7	+17.2	222.4	+ 31.5	229.2	+ 24.7
Feb	179.0	126.4	+52.6	158.0	+21.0	132.5	+ 46.5	125.1	+ 53.9
Mar	273.0	191.4	+81.6	225.3	+47.7	197.9	+ 75.1	187.3	+ 85.7
April	336.0	286.4	+49.6	286.6	+49.4	273.2	+ 62.8	286.2	+ 49.8
May	169.8	147.0	+22.8	147.5	+22.3	149.5	+ 20.3	158.1	+ 11.7
June	225.1	212.8	+12.3	202.6	+22.5	215.7	+ 9.4	216.4	+ 8.7
July	156.7	155.9	+ 0.8	145.1	+11.6	152.0	+ 4.7	160.8	- 4.1
Aug	140.2	131.2	+ 9.0	126.2	+14.0	131.4	+ 8.8	137.7	+ 2.5
Sept	65.0	64.2	+ 0.8	63.7	+ 1.3	64.3	+ 0.7	65.3	- 0.3
Oct	91.5	87.5	+ 4.0	83.7	+ 7.8	83.2	+ 8.3	88.5	+ 3.0
Nov	391.7	341.3	+50.4	342.7	+49.0	340.8	+ 50.9	362.2	+ 29.5
Dec	292.1	277.5	+14.6	257.6	+34.5	263.9	+ 28.2	280.0	+ 12.1
Jan 73	214.1	192.8	+21.3	175.0	+39.1	177.5	+ 36.6	181.7	+ 32.4
Feb	396.6	286.1	+110.5	294.0	+102.6	277.1	+119.5	289.5	+107.1
Mar	151.1	137.0	+ 14.1	144.3	+ 6.8	142.8	+ 8.3	137.4	+ 13.7
Mean difference:			+ 27.283		+ 26.525		+ 29.196		+ 21.746
Mean square error:			± 41.462		± 36.448		± 42.711		± 36.702

Wye catchment AWS gauges

Table 18 compares domain theory estimates with the observed catches (obtained by accumulating five-minute catches over a month) from the two automatic weather stations at Eisteddfa Gurig on the Wye catchment; AWS records are occasionally incomplete, and absence of data is marked by a blank in the table.

Table 16. Interpolated rainfall at Cefn Brwyn (Wye catchment, A1Y): comparison of observed monthly rainfall with estimates given by different methods. (For explanation see text)

Observed rainfall (y)		(a):	(y)-(a):	(b):	(y)-(b):	(c):	(y)-(c):	(d):	(y)-(d):
April 71	79.4	67.7	11.7	72.4	7.0	67.9	11.5	80.7	- 1.3
May	72.8	71.8	1.0	65.7	7.1	76.3	- 3.5	85.8	-13.0
June	184.3	191.8	- 7.5	183.9	0.4	179.2	5.1	227.6	-43.3
July	62.0	76.3	-14.3	57.6	4.4	61.8	0.2	60.8	1.2
Aug	201.5	205.3	- 3.8	191.0	10.5	200.6	0.9	196.4	5.1
Sept	91.7	98.6	- 6.9	88.5	3.2	89.6	2.1	95.8	- 4.1
Oct	202.5	204.4	- 1.9	185.0	17.5	199.3	3.2	210.8	- 8.3
Nov	316.3	310.3	+ 6.0	296.0	20.3	310.9	5.4	349.5	-33.2
Dec	120.2	124.6	- 4.4	123.3	- 3.1	124.2	- 4.0	116.4	3.8
Jan 72	230.3	216.8	+13.5	198.6	31.7	205.2	25.1	214.4	15.9
Feb	119.3	127.4	- 8.1	94.0	25.3	153.4	-34.1	110.7	8.6
Mar	204.7	207.7	- 2.3	196.3	8.4	219.8	-15.1	231.3	-26.6
April	272.0	271.0	+ 1.0	268.0	4.0	283.5	-11.5	279.0	- 7.0
May	120.9	121.7	- 0.8	107.0	13.9	120.5	+ 0.4	139.4	-18.5
June	174.4	176.4	- 2.0	159.8	14.6	172.7	+ 1.7	201.0	-26.6
July	125.5	136.1	-10.6	118.1	7.4	131.5	- 6.0	155.9	-30.4
Aug	105.5	110.9	- 5.4	103.8	1.7	105.7	- 0.2	117.6	-12.1
Sept	52.6	57.2	- 4.6	51.4	1.2	54.3	- 1.7	52.8	- 0.2
Oct	66.8	62.2	+ 4.6	55.6	11.2	68.1	- 1.3	84.8	-18.0
Nov	297.2	283.7	+13.5	280.6	16.6	308.2	-11.0	339.2	-42.0
Dec	224.5	222.5	+ 2.0	221.5	3.0	245.8	-21.3	273.1	-48.6
Jan 73	196.8	202.5	- 5.7	191.4	5.4	185.4	11.4	231.6	-34.8
Feb	307.7	285.9	+21.8	299.0	8.7	349.5	-41.8	383.0	-75.3
Mar	132.4	125.5	+ 6.9	109.7	22.7	137.2	- 4.8	135.8	- 3.4
Mean difference:			+ 0.154		+10.129		- 3.721		-17.171
Mean square error:			± 8.426		±13.220		±14.293		±27.250

Table 18. Comparison of domain theory estimates (a) with those observed by AWS (1) and (2): Eisteddfa Gurig, Wye catchment (Domain BIX)

	(a)	(1)	(2)	(a) - AWS mean
Sept 71	108.2	63.6		+ 44.6
Oct	207.3	157.5	158.0	+ 49.5
Nov	320.1		265.0	+ 55.1
Dec	134.6	119.0		+ 15.6
Jan 72	226.8	213.5	191.0	+ 24.6
Feb	133.4	120.5	112.0	+ 17.2
Mar	245.0	177.5		+ 67.5
April	93.1	269.0		
May	139.3	151.0	141.0	- 6.7
June	203.9	217.0	209.5	- 9.3
July	152.3	145.5	130.0	+ 14.5
Aug	123.8		157.5	- 33.7
Sept	62.0	67.0	65.5	- 4.2
Oct	71.5	80.0	78.0	- 7.5
Nov	317.3	282.0	271.5	+ 40.5
Dec	232.8	245.0		- 12.2
Jan 73	237.1			
Feb	338.0	281.5	281.5	+ 56.5
Mar	132.8			
			Mean:	+ 19.5
			Mean square error:	± 35.1

It is unwise to draw any firm conclusion from a comparison of domain theory estimates with observed rainfall at three sites only; however the following points can be made:

(i) domain theory estimates may be far from the true monthly rainfall, as at Carreg Wen in February 1973, where rainfall was underestimated by more than 110 mm. Taking an average over 24 months, domain theory estimates underestimated the true rainfall by about 27 mm per month at that site.

(ii) domain theory estimates may be reasonably near the true monthly rainfalls at certain sites, as at Cefn Brwyn. The maximum difference between domain theory estimates and observation there was 22 mm, also in February 1973. Taking an average over 24 months, the difference between observation and estimate was less than 1 mm per month. The general "closeness" of domain estimates at some sites, and their poor performance at others, suggests that each site must be considered as a special case when a decision is to be made whether or not to abandon a gauge there.

(iii) at the two sites at which trend surface estimates were also computed, domain theory estimates were to be preferred, on average, to trend surface estimates. Without further investigation, it would be unwise to conclude that domain theory estimates are always preferable, in any case, domain theory estimates may require the calculation of more parameters. Thus on the Wye and Severn catchments, domain theory required the estimation of nine and ten parameters respectively, whilst quadratic trend surface estimates required the estimation of seven. Much will depend on the number of domains used and the type of trend surface fitted.

4.5 The problem of domain definition

The rainfall surface is a continuum over each catchment as a whole, and the process of defining domains is equivalent to defining rules for the subdivision of a two-dimensional area A into elements, in each of which an ordinate to the surface is to be measured (ie point rainfall is to be observed) for the purpose of integrating the volume between the two-dimensional area and the surface above it; this volume, divided by A, gives the mean areal rainfall.

If one gauge only is sited in each element of area (domain) for the purpose of calculating mean areal rainfall, then the accuracy of the domain theory estimate depends on how closely the product of each measured point rainfall and its domain area approximates the true mean value of the rainfall surface over that domain. This mean value is, of course, always unknown; if, however, the variability in point rainfall measurements over each domain is small, then the domain theory estimate (or, indeed, any other estimate using different rules for dividing the two-dimensional map area A of the catchment into elements) is likely to be of good precision. Examination of within-domain variation in rainfall, if possible, is therefore desirable.

