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Research paper

A rigorous possibility approach for the geotechnical reliability assessment supported by external database and local experience

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ABSTRACT

Reliability analyses based on probability theory are widely applied in geotechnical engineering, and several analytical or numerical methods have been built upon the concept of failure occurrence. Nevertheless, common geotechnical engineering real-world problems deal with scarce or sparse information where experimental data are not always available to a sufficient extent and quality to infer a reliable probability distribution function.

This paper rigorously combines Fuzzy Clustering and Possibility Theory for deriving a data-driven, quantitative, reliability approach, in addition to fully probability-oriented assessments, when useful but heterogeneous sources of information are available.

The proposed non-probabilistic approach is mathematically consistent with the failure probability, when ideal random data are considered. Additionally, it provides a robust tool to account for epistemic uncertainties when data are uncertain, scarce, and sparse. The Average Cumulative Function transformation is used to obtain possibility distributions inferred from the fuzzy clustering of an indirect database. Target Reliability Index Values, consistent with the prescribed values provided by Eurocode 0, are established.

Moreover, a Degree of Understanding tier system based on the practitioner's local experience is also proposed. The proposed methodology is detailed and discussed for two numerical examples using national-scale databases, highlighting the potential benefits compared to traditional probabilistic approaches.

1. Introduction

The reliability assessment of any engineering system requires addressing four main problems: (i) to determine the input data, (ii) to adopt a suitable methodology for the reliability analysis, (iii) to define an analytical or numerical model representing the system with high-fidelity, and (iv) to give a correct interpretation of the analysis output. When dealing with geotechnical engineering problems, the first aspect of step (i) is strongly affected by the uncertain nature of the input data. The soil parameters used for the definition of the model in step (iii) are characterized by various sources of uncertainty such as inherent variability (Lumb, 1966; Manolis, 2002; Greco, 2016), measurement scatter due to limitations of the experimental techniques (Phoon and Kulhawy, 1999; Uzielli et al., 2006; Phoon et al., 2022a), and sampling or statistical error (Ching et al., 2016; Mašín, 2015) because of the limited number of soil samples used in the investigation. A more exhaustive description of the main sources of uncertainty affecting geotechnical problems can be found in Otake and Honjo (2022).

To deal with these soil uncertainties, geotechnical engineers can adopt different strategies, as pointed out by Christian (2004), namely by ignoring them, being conservative, using an observational method or quantifying their effects on structural and geotechnical safety. The latter is the purpose of the reliability analysis; according to the nature of the uncertainty, e.g., aleatoric or epistemic, the methodology of step (ii) can change considerately. Under the probabilistic framework, well-established approaches can be used: a reliability index method has been used by Cherubini (2000) for carrying out a reliability evaluation of the bearing capacity of shallow foundations; the First Order Reliability Method (*FORM*) has been used by Honjo et al. (2000) for the seismic design of a shallow foundation of a building; Monte Carlo Simulation was used by Xue and Nag (2011) to study the effects of the inclined loads on the reliability of shallow foundations; Zorzi et al. (2020) developed a reliability framework with emphasis on verifying the serviceability limit state criterion in terms of maximum allowable rotation during an extreme cyclic loading event assuming the soil parameters

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are normally distributed; Carswell et al. (2015) used a first-order method (β method) and Monte Carlo simulation (MCS) to estimate the reliability of an offshore wind turbine monopile foundation when the beta distribution has been adopted for modelling the variability of the soil properties.

Nevertheless, practical geotechnical problems are associated with sampling uncertainty resulting from sparse and scarce information. More extensively, Phoon (2018) coined the acronym *MUSIC* (or more recently *MUSIC-X*) to identify the distinctive attributes of geotechnical data, which can be described as Multivariate, Uncertain and unique, Sparse, and InComplete. Therefore, the definition of an exact probabilistic model as required by the studies previously mentioned could be doubtful in view of the limited quality and extent of the information owned by the engineer. Ching and Phoon (2019, 2020) proposed a Bayesian machine learning method, then extended with Hierarchical Bayesian Model in Ching et al. (2021), to handle *MUSIC* data.

On the other hand, the use of the probability theory, either associated with a certain expected frequency of occurrence (frequentist statistics) or with a subjective specified likelihood of occurrence (Bayesian statistics), is not the only mathematical framework that can be used for assessing the reliability of a geotechnical problem. For instance, Beer et al. (2013b) carried out a comparative reliability study between probabilistic and interval analysis on the stability of a retaining wall showing that the latter leads to conservative results and can identify extreme (low-probability-but-high-consequence) events straightforwardly. Further non-probabilistic approaches have been also applied for geotechnical reliability analyses such as adopting fuzzy numbers as nominal values (Nawari and Liang, 2000), or fuzzy sets (Pramanik et al., 2019, 2021). Cremona and Gao (1997) presented a possibilistic reliability analysis theory on the basis of a possibilistic safety index called Possibility of Failure, whilst (Feng et al., 2021) proposed a fuzzy importance sampling method for estimating failure possibilities. Moreover, hybrid approaches where different natures of the input data coexists have been developed, such as in An et al. (2016) or in Tombari and Stefanini (2019).

Mixed concepts have been used such as fuzzy probabilities (Beer, 2009), probability-boxes (Schöbi and Sudret, 2017), or possibility theory (Dubois and Prade, 2004; Dubois, 2006), and each one can be included under the general context of imprecise probabilities (Beer et al., 2013a). In particular, the possibility theory based on Zadeh's theory of fuzzy sets (Zadeh, 1965) can be seen as a simple quantitative framework for statistical reasoning with imprecise probabilities (Dubois and Prade, 2015). Hose and Hanss (2019) proved the preservation of probability–possibility consistency demonstrating that the possibility and the necessity measures can be viewed as a upper and lower probabilities. The same Authors, in Hose and Hanss (2020), addressed the problem of inferring a membership function from empirical observations; by viewing fuzzy numbers as possibility measures, they suggested a procedure for obtaining a consistent estimate of the membership function in terms of an approximation of its level sets. Membership functions can also be constructed from partitions of a dataset (see e.g., Guillaume and Charnomordic, 2004), through fuzzy (*c-means*) clusterization (Oliveira and Pedrycz, 2007). Therefore, possibility theory seems to be particularly useful to elicit a membership function from partitions of a database and can be exploited in search for a novel concept of reliability in geotechnical engineering.

1.1. Overview of the proposed approach and scope of work

The aim of this paper is to propose a novel reliability approach to deal with uncertain, scarce and sparse data in non-probabilistic fashion, without adopting neither the assumption of the probability axioms nor any probability distributions of the data. The proposed approach, based on the possibility approach (Dubois, 2006), provides a simple yet robust tool for the uncertainty propagation of quantitative data and subjective information. The key aspect of the proposed alternative definition of structural reliability is the preservation of probability–possibility consistency (Hose and Hanss, 2019; Dubois and Prade, 2015) allowing to formulate a non-probabilistic cardinality of reliability target, hereinafter named Reliability Index Value, embedding both epistemic and aleatory uncertainties. Fig. 1 shows the workflow of the proposed possibility approach for assessing geotechnical reliability. Therefore, a fully-fledged definition of possibilistic reliability characterized by a level of safety consistent with the prescriptions of the standard codes, is established (*first scientific contribution*).

Moreover, since the inputs of the proposed reliability approach are represented by possibility distributions, the estimation of their membership functions can be performed by inferring them directly from samples of datasets (see, e.g., Hose and Hanss, 2020). In this study, fuzzy (*c-means*) clusterization (Oliveira and Pedrycz, 2007) is herein adopted to generate partitions from a database, which are then transformed into possibilistic membership functions through a rigorous transformation called Average Cumulative Function (see Stefanini and Guerra, 2017; Guerra et al., 2020). Therefore, the *second contribution* of this paper is to propose a robust integration between reliability analysis and indirect regional, national or global databases of inputs (e.g., soil properties) to support the decision-making process. This can be seen as a contribution in the exploitation of the *dark data* as defined and questioned by Phoon (2020) as well as in the facilitation of the digital transformation of geotechnical engineering (Phoon et al., 2022b).