Of the domains in the Wye catchment, four (B1W, B2W, B2Y, C2X) had, by chance, duplicate gauges sited in them as shown below:-

domain:	gauge numbers:	locations:
B1W	9;2	Upper Wye; Afon Cyff
B2W	10;1	Upper Wye; Afon Cyff
B2Y	13;5	Upper Wye; Afon Cyff
C2X	12;8	Upper Wye; Afon Cyff

(No domain in the Severn contained more than one gauge). These four pairs enabled some estimate to be made of within-domain variation; using analysis of variance methods, variation amongst the eight gauge totals for each month was divided into two parts: (i) variation between the 4 domains; (ii) the average variation within domains. The larger the ratio of (i) to (ii), the better is the precision of the domain estimate likely to be. Table 19 shows the mean squares, between and within domains, together with the variance ratio ((i)/(ii)) for each of the 24 months.

The question "is the variability between raingauge readings within domains too great?" invites the questions "with what precision do you need to estimate mean areal rainfall for a month?" and "how much time and how many gauges are you prepared to divert in order to obtain such an estimate?" Answers to either question are difficult; we therefore do no more than present the values of Table 19 as they stand, with the observation that variation between domains is significantly larger than variation within domains in only 3 months of the 24, as judged by a variance ratio test (at the 5% level); this test is not a particularly sensitive one, however, being based on only 3 and 4 degrees of freedom.

Table 19. Variation between and within domains: Wye catchment

	Between domains MS:	Within domains MS($\hat{\sigma}_w^2$):	F:	Mean:	CV(=$\hat{\sigma}_w$/Meanx100%)
April 71	33.45	17.54	1.91	70.6	5.9
May	16.96	16.55	1.02	73.6	5.5
June	561.20	218.34	2.57	211.0	7.0
July	142.30	8.43	16.88*	80.6	13.6
Aug	217.02	61.81	3.51	216.5	3.6
Sept	114.28	59.20	1.93	106.0	7.3
Oct	301.09	24.88	12.10*	209.7	2.4
Nov	844.61	131.30	1.93	318.4	3.6
Dec	19.38	18.90	1.02	125.8	3.5
Jan 72	542.47	86.15	6.30	235.5	6.8
Feb	277.94	60.45	4.60	138.5	5.6
Mar	652.84	572.94	1.14	239.4	10.0
April	1395.62	719.01	1.94	271.0	9.9
May	455.86	97.85	4.66	133.8	7.4
June	767.41	219.89	3.49	196.4	7.6
July	450.16	138.97	3.24	147.6	8.0
Aug	103.06	25.14	4.10	119.9	4.2
Sept	23.56	15.17	1.55	58.8	6.6
Oct	127.18	7.88	16.13*	75.3	3.7
Nov	865.32	412.54	2.10	302.3	6.7
Dec	875.29	278.73	3.14	233.3	7.2
Jan 73	548.65	483.58	1.13	211.9	10.4
Feb	1339.07	233.90	5.72	340.0	4.5
Mar	237.77	228.46	1.04	133.9	12.7

*Significant at the 5% level.

5. GENERAL CHARACTERISTICS OF HOURLY RAINFALL ON THE WYE AND SEVERN CATCHMENTS

The overall pattern of monthly rainfall, its distribution within each catchment and its relationship to topography have been described in previous sections. A study of hourly rainfall along similar lines is not practical because there are, at present, only three separate hourly rainfall recording sites in both catchments so that topographic coverage is limited. The emphasis in this section is rather on the precision of measurement of the individual hourly values and on the precision of catchment mean hourly rainfall based on these values.

The following aspects of hourly rainfall will be considered: the serial and cross correlation present in the data; the random variation in the data; the error in predicting the rainfall at one gauge from that at another, and finally, the duration of storms and the average rainfall for given storm duration.

In connection with storm duration, there is the problem of dealing mathematically with dry periods between storms. This question has still to be resolved satisfactorily, but for the purposes of the present study, the following ad hoc procedure was adopted: data from hours during which no rain fell at any of the sites within the catchment under consideration were excluded from calculations. Thus the only zeros to be included occurred while rainfall was recorded at one or more sites in the catchment.

One major drawback in analysing short-period rainfall records is snowfall, which causes recording gauges to fill up with snow. Calculations based on hourly values recorded during winter (October to March) are therefore highly suspect, and although they are quoted, they should be treated as unreliable.

The type and appropriate symbol of the gauges used in the analysis are listed below and their locations are shown in Figure 1.

Wye catchment

Location	Type	Symbol
Eisteddfa Gurig	Dines	E
Eisteddfa Gurig	Rimco (AWS)	EG1
Eisteddfa Gurig	Rimco (AWS)	EG2
Cefn Brwyn	Dines	C
Cefn Brwyn	Artec (AWS)	CB
Esgair y Maen	Dines	F

Severn catchment

Location	Type	Symbol
Moel Cynnedd	Dines	A
Moel Cynnedd	Rimco (AWS)	T1
Moel Cynnedd	Rimco (AWS)	T2
Carreg Wen	Dines	B
Carreg Wen	Rimco (AWS)	CW2
Watershed	Dines	D

5.1 Correlation within the data

Correlation between the rainfall in one hour and that in the next is undoubtedly a characteristic of the data. The existence of serial correlation affects the precision of estimates of catchment mean rainfall; ideally, then, models representing hourly rainfall should incorporate this feature. However, as Figures 4 and 5 indicate, the serial correlation, while apparent, is not as marked as the cross correlation, and for the following analyses will be assumed to be negligible.

Given a model of the form (2.3.2), estimates of the gauge-to-gauge and residual variation may be calculated (see section 5.2). Denoting these by σ_{ε}^2 and σ_{ε}^2 respectively, the variance of the mean rainfall $\bar{x}$ from m gauges in any given time period is given by:

$$\text{var}\ (\bar{x}) = \frac{1}{m} \left[\sigma_{\varepsilon}^2\ (1 + (m - 1)\bar{\rho}) + \sigma_{\varepsilon}^2 \right], \qquad (5.1.1)$$

where $\bar{\rho}$ is the average correlation between all pairs of gauges. Thus the presence of cross correlation in the data may limit the precision attained by the mean. Intuitively, this is because the information provided by any one gauge is repeating part of the information provided by the other gauges.

In order to estimate the cross correlation present, the Dines gauges were initially considered on their own. Data from 1972 for each catchment was extracted according to the criteria described in the introduction (ie dry periods were excluded). Means, standard deviations and correlation coefficients were calculated for each month, and are shown in Table 20; again it is emphasized that the winter values are unreliable. The table also shows that the number of rainfall hours per month was greater for the Severn than for the Wye in eleven of the twelve months.

Two further points should be noted here:

(1) the quantities of Table 20 refer to values during rainfall periods (not during whole month) and (2) the correlation coefficients are, in most cases, biassed (over-estimated) because some of the basic data had been estimated by regression when observations were missing.

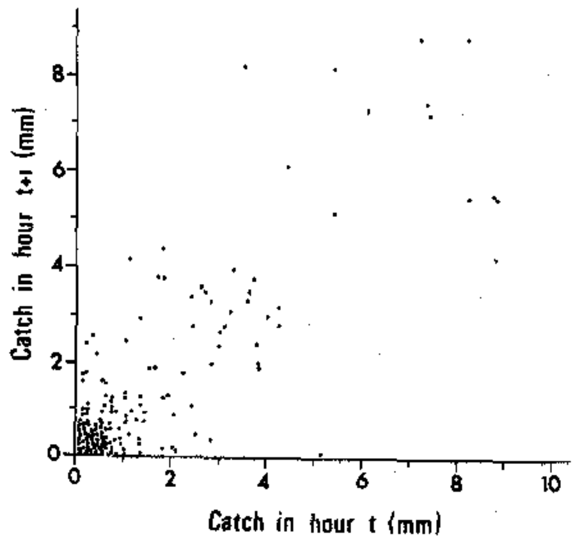

Figure 4. Carreg Wen Dines (gauge B) catch in hour t against catch in hour t + 1, November 1972.

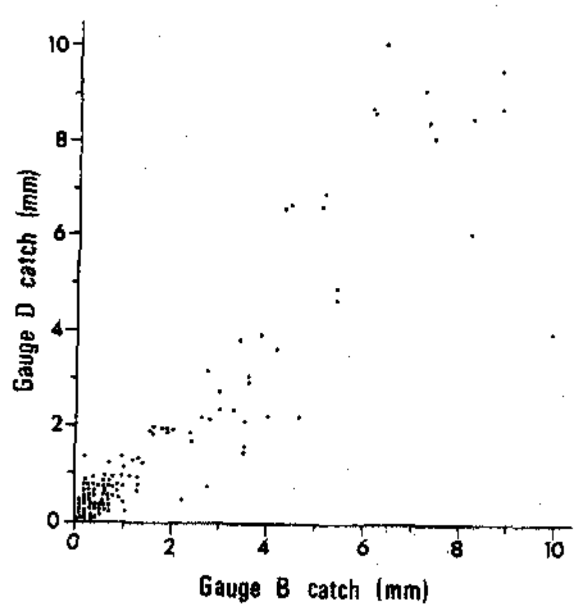

Figure 5. Carreg Wen Dines (gauge B) catch against Watershed Dines (gauge D) catch, November 1972.

Table 20. Hourly rainfall (mm). Means, standard deviations and correlation coefficients for rainfall hours in 1972

Wye catchment (Gauges C,E and F)

	Number of rainfall hours	Means* C	Means* E	Means* F	Standard deviations C	Standard deviations E	Standard deviations F	Correlation coefficients† C,E	Correlation coefficients† E,F	Correlation coefficients† C,F
Jan	235	0.81	0.78	0.84	1.25	1.14	1.23	0.86	0.87	0.99
Feb	149	0.56	0.65	0.66	0.60	0.70	0.72	0.86	0.87	0.98
Mar	182	0.82	0.90	1.02	1.26	1.36	1.52	0.86	0.87	0.98
April	211	1.11	1.18	1.33	1.39	1.52	1.62	0.84	0.89	0.88
May	189	0.62	0.78	0.68	0.84	0.99	0.86	0.86	0.91	0.96
June	238	0.60	0.72	0.81	0.76	0.85	0.95	0.83	0.83	0.95
July	129	0.86	1.03	1.14	1.40	1.59	1.70	0.94	0.96	0.99
Aug	109	0.81	0.95	1.11	1.20	1.45	1.72	0.86	0.68	0.57
Sept	69	0.72	M	0.89	1.08	M	1.14	M	M	0.93
Oct	106	0.54	0.59	0.64	0.92	1.05	1.05	0.89	0.90	0.96
Nov	282	0.68	0.75	0.80	1.12	1.38	1.32	0.89	0.92	0.98
Dec	231	0.86	0.77	0.90	1.10	1.12	1.28	0.83	0.72	0.85

Severn catchment (Gauges A, B and D)

	Number of rainfall hours	Means* A	Means* B	Means* D	Standard deviations A	Standard deviations B	Standard deviations D	Correlation coefficients† A,B	Correlation coefficients† B,D	Correlation coefficients† A,D
Jan	222	0.91	1.00	0.88	1.30	1.49	1.22	0.99	0.96	0.96
Feb	162	0.59	0.69	0.58	0.68	0.83	0.70	0.91	0.98	0.90
Mar	197	0.77	0.87	1.00	1.24	1.23	1.53	0.88	0.89	0.84
April	222	1.36	1.51	1.26	1.72	1.90	1.70	0.99	0.75	0.76
May	226	0.62	0.72	0.67	0.84	0.97	0.93	0.85	0.85	0.72
June	243	0.62	0.73	0.85	0.92	0.93	1.11	0.88	0.87	0.80
July	134	0.91	0.99	1.09	1.39	1.55	1.56	0.95	0.83	0.79
Aug	113	1.02	1.10	1.10	1.53	1.50	1.69	0.91	0.50	0.53
Sept	73	0.82	0.85	0.80	1.27	1.16	1.14	0.90	0.90	0.91
Oct	133	0.50	0.82	0.58	0.99	0.57	1.03	0.91	0.96	0.88
Nov	306	0.87	0.85	0.87	1.55	1.54	1.62	0.93	0.97	0.93
Dec	240	1.03	0.95	1.06	1.48	1.45	1.56	0.91	0.94	0.92

M denotes missing value

* Mean of rainfall during storms, not overall mean

† Inflated due to missing-value estimation

The average correlations (ρ) over the summer months (April to September, 1972) were as follows:-

Wye catchment		Severn catchment	
ρ_{CE}:	0.85	ρ_{AB}:	0.91
ρ_{EF}:	0.85	ρ_{BD}:	0.78
ρ_{DF}:	0.88	ρ_{AD}:	0.75

Average for Wye ($\bar{\rho}_w$): 0.86. Average for Severn ($\bar{\rho}_S$): 0.81.

The combination of AWS data with recording gauge data is time-consuming and it was only possible to obtain a combined sample for one month (April 1972). Even then, data for site B had to be excluded, as it was unsatisfactory. (This situation is likely to improve in the future when a comprehensive data-storage system is in operation). The correlation coefficients for all possible pairs are listed in Table 21 and are probably more realistic than those listed in Table 20 because of the exclusion of unsatisfactory data.

It has been suggested that correlation between gauges depends on their distance apart. On the basis of Tables 20 and 21, there appears to be little evidence for this hypothesis, except that (not surprisingly) those actually at the same location are more highly correlated with each other than with values showed any such relationship.

Table 21. Cross correlation between gauges. Dines recording gauge and AWS Rimco data, April 1972

Wye catchment

	C	E	F	EG1	EG2
C	1	0.68	0.85	0.86	0.83
E		1	0.68	0.73	0.76
F			1	0.93	0.91
EG1				1	0.95
EG2					1

Number of hours = 141

Severn catchment

	A	D	T1	T2	CW2
A	1	0.80	0.89	0.91	0.80
D		1	0.76	0.80	0.76
T1			1	0.93	0.77
T2				1	0.80
CW2					1

Number of hours = 196

Finally the totals over April 1972 are as follows (excluding unreliable data):

Wye catchment: number of observations = 141 (April 1972)

		Totals (mm)
Dines gauges:-		
	C:	183.30
	E:	198.81
	F:	208.68
AWS gauges:-		
	EG1:	195.99
	EG2:	180.48
Mean of EG1 and EG2:		188.84

The mean of the two AWS totals (188.84) agrees with the equivalent Dines gauge (E) total (198.81) to within 5% over 141 hourly values.

Severn catchment: number of observations = 196 (April 1972)

Dines gauges:-		Totals (mm)
	A:	211.68
	D:	221.48
AWS gauges:-		
	T1:	225.40
	T2:	227.36
	Mean of T1 and T2:	226.38
	CW2:	273.44

The mean of the two AWS totals at Moel Cynnedd (226.38) agrees with the equivalent Dines gauge (A) total (211.68) to within 7%. Such differences in catch as exist between Dines and AWS gauges may be attributed to gauge height, the Dines gauge being at standard height and the AWS gauge being at ground level.

5.2 Random variation in measured hourly rainfall

A model for the hourly values must be assumed before their precision can be estimated. The model used in section 2.3 to represent the monthly values is also appropriate here;

$$y_{ij} = \mu + g_i + \rho_j + \varepsilon_{ij}, \qquad (5.2.1)$$

where y_{ij} is the rainfall measured at gauge i in period (hour) j;

μ is the overall mean;

g_i is a disturbance associated with gauge i;

ρ_j is a disturbance associated with period j,

and ε_{ij} is a residual term.

The variance of an individual hourly values is then

$$\sigma^2 = \sigma_\varepsilon^2 + \sigma_G^2 \qquad (5.2.2)$$

where σ_ε^2 is the residual of random variance, and σ_G^2 is the variance of the g_i terms.

The combined AWS gauge and Dines recording gauge sample for April 1972 was fitted to this model and the resulting estimates are shown in Table 22. The gauge-to-gauge variation was negligible in comparison with the random (residual) variation on both catchments, although the gauge-to-gauge variation was slightly higher on the Wye than on the Severn catchment.

Table 22. Analyses of variance of hourly rainfall, Wye and Severn catchments, April 1972

Wye catchment Gauges C, E, F, EG1, EG2

Source of Variation	d.f.	s.s.	m.s
Periods (hours)	140	1505.43	10.76
Gauges	4	4.1333	1.03
Residual	560	272.355	0.49
Total	704	1781.92	2.53

Mean: 1.37

Estimate of σ_G^2 = 0.0039
Estimate of σ_ε^2 = 0.4863 } σ^2 = 0.4903

Coefficient of variation: 51%

Severn catchment Gauges A, D, T1, T2, CW2

Source of Variation	d.f.	s.s.	m.s.
Periods (hours)	195	2069.10	10.61
Gauges	4	0.7813	0.19
Residual	780	347.227	0.44
Total	979	2417.11	2.47

Mean: 1.13

Estimate of σ_G^2: 0
Estimate of σ_ε^2: 0.4452 } σ^2 = 0.4452

Coefficient of variation: 59%

The estimated variance of an individual hourly value ($\hat{\sigma}^2$) was very similar in the two catchments, and the higher variability of the Severn data observed in the monthly data was not very apparent here (possibly because no canopy gauges were included in this analysis). The coefficients of variation were 51% and 59% for the Wye and Severn catchments respectively.