Finally, it will be shown that the number of subclasses can be related to the 'degree of understanding' (see e.g., Fenton et al., 2016 or the Canadian Highway Bridge Design Code - CSA S6:19, CSA, 2019). Therefore, the proposed approach defines a three tier system, ranked as 'low', 'typical' and 'high', to establish a design value based on objective and subjective information such as the level of local experience, number of samples and engineering judgement. Therefore, the *third contribution* of this paper is to inherently embed subjective and quantitative information into the proposed reliability methodology through the concept of 'degree of understanding' (DoU); this constitutes a rigorous approach to account for several recommendations mentioned in the Eurocode 7 (BS EN 1997-2:2007, British Standards Institution et al., 2007) such as the consideration of relevant published material and data and use of local experience, which are essential for deriving reliable values of geotechnical parameters.

It is worth emphasizing that the proposed procedure is accessible to practitioners even without any prior of fuzzy theory: the database and the fuzzy partitions should be set up by third-party experts whilst the practitioners just need to define a nominal value (Prästings et al., 2019) that will be used to retrieve a design membership function also based on their local experience and subjective information. The design membership function will be just an ordered set of input data (e.g., a set of angles of internal friction of the soil) to be used sequentially into the performance function for conducting a traditional analysis, as it will be shown in the worked example in Sections 3.3 and 4. Each data input corresponds to a certain value of the membership function. The reliability is assessed by obtaining the membership value of the performance function for which the limit state is reached. This computed value is compared to the proposed target Reliability Index Value to assess the level of safety in terms of *Fail* or *Safe*.

The remainder of this paper is organized as follows. In Section 2 the proposed methodology of geotechnical reliability under the possibility framework is established. Section 3 provides a comparative example between probability and possibility reliability assessment to validate the method; moreover, two complete numerical applications of common engineering problems considering real databases are also provided in Sections 3.3 and 4.

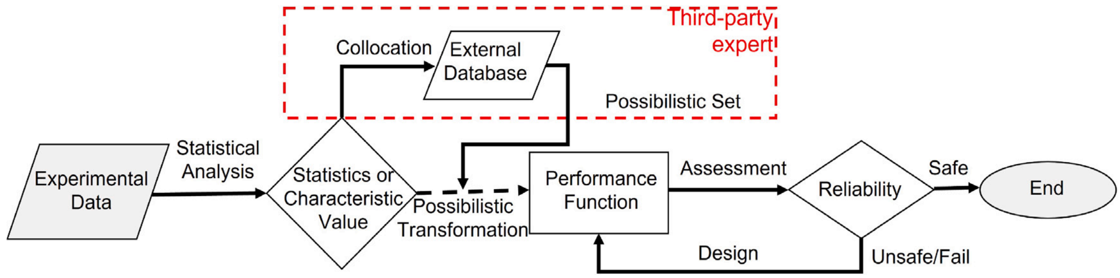


Fig. 1. Proposed possibilistic reliability with database support.

2. Proposed possibility reliability approach with database clustering

2.1. Possibilistic reliability

Let us consider a performance or limit state function, G, described as $G = f\left(X_1,\ldots,X_d\right)$, where the given deterministic or crisp function f takes as argument a vector of $d \geq 1$ real parameters $\left(X_1,\ldots,X_d\right)$ with elements in a nonempty subset $\mathbb{X} \subseteq \mathbb{R}^d$.

In case of structural safety at the Ultimate Limit State (ULS), the conventional performance function is defined as the difference between the resistance of the investigated problem, R, and the applied external load V, as follows:

$$G = R - V \geq 0. \tag{1}$$

The performance function, G, determines if the structure is safe ($G > 0$) or unsafe ($G \leq 0$):

$$\begin{aligned} G\left(x_1,x_2,\ldots,x_d\right) > 0 &\Rightarrow \text{Safe} \\ G\left(x_1,x_2,\ldots,x_d\right) \leq 0 &\Rightarrow \text{Failure} \end{aligned} \tag{2}$$

where $x_i, i = 1,2,\ldots,d$ are the parameters of the considered problem. In geotechnical engineering problems, the subset $\mathbb{X}$ contains the soil properties, e.g., $X_1 = \varphi, X_2 = \gamma, X_3 = c$, namely the angle of internal friction, the soil density and the cohesion, respectively. If these parameters are uncertain, precisely random, BS EN1990 (British Standards Institution et al., 2021) defines three levels of structural safety, classified as Level I (deterministic reliability), Level II (Reliability Index methods) and Level III (Full Probability methods). The three levels are sorted with an increasing degree of complexity and required experience (Phoon, 2023). Although the three levels should guarantee the same safety, it will be shown in Section 3.2 that this is not always occurring. The full probability analysis requires the exact knowledge of the joint probability distributions of the parameters, and it is not covered in this study.

On the other hand, when the data used for characterizing the reliability problem are affected by epistemic uncertainty, e.g., sparsity and scarcity, the adoption of the probabilistic approaches as prescribed by the standard codes (e.g. BS EN1990 (British Standards Institution et al., 2021)) might be questionable because of the high sensitivity of the failure probability to the input distribution parameters (Oberguggenberger and Fellin, 2002). To seek a more robust measure, the Possibility Theory (Dubois, 2006) is exploited in this study for deriving a novel approach to reliability assessment in geotechnical engineering.

The previously defined real-valued quantities $X_j, j = 1,2,\ldots,d$ are now expressed in terms of d fuzzy intervals (or fuzzy numbers) represented by membership functions $u_{X_j} : \mathbb{R} \rightarrow [0,1]$ such that, given two closed real intervals $[a_j,b_j]$ and $[c_j,d_j]$ with $a_j \leq c_j \leq d_j \leq b_j$, it can be defined, for $z \in \mathbb{R}$, as follows:

$$u_{X_j}(z) = \begin{cases} 0 & \text{if} \quad z < a_j \\ u^L_{X_j}(z) & \text{if} \quad a_j \leq z < c_j \\ 1 & \text{if} \quad c_j \leq z \leq d_j \\ u^R_{X_j}(z) & \text{if} \quad d_j < z \leq b_j \\ 0 & \text{if} \quad z > b_j \end{cases} \tag{3}$$

where $u^L_{X_j} : [a_j,c_j] \longrightarrow [0,1[$ is a non-decreasing right-continuous function (where 1 is not included in the interval), $u^L_{X_j}(z) > 0$ for $\in]a_j,c_j]$, called the *left side* of the fuzzy interval and $u^R_{X_j} : [d_j,b_j] \longrightarrow [0,1]$ is a non-increasing left-continuous function, $u^R_{X_j}(z) > 0$ for $z \in [d_j,b_j[$, called the *right side* of the fuzzy interval. By convention, if $d = 1$ the index j will be omitted in Eq. (3).

Therefore, the deterministic limit state function G in Eq. (1) is transformed to a fuzzy function which takes the membership functions of the inputs, $u_{X_j}, j = 1\ldots,d$ and returns the membership function u_G of the output. This can be done by using the Zadeh's extension principle (Zadeh, 1975), which is hence applied as follows:

$$u_G(g) = \underbrace{sup}_{G(x_1,x_2,\ldots,x_d)=g} \quad min\left\{u_{X_1}(x_1),u_{X_2}(x_2),\ldots,u_{X_d}(x_d)\right\} \tag{4}$$

where sup is the supremum operator, x_j, for $j = 1,2,\ldots,d$, is a value of fuzzy parameter X_j with membership $u_{X_j}(x_j)$ and $g \in \mathbb{R}$ is a dummy variable that denotes any possible value of function G on its range. In compact form:

$$u_G = G_{EP}\left(u_{X_1},\ldots,u_{X_d}\right), \tag{5}$$

where G_{EP} is a fuzzy function that relates the fuzzy inputs, $X_1,\ldots,X_d$, to the fuzzy output, G. The computational approaches to obtain u_G are described in Section 2.1.3. In analogy with the representation of Eq. (3), the membership function u_G in Eq. (5) is given in the following form:

$$u_G(g) = \begin{cases} 0 & \text{if} \quad g < a_G \\ u^L_G(g) & \text{if} \quad a_G \leq g < c_G \\ 1 & \text{if} \quad c_G \leq g \leq d_G \\ u^R_G(g) & \text{if} \quad d_G < g \leq b_G \\ 0 & \text{if} \quad g > b_G \end{cases} \tag{6}$$

where the support is defined by the interval $\left[a_G,b_G\right]$ and the core by $\left[c_G,d_G\right]$.