Analyses of variance were also carried out on data from individual storms taken from recording gauge values from both catchments; the results are shown in Table 23. The random variation was lower, and the gauge-to-gauge variation higher than in the previous analyses. This is to be expected because the latter study included data from both catchments (thereby increasing the variation in topography) and excluded the AWS data, thereby decreasing the errors of measurement, since for AWS gauges, hourly rainfall is measured only to the nearest half millimetre. The mean coefficient of variation for the 11 storms considered was 34%.

Table 23. Summary of analyses of variance of individual storms: gauges C, E, F (Wye) and A, B, D (Severn)

Date of Storm (1972)	$\hat{\sigma}_\varepsilon^2$	$\hat{\sigma}_G^2$	$\hat{\sigma}^2$	Mean	CV (percentage)
26/10 - 27/10	0.11	0.01	0.12	1.92	18
11/11 - 12/11	0.68	0.25	0.93	2.32	42
30/11 - 1/12	0.31	0.53	0.84	3.24	28
4/6 - 4/6	4.45	0.00	4.45	6.18	34
22/6 - 22/6	21.60	0.00	21.60	7.17	65
6/7 - 7/7	0.09	0.06	0.15	1.29	23
3/7 - 3/7	1.92	0.00	1.92	3.75	37
4/7 - 4/7	0.18	0.05	0.2	1.86	26
3/8 - 3/8	1.43	0.21	1.64	3.37	38
7/9 - 7/9	0.31	0.00	0.31	1.32	42
8/9 - 8/9	0.17	0.01	0.19	2.32	19

Mean CV: 34%

$\hat{\sigma}_\varepsilon^2$: estimate of random variation

$\hat{\sigma}_\varepsilon^2$: estimate of gauge-to-gauge variation;

$\hat{\sigma}^2$: estimate of combined variation

A surprising aspect of the analyses of variance was the frequency with which the mean square for gauges was less than the residual mean square. When this happens, a negative estimate is implied for σ_G^2, and in practice, $\hat{\sigma}_G$ is set to zero. The only explanation (in terms of the model) found for this phenomenon, is that $E[\varepsilon_{i,j}, \varepsilon_{i+1,j}] = \rho_\varepsilon^2 \sigma_\varepsilon^2$ ($\rho_\varepsilon<0$), ie the residuals exhibit negative serial correlation. However, this is unlikely to be the case, so the approach was not pursued.

The number of gauges (n) required to estimate a catchment mean with given precision d was calculated, as before, by the formula:

$$n = 4 \left(\frac{CV}{d}\right)^2,$$

where CV is the coefficient of variation of the individual measurements. The results are shown in Table 24.

Table 24. Numbers of gauges required to estimate hourly mean rainfall with given precision

	Estimate to lie within X% of true areal mean rainfall.			
	X = 50%	X = 30%	X = 20%	X = 10%
Approx. number of gauges (Wye: CV = 50%)n:	4	11	25	100
Approx. number of gauges (Severn: CV = 60%)n:	6	20	36	144
Approx. number of gauges (both catchments, Dines recorders only: CV = 35%)n:	2	5	12	49

5.3 Prediction of point values

In processing rainfall data, it is often necessary to fill in missing values. This means that point values have to be predicted from an adjacent or 'reference' gauge by ratio estimation or by regression. Sometimes the total rainfall over the missing period may be obtained from monthly totals, which may be available even though individual hourly values are not. If this is the case, a modified version of least squares regression using constrained minimization may be applied to the data to obtain estimates which sum to the required total.

The method was tried on 1971-72 Dines data for Carreg Wen and Moel Cynnedd; two months were chosen and analysed separately: June, a relatively wet month in both years, and September, a relatively dry month. The 1971 data was used to derive the relationship between the hourly values at the two sites, given the monthly totals in 1972. The monthly total at Carreg Wen in 1972 was then split up (or disaggregated) into estimates of the hourly values, which were then compared with those actually observed.

Dry spells were omitted from this analysis by the following procedure.

1. 1971 data was used to calculate the regression constants if and only if, non-zero rainfall was observed at Moel Cynnedd (this meant that a few non-zero values at Carreg Wen were omitted).

2. 1972 data was used to accumulate monthly totals independently for each site, ie all non-zero values were accumulated, whether or not they occurred at the same time. This procedure was adopted so as to conform to the practical situation where a monthly storage gauge total is to be split up into hourly values which are assumed to be concurrent with those at the reference gauge.

The regression lines obtained using the 1971 individual values and the 1972 totals are shown in Figure 6, where the values measured in 1972 are also shown. The sums of squares of differences between the observed and predicted values in 1972 were 0.35 and 0.33 for June and September, ie an error of ± 0.6 mm on an individual hourly value for both months.

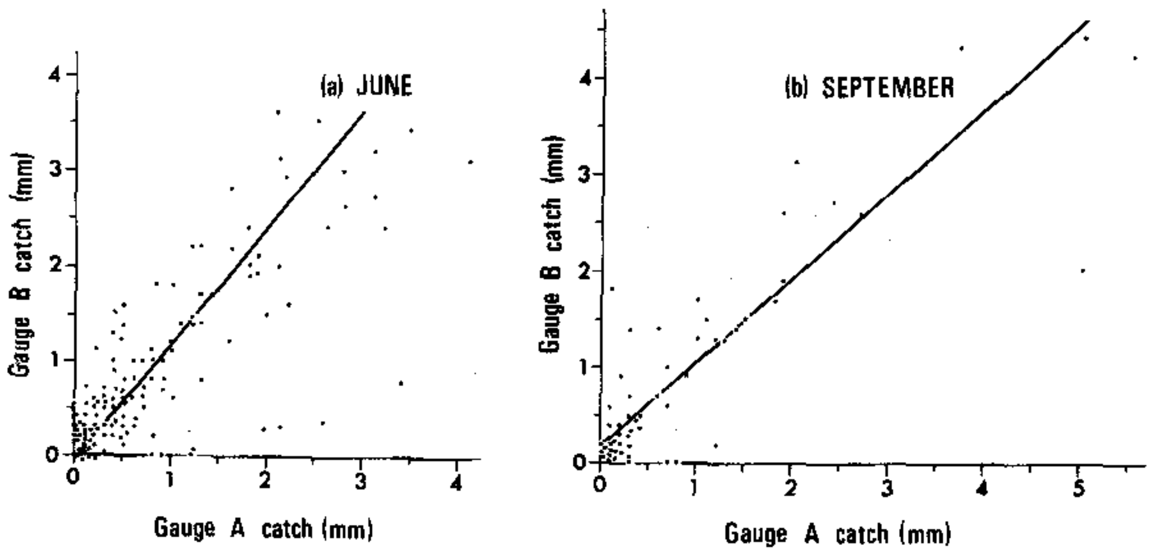

Figure 6. Regression lines based on 1971 hourly rainfall and 1972 monthly totals from Moel Cynnedd Dines (*gauge A*) *on a Carreg Wen Dines* (*gauge B*), together with observed 1972 hourly rainfall.

A second context in which point values require to be predicted arises as follows: the precision with which the catch at *any one gauge is* predicted by the other gauges in the catchment may be used as an indicator of the information which that gauge is providing towards the catchment mean. A gauge catch which is predictable might well be considered to be to a certain extent redundant. Table 25 shows the multiple correlation coefficient and residual error, when each gauge is regressed on the others in the catchment.

Table 25. Multiple correlation coefficients (ρ) and residual error (σ): each gauge against remainder in catchment, April 1972

	Wye			Severn	
Gauge	ρ	σ	Gauge	ρ	σ
C	0.93	0.74	A	0.95	0.56
E	0.87	1.11	D	0.89	0.85
F	0.96	0.61	T1	0.96	0.59
EG1	0.98	0.42	T2	0.97	0.51
EG2	0.98	0.43	CW2	0.90	0.88

5.4 Duration of storms

A storm is defined as a period during which rainfall was continuously observed. The data was divided into winter (October to March) and summer (April to September) although the problem of snowfall, mentioned earlier, renders some of the results for winter meaningless. The frequencies of storms for given durations were extracted, along with the intensity corresponding to each duration (the intensity being expressed as mean rainfall per hour). The results are shown in Tables 26 and 27 and, for summer months only, in Figures 7 and 8.

The relatively high number of storms lasting longer than 20 hours is probably due to the practice of dividing daily totals into 24 when hourly values are missing, thus spreading out the rainfall into storms of apparently long duration. The intensities for storms lasting longer than 20 hours have been omitted from Figure 8 for that reason.