To derive a safety measure based on the membership function u_G of Eq. (6), a criterion to separate the safe from the unsafe domain is established. Criteria based on several definitions of the cardinality of a fuzzy set might be defined; nevertheless, in this study, a cardinality calibrated to be consistent with the prescribed values provided by BS EN1990 (British Standards Institution et al., 2021) is proposed. As demonstrated in Dubois and Prade (2016), a probabilistic cumulative distribution function (CDF) can be transformed into a possibility distribution u_G, consistent with the density function f_X, as described in the next Section 2.1.1. Therefore, whilst the probability assessment requires that the computed failure probability is lower than a pre-set target value P_0:

$$P_f = \int_{G(X)\leq 0} f_X(X)dX < P_0, \tag{7}$$

the proposed approach establishes a possibilistic target value, named Target Reliability Value, as a limit value on the membership function of the performance function, u_G.

2.1.1. Possibility measures and probability-to-possibility transformations

Generally, a membership function $u : \mathbb{X} \to [0,1]$ can represent a possibility measure on $\mathbb{X}$ with an associated *possibility distribution* $U : \Omega(\mathbb{X}) \to [0,1]$ with $\Omega(\mathbb{X})$ being the family of subsets of $\mathbb{X}$. For a subset $A \in \Omega(\mathbb{X})$, $U(A)$ is defined by $U(A) = sup\{u(x)|x \in A\}$ and has the following essential properties:

1. $U(A \cup B) = max\{U(A), U(B)\}$ for all $A, B \in \Omega(\mathbb{X})$;
2. $U(\mathbb{X}) = 1$,
3. $U(\emptyset) = 0$.

It can be observed that the possibility measure is defined in analogy with the probability measure, without relying upon the determination of a σ-field, set of all possible outcomes (Hose and Hanss, 2019).

To derive a possibility measure from Eq. (6), a useful representation (see Stefanini and Guerra, 2017; Guerra et al., 2020) of a fuzzy number u is through the λ-Average Cumulative Function (λ-ACF for short) of u, with $\lambda \in [0,1]$, stated as follows:

$$F_u^{(\lambda)}(x) = (1-\lambda)F_u^L(x) + \lambda F_u^R(x) \tag{8}$$

which is defined as the convex combination of two monotonic functions F_u^L and F_u^R, for all $x \in \mathbb{R}$:

$$F_u^L(x) = \begin{cases} 0 & if \quad x < a \\ u^L(x) & if \quad a \le x < c \\ 1 & if \quad x \ge c. \end{cases} \tag{9}$$

and

$$F_u^R(x) = \begin{cases} 0 & if \quad x \le d \\ 1 - u^R(x) & if \quad d < x \le b \\ 1 & if \quad x > b. \end{cases} \tag{10}$$

For $\lambda = \frac{1}{2}$ we denote $F_u^{(\frac{1}{2})}(x)$ by $F_u(x) = \dfrac{F_u^L(x) + F_u^R(x)}{2}$. Therefore, there exists a unique membership function u with AC function F_u such that, for all x and all $\alpha \in]0,1]$, we have (see Guerra et al. (2020)):

$$u_\alpha^- = (F_u)^{-1}\left(\frac{\alpha}{2}\right) \text{ and}, u_\alpha^+ = (F_u)^{-1}\left(1 - \frac{\alpha}{2}\right). \tag{11}$$

where u_α^+ and u_α^- are the end-points of the α-cuts. The α-cuts of the fuzzy interval u are the compact intervals $[u]_\alpha = \{x|\mu(x) \ge \alpha\} \subset \mathbb{R}$ with $\alpha \in]0,1]$, denoted by $[u]_\alpha = \left[u_\alpha^-, u_\alpha^+\right]$; for $\alpha = 0$ we define $[u]_0$ to be the closure of interval $\{x|\mu(x) > \alpha\}$, assumed to be bounded.

In such a way, u is determined by the pair $u = \left(u^-, u^+\right)$ of functions $u^-, u^+ : [0,1] \longrightarrow \mathbb{R}$. The representation of a possibility distribution as ACF can be effectively used for performing arithmetic operations between fuzzy sets (Stefanini and Guerra, 2017).

A further advantage of the λ-ACF is to produce a direct probability-to-possibility transformation T, given by Eq. (14), and its inverse T^{-1}, given by Eq. (8), as suggested in Guerra et al. (2020). Let us denote by $\mathcal{U}$ the collection of all possibility distributions U (or measures u) on $(\mathbb{X}, \Omega(\mathbb{X}))$ and by $\mathcal{P}$ the collection of all probability distributions P (or density p) on the same space $(\mathbb{X}, \Omega(\mathbb{X}))$. A probability-to-possibility transformation is any map $T : \mathcal{P} \to \mathcal{U}$ such that for $P \in \mathcal{P}$ it is $U = T(P) \in \mathcal{U}$ that transforms a probability distribution P into a possibility distribution U.

It is important to remark that T is an *admissible* transformation (see Jin et al. (2019)); i.e., it has the following properties, listed below in the case of a discrete set $(\mathbb{X})$:

1. T is *bijective*, i.e., the inverse T^{-1} exists: $T^{-1}(U) = P$ if and only if $T(P) = U$;
2. T is *consistent*, i.e., if $P \in \mathcal{P}$ and $U = T(P)$ we have $U(A) = max\{U(x)|x \in A\} \ge sum\{P(x)|x \in A\}$ for all $A \in \Omega(\mathbb{X})$.
3. T is *support-preserving*, i.e., $P \in \mathcal{P}$ and $U = T(P)$ we have $P(x) = 0$ if and only if $U(x) = 0$ for all $x \in \mathbb{X}$.

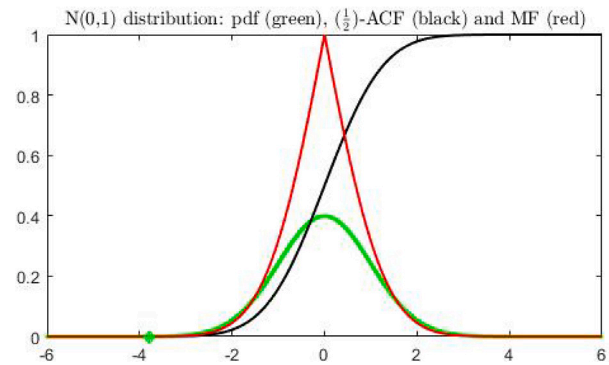


Fig. 2. Standardized normal distribution: probability density function (green), ($\frac{1}{2}$)-AC function (black) and corresponding Membership Function (red). The marked point is the target $-beta^* = -3.8$ for the Ultimate Limit State at 50 years corresponding to the failure probability 7.2348×10^{-5}; the membership value of -3.8 is $\alpha^* = 1.44696088 \times 10^{-4}$.

4. T is *co-monotonic*, i.e., if $P \in \mathcal{P}$ and $U = T(P)$ we have $P(x) \ge P(y)$ if and only if $U(x) \ge U(y)$ for all $x, y \in \mathbb{X}$.

Usually, a transformation T and its inverse T^{-1} are called Direct and Reverse, respectively. The ACF-based transformation is admissible and has the additional *core*-preserving property, that is:

$$P(x^*) = max\{P(x), x \in \mathbb{X}\} \text{ if and only if } U(x^*) = 1 \tag{12}$$

if $P \in \mathcal{P}$ and $U = T(P)$.