Table 26. Duration of storms: frequency and mean rainfall (mm hr^{-1}) April - September 1972

Gauge	C		E		F		A		B		D	
Duration (hours)	Mean	freq	Mean	freq	Mean	freq	Mean	freq	Mean	freq	Mean	freq
1	3.99	100	4.21	75	3.85	78	2.56	87	3.71	79		
2	2.88	37	3.90	41	3.38	33	3.08	41	3.29	30		
3	2.15	28	2.32	30	1.71	32	2.51	28	2.22	24		
4	1.44	11	1.40	17	1.05	19	1.92	15	2.41	15		
5	2.11	6	2.70	7	2.70	7	2.14	10	2.09	8		
6	1.72	14	1.93	11	2.24	10	1.41	5	2.00	12		
7	1.35	5	2.02	6	1.38	7	1.23	6	1.45	4		
8	1.20	7	1.86	10	1.52	8	1.51	6	1.29	10		
9	1.36	2	2.12	2	2.06	2	1.80	3	2.04	2		
10	0.93	3	0.40	1	1.28	4	1.41	3	1.39	6		
11	1.33	2	1.20	5	1.52	4	1.09	5	1.33	6		
12	0.90	2			1.47	3	0.57	4	0.71	3	Missing data	
13	1.77	2	1.40	2	1.76	1	2.00	1	0.40	1		
14	0.24	1	1.74	1	1.13	2	1.41	3	1.38	2		
15	1.05	2			0.97	2	1.40	2	1.41	2		
16	1.40	1	1.01	1			1.40	1	1.55	1		
17									1.22	1		
18												
19	0.91	2					0.86	1				
20			1.51	1	1.44	1	1.54	1				
21			0.62	1								
22			0.60	1	0.69	2			0.85	1		
23			1.12	1			0.31	1	0.40	1		
24	0.52				0.67	1			1.28	1		
25							0.76	1	0.84	1		
26			0.52	1								
27					0.94	1						

Table 27. Duration of storms: frequency and mean rainfall (mm hr^{-1}) January - March, October - December, 1972

Gauge:	C		E		F		A		B		D	
Duration (hours)	Mean	freq	Mean	freq	Mean	freq	Mean	freq	Mean	freq	Mean	freq
1	2.64	86	2.72	97	2.39	99	2.69	114	2.05	87	2.99	103
2	2.74	45	2.61	48	2.71	56	2.30	46	2.74	43	3.28	51
3	1.90	36	1.95	32	2.30	31	2.13	26	2.44	29	1.79	28
4	1.54	26	1.82	25	1.68	22	1.73	22	1.62	21	2.08	22
5	1.62	16	1.17	23	1.24	18	1.08	19	1.19	22	1.49	18
6	0.96	14	1.28	11	1.18	10	1.58	11	1.49	10	1.12	8
7	1.12	6	0.89	6	1.88	7	0.87	10	1.15	8	0.97	7
8	1.64	6	1.55	6	1.11	4	1.49	6	0.66	7	1.62	6
9	1.79	4	1.60	6	1.52	7	0.98	10	1.44	11	1.59	8
10	1.25	4	1.21	2	1.00	2	1.53	3	2.46	1	1.06	3
11	2.27	2	1.30	4	1.52	5	1.28	3	1.50	6	1.16	4
12	1.01	4	0.86	2	0.99	3	1.54	1	0.13	1	1.52	1
13	1.29	3	2.08	1	2.37	1	2.01	2	1.99	2	2.25	2
14	0.76	1	1.59	3	1.57	3	1.10	1	0.82	2	0.93	2
15	2.18	1	1.34	2	1.33	2	0.83	1	0.83	3		
16					0.29	1	1.02	3	1.45	2	1.76	2
17	0.76	2										
18					0.29	1	0.33	1			0.28	1
19							1.63	1			1.85	1
20												
21	0.27	1										
22	0.69	1			0.29	1	1.07	2	1.22	1	1.09	2
23												
24									0.15	1		
25									0.71	1		
26												
27											0.21	1
28							0.14	1				
29			0.49	1					0.47	1		
30			0.33	1	0.42	2	0.31				0.31	1

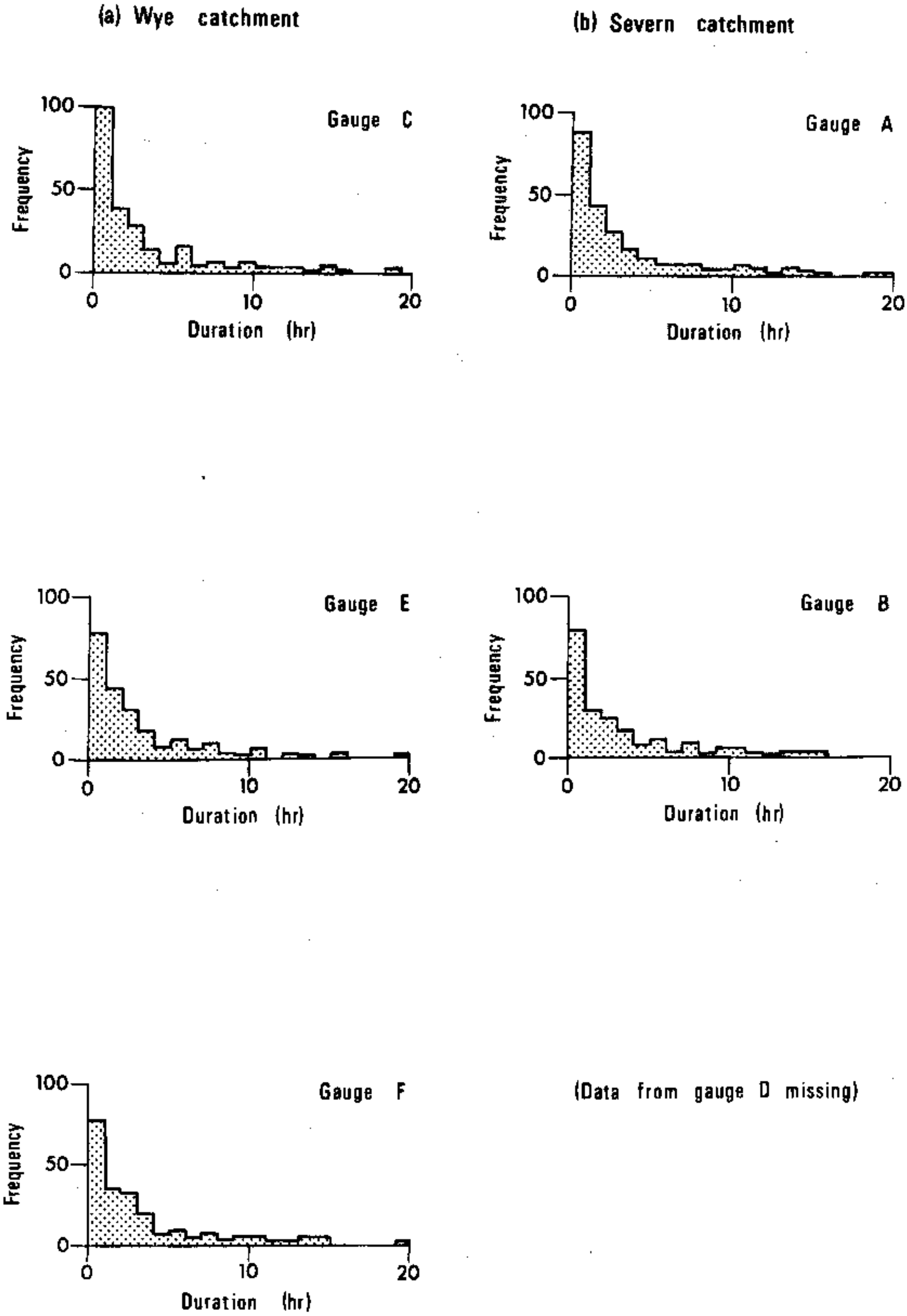

Figure 7. Storm-duration frequency distribution, Wye and Severn catchments, Summer months (April to September) only

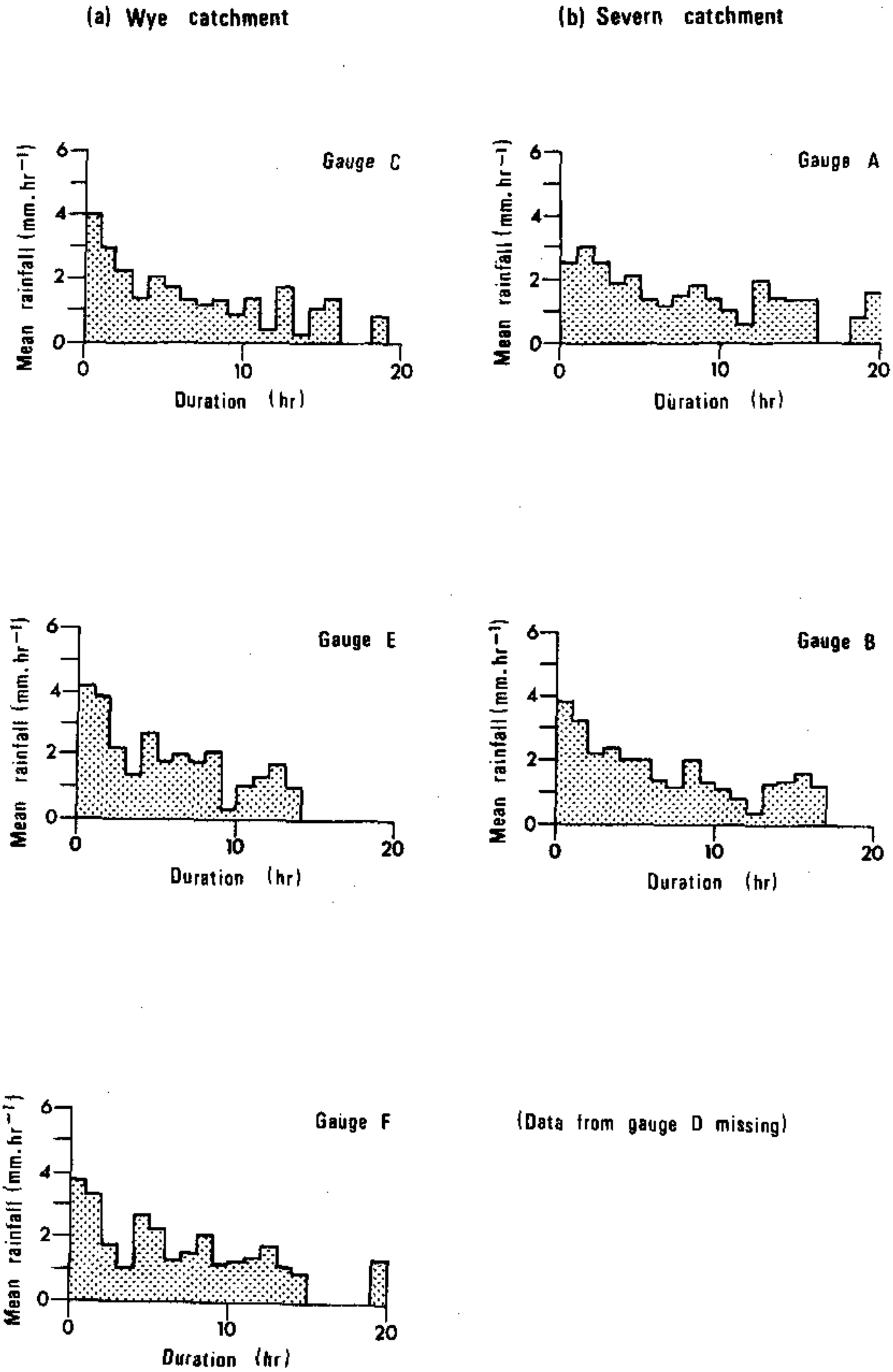

Figure 8. Mean rainfall against storm duration, Wye and Severn catchments, Summer months (April to September) only.

6. SUMMARY AND CONCLUSIONS

Much of the content of this report is of a descriptive nature and was designed to place on record the analyses undertaken and the methods used, whether or not clear-cut results emerged. Such conclusions as did emerge are as follows:

Differences between Wye and Severn rainfall

(1) For the period April 1971 - March 1973, there is no conclusive evidence that monthly precipitation differed for the Wye and Severn. Both these years had rather fewer days on which snow fell than the calendar years 1969 and 1970, when the differences in monthly precipitation, during months when snow fell, were much greater (41.9 ± 6.35 mm, averaged over 9 months). This difference may be (i) due to underestimation of the Severn precipitation by the canopy gauges of the Severn; (ii) due to overestimation of Wye precipitation, for some reason, during months of snowfall; or (iii) a real difference; (iv) due to some combinations of (i), (ii) and (iii).

(2) Averaged over the years 1969-1972 and over whatever gauges existed on the catchments at a particular time, the arithmetic mean precipitation on the Wye was 2288 mm, and on the Severn 2289 mm. The closeness of these estimates is especially remarkable in view of the result (1) above.

Random variation and precision of mean areal estimates

(3) For the period April 1971 - March 1973, the random variation - as measured by the period x gauge interaction (in measured monthly precipitation at a particular gauge) was less for gauges on the Wye than for the Severn on eleven months out of twelve. This may be associated with greater turbulence around the canopy-level gauges of the Severn: (Mean value of the coefficient of variation was approximately 7.0% for the Wye, 10.0% for the Severn).

(4) Sampling investigations based on the monthly precipitation recorded by the period gauges on the Wye and Severn suggest that the precision of (Thiessen) mean monthly areal rainfall is but little improved if the number of period gauges is increased beyond five for the Wye, or nine for the Severn.

(5) Little difference was found between the estimates of mean areal monthly rainfall obtained by various methods.

Validity of domain theory

(6) For both Wye and Severn catchments, the catch by a period gauge increased with altitude, and there was no evidence that gauges in the same altitude class, but different catchments, recorded differing amounts of precipitation.

(7) The association between monthly rainfall and domain slope was rarely statistically significant. Nevertheless, certain patterns emerged- taking the two-year average for each month of the year, the monthly rainfall caught by gauges in the slope domain 0^o - 9^o was always greater than the arithmetic mean of all gauges, and the catch by gauges in the slope domain 10^o - 19^o was always less than the mean. This result could not be easily explained in terms of sloping gauge orifices, nor in terms of unequal distribution of gauges in different slope classes.

(8) No strong relation emerged between monthly rainfall and aspect class, but again certain consistencies were observed from month to month. Gauges facing North East (ie with aspect in the first compass quadrant) tended to have catches greater than the arithmetic mean for the whole catchment, whether they were in the Wye or Severn. In the Wye, also, South East facing gauges tended to catch less than the mean, over all gauges, for the catchment.

(9) With the existing networks of period gauges in the catchments, it is probable that, in nine months out of ten, the true mean areal monthly rainfall is estimated to within 5% of its value.

Characteristics of hourly rainfall

(10) Concerning hourly rainfall, serial correlation in the catch recorded by Dines gauges was less apparent than cross-correlation between pairs of gauges. For summer months (April to September 1972) the cross correlation between hourly catches was about 0.86 for the Wye, and about 0.81 for the Severn. There were insufficient gauges to detect any decrease in correlation with increasing distance separating them;

(11) Using both Dines recording gauges and the Rimco gauges of the automatic weather stations, the coefficient of variation for hourly rainfall - calculated for the month of April 1972, only, and calculated only from rainy periods (ie periods in which at least one gauge, recorded rainfall) - was 51% on the Wye, and 59% on the Severn. These values suggest that if it were necessary to estimate mean areal hourly rainfall to within 20% of its true value, about 25 hourly gauges would be required for the Wye, and about 36 for the Severn.

(12) The estimated numbers of gauges in (11) were derived using a coefficient of variation calculated for one month only, April 1972. Using instead a coefficient of variation (35%) calculated from 11 storms distributed throughout the whole period April 71 - March 73, and using only the Dines records from both catchments, 12 Dines recorders would be required to estimate mean areal hourly rainfall over both catchments to within 20% of its true value, and about 49 would be required to estimate it within 10%.

APPENDIX 1

Installation of ground level and canopy level gauges

Once the site is located on the ground from a randomly determined position on the map within each domain to be gauged, its slope and aspect aspect are checked to ensure that they correspond to those of the domain. If they do not, the prospective site is moved the minimum distance for the requirements to be met.

Canopy level gauges

Canopy level gauges are used where the gauge sites occur in the forest. The gauge is funnel shaped, mast-mounted so that its funnel orifice is horizontal, and is at mean canopy height or above. The mast has a concrete base black and there are another three concrete blocks into which are cast 1 cm (½-inch) reinforcing hoops for securing the guy ropes. (Figure 9).

Ground level gauges

Ground level gauges are used in grassland areas. The gauge is set in a pit with its orifice parallel to the angle of slope of the domain. The angle of ground slope and aspect for the gauge is taken from the appropriate 125 m grid square on the 1:5000 map used to delimit the raingauge domains. The pit and finally the gauge orifice are aligned accordingly.