For any $\lambda \in]0,1[$, there exists a bijective correspondence between the family of all λ-AC functions $F_u^{(\lambda)}$ and the family of the probabilistic CDFs F_X of a real random variables X. For a fixed $F \equiv F_u \equiv F_X$, the function

$$F^{-1}(\alpha) = \inf\{x|F(x) \ge \alpha\} \text{ for all } \alpha \in]0,1] \text{ and } F^{-1}(0) = a \tag{13}$$

is called the quantile function of F.

Remark that for the value of $\lambda = 1/2$, the membership function u, associated to a given AC function, $F : \mathbb{R} \to [0,1]$, can be obtained easily as $u(x) = 2 \cdot min\{F(x), 1 - F(x)\}$; in this case the core of u coincides with the median of the distribution F. Alternatively, the value of $\lambda \in]0,1[$ can be calibrated such that the core of the possibility distribution u coincides with the modal value, $\hat{x}$, of the probabilistic F, (more details in Guerra et al. (2020)):

$$u(x) = min\left\{\frac{F(x)}{1-\hat{\lambda}}, \frac{1-F(x)}{\hat{\lambda}}\right\} \tag{14}$$

where $\hat{\lambda} = 1 - F(\hat{x})$.

It is worth mentioning that a probability–to–possibility transformation, for a unimodal probability distribution P, can be obtained by considering the value $\hat{x}$, corresponding to the modal (of highest probability) element to be considered the most natural nominal value (see e.g., Dubois et al., 2004). On the other hand, an interval I_α, for a fixed confidence level $\alpha \in]0,1[$, is such that $P(I_\alpha) = 1-\alpha$ and $P(I_\alpha^c) = \alpha$ has the meaning of a risk level, i.e., the probability for the variable X to be outside I_α. Consequently, a family $\left\{\hat{I}_\alpha : \alpha \in]0,1[\right\}$ of nonempty and nested intervals containing $\hat{x}$ induces the fuzzy membership $\hat{u}$ such that its α-cuts are $[\hat{u}]_\alpha = \hat{I}_\alpha$, i.e., for all α,

$$\hat{u}(x) = sup_\alpha\left\{1 - P(\hat{I}_\alpha) : x \in \hat{I}_\alpha\right\}. \tag{15}$$

Clearly, each measurable set A has $P(A) \le \hat{U}(A)$, i.e., P is consistent with $\hat{U}$ (see also Dubois and Prade (1990)). If the sets $\hat{I}_\alpha$ above have minimal length then $\hat{u}$ is maximally specific, satisfies the consistency principle and the order preservation condition ($\hat{u}(x') \ge \hat{u}(x'') \Leftrightarrow p(x') \ge p(x'')$).

Therefore, for all $x \le \hat{x}$ and for all probabilities P of the credal set (Dubois and Prade, 2016) of u (or U), the following property holds:

$$P(]-\infty, x]) \le (1-\hat{\lambda})u(x). \tag{16}$$

Specifically, if a reliability condition of the form $P(]-\infty, x]) \leq \epsilon$ is to be tested for a fixed $x \leq \hat{x}$, where ϵ is a small value (e.g., the failure probability P_0), the safety test can be performed on $u(x)$, as follows:

$$u(x) \leq \frac{\epsilon}{(1-\hat{\lambda})} \Longrightarrow P(]-\infty, x]) \leq \epsilon. \tag{17}$$

Therefore, Eq. (17) can be exploited to define a safety criterion for the fuzzy-valued performance function G, in terms of its membership function $u_G : \mathbb{R} \to [0,1]$ of Eq. (5) by extending the reliability problem of Eq. (2) to fuzzy numbers:

$$\begin{aligned} &u_G(0) < \alpha^* \Rightarrow \text{Safe} \\ &u_G(0) \geq \alpha^* \Rightarrow \text{Failure;} \end{aligned} \tag{18}$$

here, the maximum acceptable possibility level to guarantee structural safety, herein called Target Reliability Value (TRV), α^* is determined as:

$$\alpha^* = \frac{P_0}{(1-\hat{\lambda})}. \tag{19}$$

It is worth emphasizing that the proposed possibility TRV represents an upper bound value for all probabilities P of the credal set of u, hence the proposed methodology can be seen as conservative with respect to the probabilistic Level II approach. For a membership function constructed from a normal distribution and with core coincident with the median of the distribution, $\alpha^* = 2P_0$ is obtained.

A similar approach was recently suggested by Guerra and Sorini (2020) for assessing the risk management in real financial markets; the ACF transformation of Eq. (13) is used to define a possibilistic (fuzzy) *Value at Risk* (VaR) measure, given, for a fixed membership value α^*, by

$$VaR_{ACF}(\alpha^*) = (F_u)^{-1}(\alpha^*). \tag{20}$$

The relevant advantage of the proposed possibility reliability for Eq. (18) lies in the simplicity of its computation: once the parameters are represented as fuzzy numbers and the membership function of G is obtained as in Eq. (6), the condition $G = 0$ is verified when the associated degree of membership $u_G(0)$ is lower than a given target α^* and consequently the structural design can be considered to be *safe*; otherwise, it is considered to *fail*.

2.1.2. Consistency of the target reliability value with the prescribed standards

The Target Reliability Value, α^*, required for the reliability assessment of Eq. (18), which dictates the required level of safety on the proposed possibility approach is defined to be consistent with the prescribed standards. In particular, under the probability Level II approach, EN1990 (British Standards Institution et al., 2021) proposes the evaluation of the reliability index β, which is a measure of the distance between the mean performance and the failure boundary, defined as follows:

$$\beta = \frac{\mu_R - \mu_V}{\sqrt{(\sigma_R^2 + \sigma_V^2) - 2 \cdot Cov[R, V]}} \tag{21}$$

where $\mu_{R,V}$ and $\sigma_{R,V}$ is the mean and standard deviation of the resistance and load, respectively. The term $Cov[R, V]$ is the covariance between R and V which yields 0 if the two random variables are independent.

The probabilistic reliability index β should be greater than a preset value, the target β^*, given by the structural code and reported in Table 1, for 3 classes of consequences, i.e. RC3, RC2 and RC1, corresponding to i.e. high, medium, low consequences, respectively.

In Table 1 are also reported the corresponding target failure probabilities, P_0, obtained through the following expression:

$$P_f^* = \Phi(-\beta^*) \tag{22}$$

where Φ is the cumulative distribution function of the standardized Normal distribution. For instance, in case of $\beta^* = 3.8$, the target probability of failure is $P_f^* = 7.2348 \times 10^{-5}$.

Table 1
Recommended minimum values for reliability index β (probability of failure in parenthesis) for ULS (British Standards Institution et al., 2021).

Reliability class	1-year reference period	50-year reference period
RC1	5.2 (9.9644×10^{-8})	4.3 (8.5399×10^{-6})
RC2	4.7 (1.3008×10^{-6})	3.8 (7.2348×10^{-5})
RC3	4.2 (1.3346×10^{-5})	3.3 (4.8342×10^{-4})

Table 2
Proposed recommended values for the Target Reliability Value α^* for ULS.

Reliability class	1-year reference period	50-year reference period
RC1	1.993×10^{-7}	1.708×10^{-5}
RC2	2.602×10^{-6}	1.447×10^{-4}
RC3	2.669×10^{-5}	9.668×10^{-4}

In case of the standard Gaussian distribution $N(0,1)$ the maximal specificity transformation u_N (in closed form) is such that for any negative $x^* < 0$ with $U_N(x^*) = \alpha^*$ it is $P(]-\infty, x^*]) \leq \beta^*$ where $\beta^* = P(x^*)$.

Therefore, in this paper, the Target Reliability Value is derived in order to guarantee a level of safety comparable to the probabilistic reliability of Table 1, by adopting the probability to possibility transformation.

Therefore, from the cumulative distribution function of the standardized normal distribution $N(0,1)$ used to calculate the probability of failure through Eq. (22), a fuzzy number can be elicited through the ACF transformation. Fig. 2 shows the PDF of the standard $N(0,1)$ by a green-coloured curve and its cumulative distribution function Φ by a black-coloured curve, which is assumed to coincide with the AC function, F, appearing in Eq. (14) and giving, as the left side of Eq. (14), the resulting membership function $u_{N(0,1)}$, pictured in red colour. Therefore, it is possible to derive the value α^* of the membership function $u_{N(0,1)}$, corresponding to the point of the probabilistic target reliability index $-\beta^*$ (e.g. $\beta^* = 3.8$), as marked in Fig. 2; this computed value is referred to as the Target Reliability Value α^* and given in Table 2 for the three reliability classes RC1, RC2 and RC3 and for the 1-year or 50-year reference periods.

It is worth emphasizing that many probability-to-possibility transformations are introduced by several scholars in the literature. An interesting setting helping to choose the one or other among them, has been proposed recently by Jin et al. (2019), where the class of so-called Arising Accumulation Transformations (AAT and its inverse RAT) introduced by Dubois and Prade (see, e.g., Dubois and Prade (2016) and Dubois et al. (2004)) is analysed. An empirical comparison of different transformations is not the direct scope of this paper but is clear that the proposed calibration of the Target Reliability Value can be performed through a different transformation, without altering the mathematical construction in a significant way. For example, using the AAT as a substitute of ACF, a corresponding α^*_{AAT} will be obtained; as AAT has a higher uncertainty degree with respect to ACF (see e.g., Jin et al., 2019), a value of $\alpha^*_{AAT} = 2.363 \times 10^{-3} > \alpha^*_{ACF}$ for the 50-years RC2 case of Table 2.

2.1.3. Computational approach to the extension principle

By exploiting the Zadeh's extension principle (Zadeh, 1975), the performance function G of Eq. (1) is extended to a fuzzy interval $u_G = G_{EP}\left(u_{X_1}, \dots, u_{X_d}\right)$ having α-cuts $[G]_\alpha = [G^-_\alpha, G^+_\alpha]$, obtained by solving the box-constrained global optimization problems, for $\alpha \in [0,1]$,

$$\begin{cases} G^-_\alpha = \min\left\{ G(x_1, \dots, x_d) | x_j \in [u_{X_j}]^\alpha,\ j = 1, \dots, d \right\} \\ G^+_\alpha = \max\left\{ G(x_1, \dots, x_d) | x_j \in [u_{X_j}]^\alpha,\ j = 1, \dots, d \right\} \end{cases}. \tag{23}$$

The optimization (23) can be solved for a finite set of N values of α, say $0 = \alpha_1 < \alpha_2 <, \dots, < \alpha_N = 1$, and in general, a relatively small value for N is adopted, depending on the required precision in constructing a good approximation of the whole membership function; the values $\alpha_1 = 0$ and $\alpha_N = 1$ are always included, in order to obtain the exact support and core of the membership function of the fuzzy number G_{EP} as in Eq. (6) and deduce its associated possibility measure by the ACF transformation of Eq. (14). For a general (non monotonic) function G, it is necessary to solve globally the *min* and *max* problems in Eq. (23) for the N different values of the selected α_j. This can be done simultaneously with a single call to an optimization routine, e.g., Hanss (2005) adopts its so-called transformation method, while Stefanini et al. (2008) adapted two versions of the differential evolution (DE) algorithm.

The computational complexity is reduced significantly when the problem is described through a monotonic function, namely when the performance function G is monotonic increasing or decreasing with respect to its parameters. This implies that the Eq. (23) can be globally solved by simply evaluating function G at the extreme points of intervals $[u_{X_j}]^\alpha$, $j = 1, \dots, d$, corresponding to the required α-cuts with $\alpha \in \{\alpha_1, \alpha_2, \dots, \alpha_N\}$, i.e., with a single computation of G for each minimization and maximization. Therefore the total number of function evaluations depends only on the discretization of the membership function.

2.2. Robust embedding of databases and local experience

The fuzzification of the performance function in Eq. (5) requires defining the parameters or inputs of the considered engineering problem, i.e., $X_1, \dots, X_d$, as possibility distributions, in which the vagueness and uncertain nature is expressed by membership functions, $u_{X_1}, \dots, u_{X_d}$.

Given a database of d-dimensional tuples, $\mathbf{x}_i = (x_{i,1}, \dots, x_{i,d})$, $i = 1, 2, \dots, m$, with m records for each of the given d attributes, the estimation of the membership function for each of the d parameters taken independently, is achieved through the three-step procedure described below. In the rest of this section, the procedure is developed for a given input X_j, hence, the j index is omitted, denoting with $x_1, x_2, \dots, x_m$ the available values instead of using the double index notation $x_{1,j}, x_{2,j} \dots, x_{m,j}$. The data x_i are assumed to be elements of a real interval $[a, b]$ such that $a \le x_i \le b$, $i = 1, 2, \dots, m$, i.e., $a = min\{x_i | i = 1, \dots, m\}$ and $b = max\{x_i | i = 1, \dots, m\}$.

2.2.1. Step 1: Fuzzy clustering and partitioning

The first step entails to perform the clustering of the database, hence decomposing the data into subgroups; clustering is essentially an optimization approach that consists in determining a family of n_c classes $C_1, \dots, C_{n_c}$ and a numerical matrix $U = [u_{i,k}]$ with $u_{i,k} \in [0, 1]$, in order to minimize an objective function defined in several ways (see, e.g., Bezdek (1981), Döring et al. (2006) and Oliveira and Pedrycz (2007)) based on some distance measure, $d(x_i, C_k)$, between the ith record and the kth class (cluster); for instance,

$$\text{minimize } J(U, C_k, \beta) = \sum_{i=1}^{m} \sum_{k=1}^{n_c} u_{i,k}^{\beta} d_{i,k}^2 \tag{24}$$

where the component $u_{i,k}$ of matrix U measures the degree of membership of x_i to class C_k and $\beta \ge 1$ is a fixed fuzzification coefficient. For $\beta = 1$, the approach is consistent with the deterministic *k-means* or *hard* clustering. Usually, each cluster C_k is identified in terms of a computed (representative) prototype value $c_k \in [a, b]$, $k = 1, 2, \dots, n_c$ and the distance $d_{i,k}$ is the Euclidean one.

The matrix U represents a (discrete) n_c-dimensional fuzzy set such that its kth column, $k = 1, 2, \dots, n_c$, is a fuzzy set on the m records in the data set. From the properties of U, giving a fuzzy clustering of the data set, no cluster is empty and each datum has total membership 1, i.e., the following two conditions hold true:

$$\sum_{i=1}^{m} u_{i,k} > 0 \text{ for all } k \text{ , } \sum_{k=1}^{n} u_{i,k} = 1 \text{ for all } i.$$

The fuzzy clustering (*fuzzy c-means*) can be carried out through the Alternating Optimization (AO) algorithm as described in Bezdek (1981) and Oliveira and Pedrycz (2007). The algorithm is implemented in the MATLAB (MATLAB, 2022) routine ***fcm***.