The pit is constructed in the following manner: a square pit of side approximately 130 cm (50 inches) is dug in such a way that its diagonal is aligned with the map aspect; this is to allow for drainage if necessary. A wooden frame is inserted and once the gauge is installed the pit may be filled with gravel to a depth of 30 cm (1 foot) below the surface, which is covered by a non-splash grid. A preferred alternative for Plynlimon is to fit a false floor in the frame 30 cm (1 foot) below the grid, leaving a void beneath, since the local acidic soils attack the solder in the gauge seams and, eventually, cause leaks. (Figure 10).

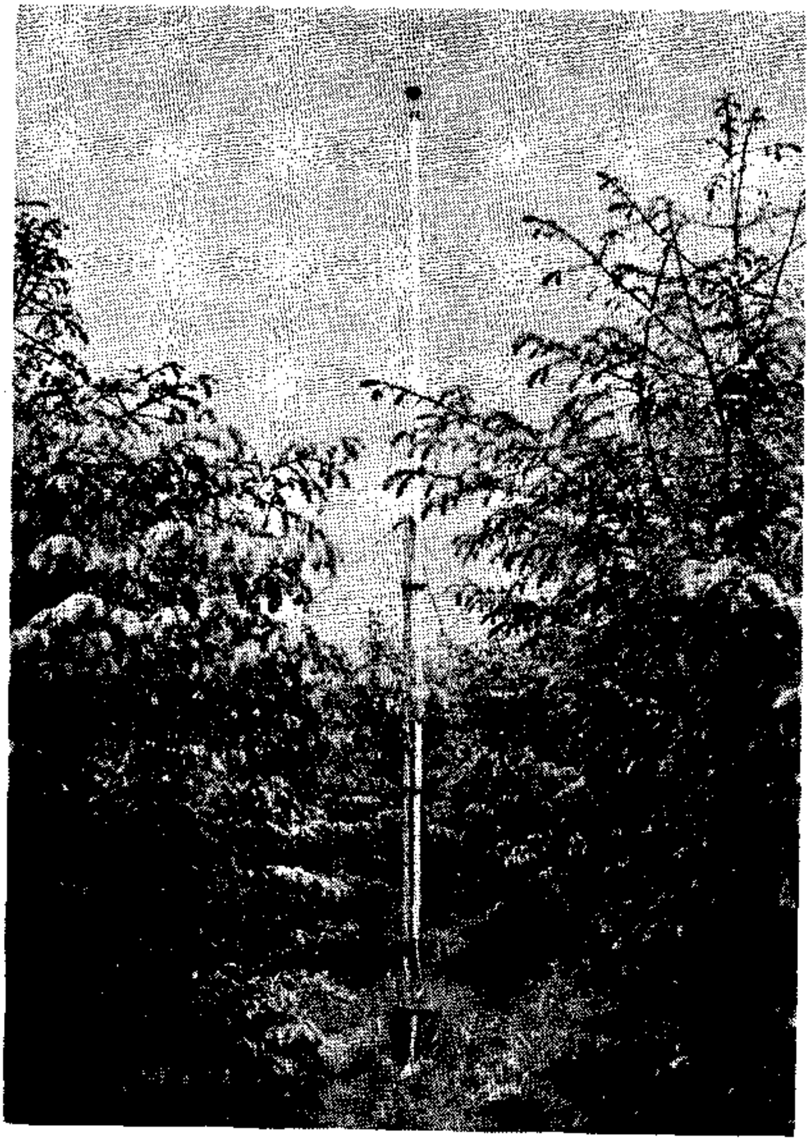

Figure 9. Canopy level raingauge

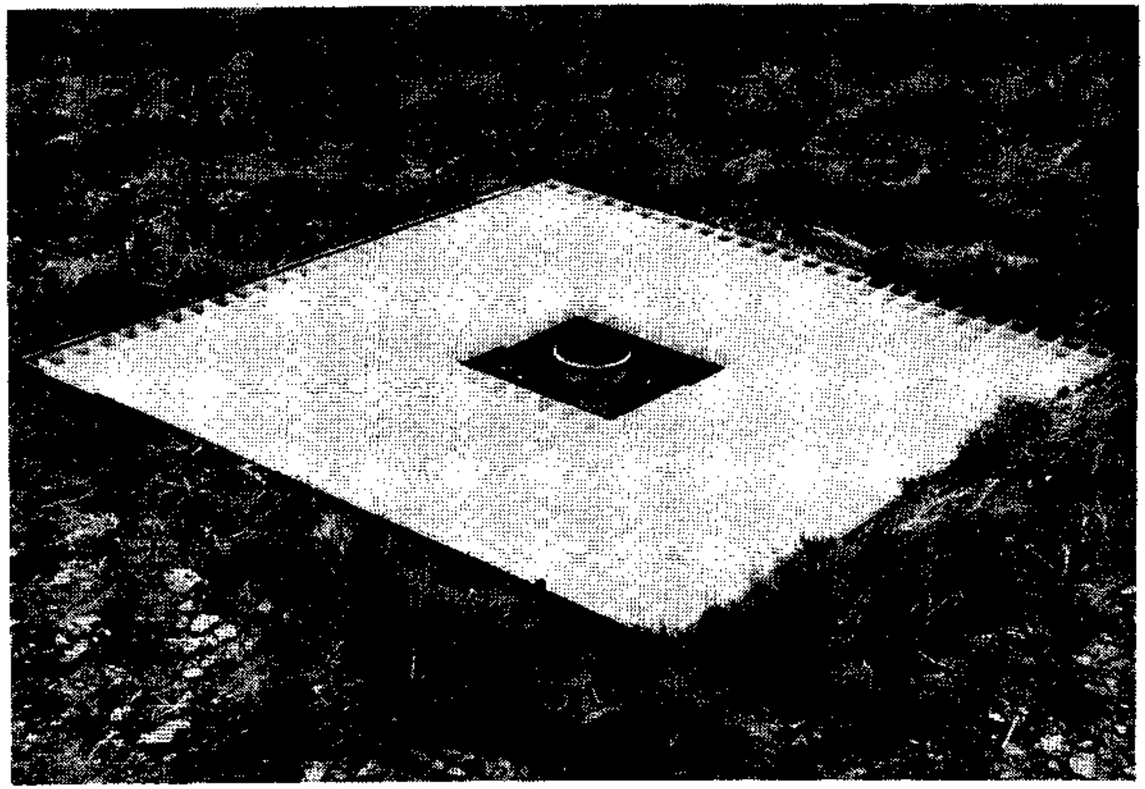

Figure 10. Ground level raingauge

APPENDIX 2

Monthly rainfall data, Wye and Severn catchments, April 1971 - March 1973

Wye catchment

GAUGE NO	1 B2W	2 B1W	3 B1X	4 A1Y	5 B2Y	6 A2Y	7 A2Z	8 C2X	9 B1W	10 B2W	11 B2Z	12 C2X	13 B2Y	14 B2X	15 C2W	16 C2Y	17 D1X	18 D1Y	19 A2X	20 A2W	28 Cefn Brwyn
1971																					
April	77.1	79.0	73.5	62.2	68.3	63.9	67.5	64.8	69.5	69.8	73.0	66.8	68.8	69.5	57.9	78.6	68.0	75.1	73.4	72.0	79.4
May	70.9	68.1	69.1	71.9	71.3	70.1	70.4	76.0	79.0	72.3	71.6	79.4	71.6	75.9	78.9	82.5	89.4	91.6	70.5	78.0	72.8
June	227.0	232.4	210.5	184.5	188.6	177.2	173.5	225.8	216.2	188.8	182.0	220.8	188.5	189.0	201.1	186.6	178.3	221.9	183.0	198.5	184.3
July	75.8	76.1	81.8	80.5	77.6	67.6	58.6	93.1	82.5	74.4	64.2	92.0	70.9	67.5	92.2	81.4	94.4	85.5	69.8	66.0	62.0
Aug	221.5	215.2	215.1	214.5	211.4	192.3	201.5	235.0	217.0	203.5	204.4	226.8	201.4	213.2	225.0	206.8	239.9	236.4	201.2	200.1	201.5
Sept	135.6	105.4	108.2	104.5	101.2	89.0	87.9	118.0	107.2	97.0	89.9	112.0	91.8	93.2	105.5	92.5	108.5	108.0	91.0	95.9	91.7
Oct	202.9	212.0	200.6	206.5	197.5	192.0	193.9	228.5	219.0	195.5	205.0	220.0	202.3	204.0	234.0	220.2	225.5	234.5	198.0	200.4	202.5
Nov	313.1	355.0	303.2	316.9	291.3	261.5	290.0	326.9	326.5	301.7	306.3	337.6	300.0	314.5	286.8	311.9	342.5	355.6	314.2	320.1	316.3
Dec	128.2	134.4	134.5	128.8	124.1	112.3	121.6	126.8	125.0	121.1	128.5	126.4	120.6	128.9	124.3	114.6	131.7	131.5	124.7	124.9	120.2
1972																					
Jan	235.6	224.5	224.5	227.0	227.5	205.5	206.0	251.5	218.5	219.5	219.5	266.5	240.5	208.5	249.5	242.2	250.0	239.5	212.0	212.5	230.3
Feb	137.8	132.3	140.7	135.0	132.0	127.5	142.2	146.5	135.4	150.0	132.2	156.8	117.2	161.0	193.7	177.9	252.5	251.7	120.3	124.0	119.3
Mar	254.9	274.9	232.5	208.0	218.5	187.5	200.0	263.2	229.5	206.5	226.5	249.8	218.0	238.0	228.2	223.1	287.8	296.4	204.0	224.1	204.7
April	268.5	296.5	260.0	278.5	266.6	232.7	270.6	287.0	283.0	196.2	315.2	285.0	285.0	300.2	235.0	227.0	306.5	328.5	264.7	280.5	272.0
May	128.5	129.6	128.8	124.4	131.9	108.0	120.8	156.5	139.5	103.4	131.0	149.1	131.9	144.1	142.3	142.9	171.0	155.5	117.6	126.3	120.9
June	203.2	204.0	192.1	183.2	173.0	161.5	164.0	224.2	200.7	163.5	184.0	218.6	184.0	203.5	205.4	195.0	227.3	214.0	168.5	186.0	174.4
July	155.5	158.2	147.2	139.9	131.0	120.2	117.3	167.0	151.2	123.2	123.2	162.7	132.0	143.3	155.0	140.0	163.9	161.0	124.8	142.4	125.5
Aug	122.5	122.0	119.8	114.6	113.7	101.6	105.0	131.2	120.5	108.8	119.8	127.9	113.2	122.5	119.5	126.8	137.0	132.5	109.2	113.2	105.5
Sept	56.0	56.0	62.5	57.6	55.5	54.0	55.8	59.4	63.8	56.5	57.5	67.1	56.5	59.5	65.1	58.0	66.8	66.2	57.5	54.0	52.6
Oct	74.6	74.0	71.8	60.0	68.4	64.0	67.6	86.3	76.0	67.0	69.7	87.0	69.3	76.2	88.0	83.7	94.2	91.5	56.8	70.2	66.8
Nov	304.0	327.0	292.0	288.5	280.0	252.0	278.0	319.5	321.4	258.0	301.0	311.5	297.0	316.0	288.6	273.4	348.5	320.4	284.0	320.4	297.2
Dec	225.4	238.3	212.6	234.0	215.0	207.0	268.2	244.0	215.0	223.0	237.9	284.5	221.4	247.7	211.0	221.0	248.8	273.0	210.0	223.0	224.5