It is well known that, except for special cases, the columns of matrix U do not produce fuzzy numbers, but only fuzzy sets with non-monotonic left and right branches as instead required in Eq. (3); an additional step is then required, to modify U such that each class C_k corresponds to the membership function of a fuzzy number, hence to a possibility distribution. From the clustering matrix $U = [u_{i,k}]$, to the ith data and for each of the $k = 1, \dots, n_c$ classes, a (finite) *fuzzy n_c-partition* of the interval $[a, b]$ in terms of a family of n_c fuzzy numbers $\{A_k^{n_c}, k = 1, 2, \dots, n_c\}$ is sought. In this paper, the so-called *Ruspini condition* (see, e.g., Holčapek et al., 2015; Novák et al., 2016) is satisfied for any pair of two consecutive partition classes, the kth and $(k+1)$-th; namely, $A_k^{n_c}(x) + A_{k+1}^{n_c}(x) = 1$ for all $k = 1, 2, \dots, n_c - 1$ and all $x \in [a, b]$. The usefulness of a fuzzy partition is that each observed datum x_i has either one membership value $A_k^{n_c}(x_i) = 1$ (and it belongs to the core of the fuzzy number $A_k^{n_c}$ representing the kth cluster), or exactly two positive values $A_k^{n_c}(x_i)$ and $A_{k+1}^{n_c}(x_i) = 1 - A_k^{n_c}(x_i)$, the others being zero. Furthermore, the centroids $c_1, c_2, \dots, c_{n_c}$ obtained in the fuzzy clustering fully characterize the cores and supports of the fuzzy numbers $A_k^{n_c}$ in the Ruspini partition; in particular, on the left side of the interval $[a, b]$, $A_1^{n_c}$ has core $[a, c_1]$ and support $[a, c_2]$, each intermediate $A_k^{n_c}$, for $k = 2, \dots, n_c - 1$, has core c_k (a singleton) and support $[c_{k-1}, c_{k+1}]$, and on the right side of $[a, b]$, $A_{n_c}^{n_c}$ has core $[c_{n_c}, b]$ and support $[c_{n_c - 1}, b]$.

Associated with a Ruspini partition there is a semantic interpretation of the family of numbers $A_k^{n_c}$ in terms of the *level of granularity* of a variable X on interval $[a, b]$, expressed by the number n_c of clusters. For example, a partition into three classes, with fuzzy numbers A_1^3, A_2^3, A_3^3, can be interpreted as A_1^3 = 'value of X is Low', A_2^3 = 'value of X is Medium', A_1^3 = 'value of X is High'; similarly, a partitioning with six classes may correspond to a semantics A_1^6 ='Very Low X', A_2^6 ='Low X', A_3^6 = 'Low-to-Medium X', A_4^6 = 'Medium-to-High X', A_5^6 = 'High X', and A_6^6 ='Very High X'.

Therefore, by increasing the number n_c of classes, the granular interpretation of the variable X in $[a, b]$ becomes finer and hence, its degree of uncertainty decreases. This property is here exploited to embed the local experience and engineering judgement for the assessment of the reliability; the level of confidence in the experimental data and in the geotechnical design is hereinafter called 'Degree of Understanding' (DoU) in analogy with the concept proposed by Fenton et al. (2016) or by the Canadian Highway Bridge Design Code - CSA S6:19, CSA (2019). The DoU is used to define a tier system which implements the recommendations of the BS EN1997 (British Standards Institution et al., 2007) for determining reliable geotechnical parameters by considering local experience, sample size and comparison with published data. If the level of confidence or DoU is 'high', a higher number of classes are used so to reduce the epistemic uncertainty and therefore, giving more weight to the experimental data; on the other hand, for low levels of confidence or 'low' DoU, a lower number of classes will be used, leading to fuzzy partitions with a level of epistemic uncertainty consistent with the dispersion of the database.

2.2.2. Step 2: Determine empirical membership functions

The *second step* consists in obtaining, for the n_c classes of the Ruspini partition, the empirical membership functions extracted from the observed values $\{x_i | i = 1, 2, \dots, m\}$ in the clustered database. For each kth Ruspini membership function $A_k^{n_c}$, the associated empirical membership function u_k^{ACF}, $k = 1, \dots, n_c$, through the ACF transformation of the subset of data belonging to the support of each $A_k^{n_c}$, $k = 1, 2, \dots, n_c$, as described by Eq. (14). The transformation will generate a possibility distribution consistent with the empirical cumulative distribution of the considered subset of data.

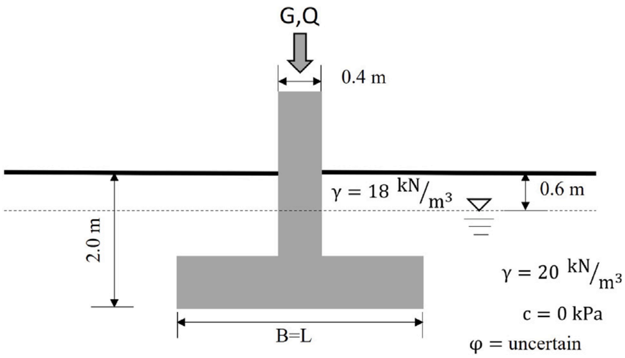


Fig. 3. Square Pad Foundation under vertical load.
Source: Readapted example from Orr and Farrell (2011).

2.2.3. Step 3: Derivation of the design Fuzzy membership function

The final step of our procedure is to obtain the fuzzified version of the input parameter $\hat{X}$, i.e., its membership function $u_{\hat{X}}$ on the range $[a,b]$, referred to as Design Membership Function. The Design MF is derived by considering two consecutive fuzzy numbers $A_{\hat{k}}^{n_c}$ and $A_{\hat{k}+1}^{n_c}$ of the Ruspini partition $\left\{A_k^{n_c} \mid, k=1,2,\dots,n_c\right\}$ obtained in *Step 1*; these are selected from the database partitions according with the position of a fixed parameter $\hat{x}$. In this paper, $\hat{x}$ is the characteristic or nominal value (Nawari and Liang, 2000; Länsivaara et al., 2022) of the considered geotechnical parameter.

Finally, a normalized convex combination is used for merging two adjacent empirical membership functions as follows:

- if the position of $\hat{x}$ is $\hat{x} \notin core(A_{\hat{k}}^{n_c}) \cup core(A_{\hat{k}+1}^{n_c})$ and $\hat{x} \in supp(A_{\hat{k}}^{n_c}) \cap supp(A_{\hat{k}+1}^{n_c})$, for $\hat{k}=1,\dots,n_c-1$, then:

$$u_{\hat{X}} = \frac{w u_{\hat{k}}^{ACF} + (1-w) u_{\hat{k}+1}^{ACF}}{\max(w, 1-w)} \tag{25}$$

  where the weights w and $1-w$, are given by $w = A_{\hat{k}}^{n_c}(\hat{x})$ (or, equivalently, $1-w = A_{\hat{k}+1}^{n_c}(\hat{x})$);
- if $\hat{x} \in core(A_{\hat{k}}^{n_c})$ (and consequently $1-w=0$, $w=1$), then, set $u_{\hat{X}} = u_{\hat{k}}^{ACF}$;
- if $\hat{x} \in core(A_{\hat{k}+1}^{n_c})$ (and consequently $1-w=1$, $w=0$), then, set $u_{\hat{X}} = u_{\hat{k}+1}^{ACF}$.

The obtained fuzzy number, $u_{\hat{X}}$, defines the Design Membership Function to be used as an argument of the fuzzy performance function of Eq. (5), to assess the possibilistic reliability in Eq. (18). The Design Membership Function embodies the aleatory randomness of the considered parameter as well as the epistemic information extracted from the database through the core, the support and the shape of its membership function.

3. Possibility reliability assessment of shallow foundations

3.1. Problem description

In this section, the proposed methodology is applied to the common engineering problem of safety assessment of a square pad foundation under vertical loads. The design of the footing size is performed with regard to the Ultimate Limit State, and then, the structural reliability is assessed through traditional probability methods as well as the proposed possibility approach. The investigated problem, as illustrated in Fig. 3, is re-adapted from the worked example in Orr and Farrell (2011) by considering the angle of internal friction as an uncertain parameter.

The bearing resistance per unit of area of square shallow foundations of dimensions $B \equiv L$ can be estimated through the trinomial equation as follows:

$$q_{ult} = c N_c s_c + q_s N_q s_q + 0.5 \gamma_1 B N_\gamma s_\gamma \tag{26}$$

where q_{ult} is the ultimate bearing capacity of the footing, c is soil "effective" cohesion, γ is the unit weight of the soil, q_s is the surcharge pressure and B is the width of the foundation; N_c, N_q, N_γ are the bearing capacity factors and s_c, s_q, s_γ are the shape factors, as prescribed by BS EN1997 (British Standards Institution et al., 2007). It is worth noting that an interesting approach to capture the uncertainty in the modelling the considered physical engineering problem is based on the introduction of a model factor (see e.g., Tang and Phoon, 2021; Phoon et al., 2022a) to modify Eq. (26); nevertheless, this paper is limited to sources of uncertainties affecting the parameters of the given model, and hence, the model factor is neglected in this study without compromising its mathematical relevancy.