1973																					
Jan	229.5	214.3	248.5	200.5	191.6	170.0	179.4	226.6	246.2	177.7	202.4	214.1	195.0	194.0	201.3	173.2	209.5	200.4	185.9	217.0	196.8
Feb	348.1	338.5	306.9	292.0	302.0	251.5	331.6	378.6	365.5	335.0	318.0	347.6	305.1	321.0	292.0	265.0	408.0	409.0	337.0	341.0	307.7
March	146.0	148.0	123.6	133.5	117.0	106.7	112.3	156.7	134.3	115.2	123.8	131.1	123.0	128.5	130.4	110.5	166.3	199.0	116.4	136.2	132.4

APPENDIX 2 (Contd.)

Severn catchment

GAUGE NO	1 DIX	2 C2W	3 B2X	4 B2Z	5 BIZ	6 AIW	7 D2X	8 DTW	9 CIW	10 CIY	11 B2X	12 B2W	13 A2Y	14 CIX	15 AIX	16 C2X	17 ATY	18 A3Z	29 Carreg Wen	30 Moel Cynnedd
1971																				
April	80.4	78.4	73.5	74.2	69.2	68.2	71.3	79.6	73.8	72.0	70.2	73.8	69.0	70.8	66.0	58.9	66.4	65.2	79.4	68.8
May	88.9	77.5	73.0	73.6	74.3	74.5	81.0	74.1	82.5	83.5	68.0	77.2	66.8	80.8	68.8	74.0	65.5	69.8	81.5	64.7
June	220.8	232.5	181.2	193.8	178.5	188.2	201.5	207.9	209.0	234.2	181.4	209.6	151.1	221.7	165.0	172.9	147.4	165.8	222.3	165.2
July	83.4	81.6	61.2	67.0	64.8	68.5	78.0	79.4	79.0	82.5	64.3	78.0	59.2	77.5	63.8	66.1	56.5	68.2	80.7	63.1
Aug	242.0	243.0	207.0	213.8	206.0	217.0	225.9	227.4	232.9	234.2	207.3	230.2	180.5	248.6	194.2	206.6	192.8	211.5	233.3	203.6
Sept	109.5	102.3	81.7	91.1	86.9	83.5	97.5	100.5	100.5	101.5	82.7	94.6	64.9	107.1	72.1	86.5	67.2	84.7	97.7	75.2
Oct	223.0	238.0	218.2*	212.9	203.4	216.0	211.0	236.5	236.0	230.0	191.6	224.3	168.7	220.7	174.2	162.9	229.7	202.2	242.4	212.8
Nov	368.7	354.9	282.0	336.6	293.1	292.7	331.3	368.9	324.0	312.5	286.3	324.3	236.4	315.0	245.4	256.6	339.5	269.0	399.8	294.8
Dec	139.5	140.4	154.8	128.4	125.2	121.7	129.5	138.2	131.6	132.5	112.0	128.5	92.5	128.4	99.2	118.2	166.0	115.6	144.5	118.3
1972																				
Jan	265.5	233.0	228.5	203.4	199.3	225.7	243.4	264.0	262.0	271.0	191.6	228.2	187.2	207.5	189.0	215.5	270.8	210.5	253.9	222.0
Feb	242.7	122.2	111.0	128.0	122.0	130.0	225.9	261.3	187.3	144.7	96.5	132.5	117.5	109.0	116.0	119.1	153.0	107.2	179.0	128.2
March	295.6	204.3	198.2	218.5	196.0	189.9	278.3	302.6	227.6	249.9	165.4	202.8	203.2	181.5	149.5	178.6	225.5	170.5	273.0	198.4
April	330.0	284.0	357.0	355.3	369.7	301.4	273.0	304.5	290.0	292.0	281.2	322.6	238.6	306.8	242.2	224.5	368.4	279.9	336.0	302.9
May	159.0	170.3	155.4	141.8	129.2	152.5	144.9	150.0	147.2	146.6	132.0	159.0	124.1	157.5	122.2	136.5	160.5	136.2	169.8	144.8
June	227.2	229.2	257.2	182.0	171.0	187.5	229.2	214.8	212.8	195.3	181.4	210.2	152.2	229.5	153.7	190.0	163.3	178.8	225.1	174.8
July	175.0	166.4	153.8	135.2	127.0	130.1	154.2	155.9	153.3	144.8	129.5	148.6	108.2	168.2	112.5	137.0	107.0	131.4	156.7	125.2
Aug	142.0	144.5	132.8	120.3	115.6	121.6	123.0	127.0	131.0	125.6	115.0	134.1	109.5	138.2	103.8	123.0	106.0	115.3	140.2	119.4
Sept	65.5	68.4	66.0	58.0	57.5	57.2	62.3	65.5	63.0	63.7	63.2	61.5	62.9	66.2	58.6	61.2	62.6	53.0	65.0	61.6
Oct	101.5	93.3	86.3	73.9	73.5	74.5	86.6	91.5	89.0	86.5	67.8	78.8	65.0	86.5	66.0	87.0	80.0	72.3	91.5	66.5
Nov	371.2	381.3	293.0	317.0	299.0	330.0	342.0	409.7	345.3	357.2	303.5	347.0	255.1	330.2	277.0	357.2	320.0	278.0	391.7	322.7
Dec	286.0	304.0	284.1	255.6	242.4	273.5	237.0	239.5	266.0	268.0	245.0	273.3	224.0	262.0	204.8	234.0	243.0	244.5	292.1	270.8
1973																				
Jan	197.5	206.5	219.2	195.0	181.4	174.5	212.8	130.8	193.2	198.6	120.4	179.5	136.6	178.9	141.4	191.5	151.5	165.5	214.1	158.8
Feb	421.0	295.0	278.4	320.9	280.5	255.3	314.0	408.6	326.5	315.0	243.5	275.0	194.3	257.0	207.9	253.0	211.5	243.1	396.6	257.5
March	212.6	158.0	178.2	136.2	114.4	131.1	187.8	186.3	140.3	127.3	125.2	133.2	96.2	129.5	97.5	136.2	105.8	99.0	151.1	112.2

*Estimated value

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