The following expressions for the bearing capacity factors and shape factors (for non-inclined vertical loads and square shape), are considered:

$$\begin{aligned} N_q &= \exp(\pi \cdot \tan(\phi)) \cdot (\tan(\pi/4 + \phi/2))^2 \\ N_c &= (N_q - 1) \cdot \cot(\phi) \\ N_\gamma &= 2 \cdot (N_q - 1) \cdot \tan(\phi) \\ s_q &= 1 + 1 \cdot \sin(\phi) \\ s_c &= \frac{s_q \cdot N_q - 1}{N_q - 1} \\ s_\gamma &= 0.7 \end{aligned} \tag{27}$$

The angle of internal friction, ϕ, and the cohesion, c, of the soil, governing the ultimate bearing capacity of the foundation, have to be considered as nominal or characteristic values or design values according with BS EN1990 (British Standards Institution et al., 2021). The resistance of the shallow foundation is obtained as follows:

$$R = q_{ult} \cdot B^2 \tag{28}$$

The structural safety at the Ultimate Limit State (ULS) described by the function $G = f\left(X_1, \dots, X_d\right)$ of Eq. (1) is hence assessed by comparing the resistance given by Eqs. (26)–(28) to the external load V.

In the following sections, two numerical applications will be conducted. In the first one, the frictional angle is assumed as an ideal random variable so to highlight the consistency of the proposed possibility approach with the prescribed probability values reported in EN1990 (British Standards Institution et al., 2021). In the second worked example, the values of the angle of internal friction are obtained from a limited number of real experimental direct shear testing conducted by three different technicians, and the reliability assessment is performed by considering subjective information as well as an real database.

The remaining parameters of the problem are considered certain/crisp; Fig. 3 shows the values of the soil parameters (e.g. surcharge pressure $q = 18\,\mathrm{kN/m^3} \cdot 0.6\,\mathrm{m} + 20\,\mathrm{kN/m^3} \cdot 1.4\,\mathrm{m} = 25.1\,\mathrm{kN/m^2}$ and $\gamma = 20\,\mathrm{kN/m^3}$) and geometrical dimensions (e.g. foundation depth $= 2\,\mathrm{m}$); on the other hand, the size of the foundation ($B = L$) is designed consistently with the Level I semi-probability approach. To keep the focus on the soil uncertainty, the vertical unfavourable loads, i.e. the permanent load, $G = 935.9\,\mathrm{kN}$, and the variable load, $Q = 300\,\mathrm{kN}$ are considered deterministic through the application of the partial load factors according the Design Approach 1 (DA1-1 and DA1-2) prescribed by the UK National Annex to EN 1997-1. It is worth emphasizing that the proposed possibility methodology can also be directly applied when the remaining parameters are considered as fuzzy sets; on the other hand, if any of them are considered purely random or aleatory, a hybrid approach should be used as done in Tombari and Stefanini (2019).

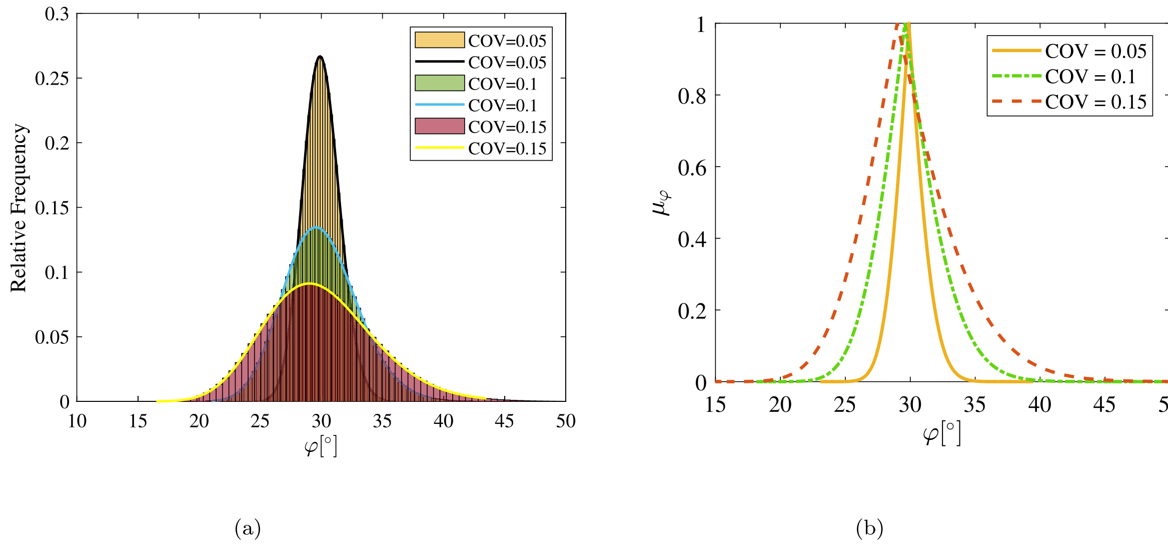


Fig. 4. Representation of the sets of angles of internal friction for 3 different COVs as (a) Histograms and Probability Distribution Functions (b) Fuzzy Membership functions.

3.2. Comparative study: Probability and possibility reliability assessment

The following application aims to demonstrate the validity of the proposed method to assess the reliability of the foundation design by maintaining the consistency between Probability and Possibility theory for the definition of the target safety criterion. The methodology of Fig. 1 is applied without considering the use of an indirect database to allow a direct comparison with the probability approach.

Let us consider the problem depicted in Fig. 3 where the uncertain angle of internal friction ϕ is ideally described through a log-normal probability distribution function with mean value $\bar{\phi} = 30°$ and variable standard deviation σ_ϕ obtained by equally ranging 15 Coefficients of Variation, $COV = \sigma_\phi/\bar{\phi}$ from 0.01 to 0.15. A large number of samples, $N = 1 \times 10^7$ samples have to be generated in order to reach the convergence at the target probability of failure through Eq. (7).

3.2.1. Probabilistic Level I and Level II reliability

Once the distributions of the angle of internal friction have been obtained for each considered COV, the size of the square foundation, $B = L$, is calculated according with the Level I semi-probabilistic approach. Each characteristic value is computed as 5% fractile of the related probability distribution function (PDF). PDFs of the generated angles of friction for 3 selected COVs are plotted in Fig. 4a. In Fig. 5a is depicted the variation of the characteristic value and the designed footing width as a function of the COV. To assess the level of safety, the Level II-β approach is first applied. The reliability index, β, calculated through Eq. (21), is plotted in Fig. 5b.

Since the partial safety factor used in the Level I design is constant and invariant with respect to the actual dispersion of the data, a uniform level of safety cannot be guaranteed. Although the foundation has been properly designed, Fig. 5b shows that the Reliability Index, β is lower than the target, β^* of Table 1 for low COVs, and greater than the required level of safety for high COVs, leading to over- or under-conservative design depending on the dispersion of the input parameters.

Moreover, because of the nonlinear dependence between the angle of friction and the bearing resistance, the resulting probability distribution function is not lognormal distributed. Fig. 6a shows the normalized histograms of bearing resistance values, R, in comparison with inferred log-normal distributions; differences can be observed even for low COVs at the tails of the PDFs. The results in terms of the computed probability of failure compared to the target one are depicted in Fig. 6b; the divergence with the probability of failure calculated from the β values evidences the impact of the non-gaussianity of the resulting distribution.

3.2.2. Possibility and probabilistic reliability

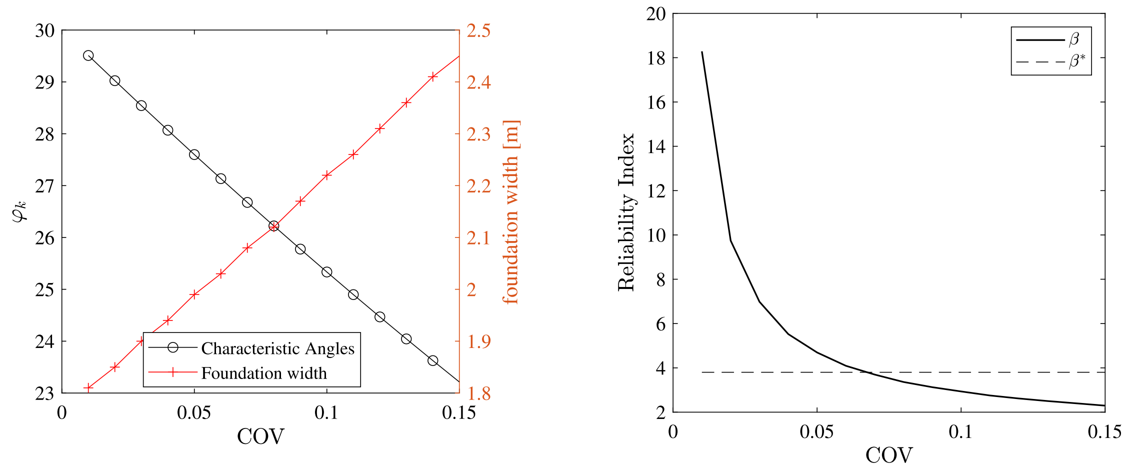
The previously generated distributions of angles of internal friction are transformed into fuzzy sets through the ACF transformation of Eq. (14). As mentioned in Section 2.2, the proposed procedure can be still applied without an indirect database by taking the whole dataset as an individual partition. The membership functions of the angle of internal friction, u_ϕ, are illustrated in Fig. 4b for the same selected COVs of Fig. 4a. The support width of each membership function is directly proportional to the coefficient of variation, hence, the larger is, the more dispersed are the data, whilst the core represents the median of the data. The skewness and the shape of the membership function can also provide information about the propagation of the uncertainty.

Once the membership functions of the input data are obtained, Eq. (1) is fuzzified as described in Section 2.1. The membership functions of the performance function or ultimate limit state are hence obtained through the computational approach described in Section 2.1.3; since the function is monotonic, the computational procedure is simplified as follows:

1. The membership function of the internal friction angle, u_ϕ, is given as a ordered pair (ϕ, u_ϕ) where ϕ is sorted in increasing order and $u_\phi \in]0\ 1]$;
2. The deterministic performance function of Eq. (1) is solved sequentially for each ϕ, and each value of the function, $G(\phi)$ is associated with the corresponding u_ϕ;
3. The membership function of G, u_G, is then recovered from all the ordered pairs, (G, u_ϕ).

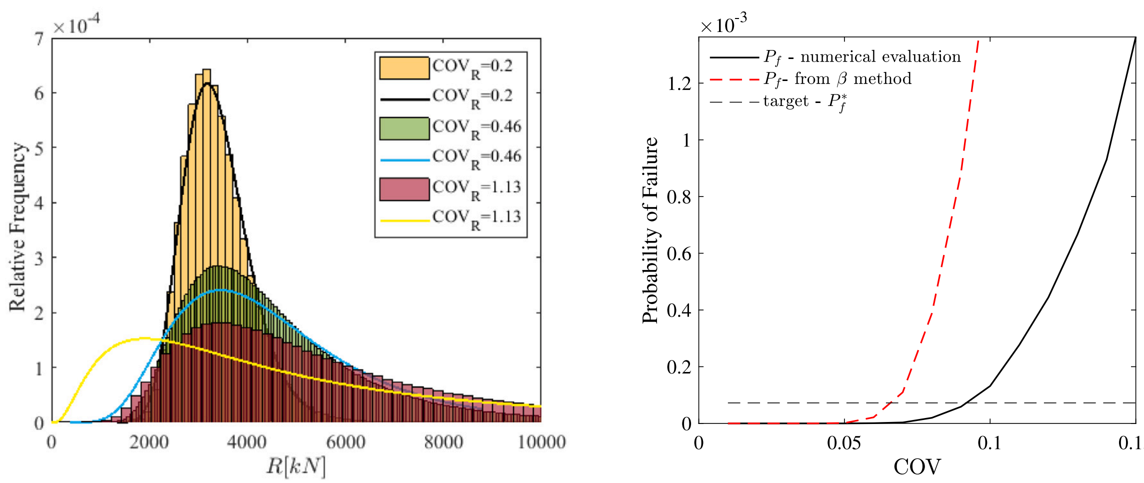
Fig. 7a shows the computed membership functions of the performance function for three selected values of COVs of the input data, namely COV $= 0.05, 0.1, 0.15$. Therefore, by adopting the criterion proposed in Section 2.1.2, the value of the membership function, $u_G(g = 0)$, is compared with the target fuzzy reliability index, $\alpha^*_{cut} = 1.447 \times 10^{-4}$, consistent with the probability of failure for a 50-year reference period in the probability framework. Fig. 7b shows a close-up of the membership functions around the $G = 0$ compared to the proposed target Reliability Index Value.

The safe/unsafe test is repeated for each considered COV; results are shown in Table 3. In the same figure, the probability of failure is also numerically computed for the same sets of data assessed against the target probability of failure of Eq. (22); therefore, it is demonstrated that the proposed possibility approach provides with a safe-failure outcome consistent with the probability test, as initially aimed. It is worth emphasizing that $N = 1 \times 10^7$ samples had to be used to have



(a) (b)

Fig. 5. (a) Characteristic angle of internal friction and foundation width B for each generated distribution; (b) Level II - Reliability β Index.



(a) (b)

Fig. 6. (a) Histograms of the calculated bearing resistances for 3 different COVs, the continuous curves represent the log-normal Probability Distribution Functions obtained from the mean and standard deviation values computed from the data; (b) Probability of Failure calculated through numerical and *beta* methods.

a convergence result for the probability test; on the other hand, a reliable fuzzy set can be obtained by using only a few data along with the support of an indirect, regional or national, database and local experience as shown in the following application. It is worth mentioning that the number of function evaluations to perform the proposed possibilistic approach is related to the discretization of the membership function, which can be refined around $u_G(g = 0)$ to reduce the computational complexity.

Table 3
Comparison of possibility and probability reliability assessment for several COVs of the angle of internal friction.

Reliability measure	COV_φ	0.01	0.05	0.09	0.1	0.15
Probability	P_f	0	0	6.6e−5	2.78e−4	3.04e−3
	Test - Eq. (7)	Safe	Safe	Safe	Fail	Fail
Possibility	$u_G(0)$	0	0	1.414e−4	1.294e−4	1.3478e−3
	Test - Eq. (18)	Safe	Safe	Safe	Fail	Fail

3.3. Possibilistic reliability using experimental data

3.3.1. Experimental data

In this section, the soil of the geotechnical problem described in Section 3.1 is characterized through laboratory testing. The values of the angle of internal friction, ϕ, are obtained from experimental direct shear tests carried out by three independent technicians (T1, T2, T3). A dry silica sand, composed by mixing rounded grains of size corresponding to fraction B (particle size between 1.18 mm–600 μm) and fraction C (particle size between 600 μm–300 μm) in a ratio 1:2, has been tested. Each soil specimen is prepared by dry tamping layers of 10 mm with 25 blows per layer, until the rectangular shear box of 100 mm × 100 mm area and 48 mm depth has been filled. The procedure has been followed by each technician in order to reach the same compaction for every sample. The test is conducted at three increasing loading stages (490.5 N, 981 N and 1471.5 N), and the angle of internal friction is derived by least-square fitting of the Mohr–Coulomb model, in which the failure criterion is determined as a linear function of the normal

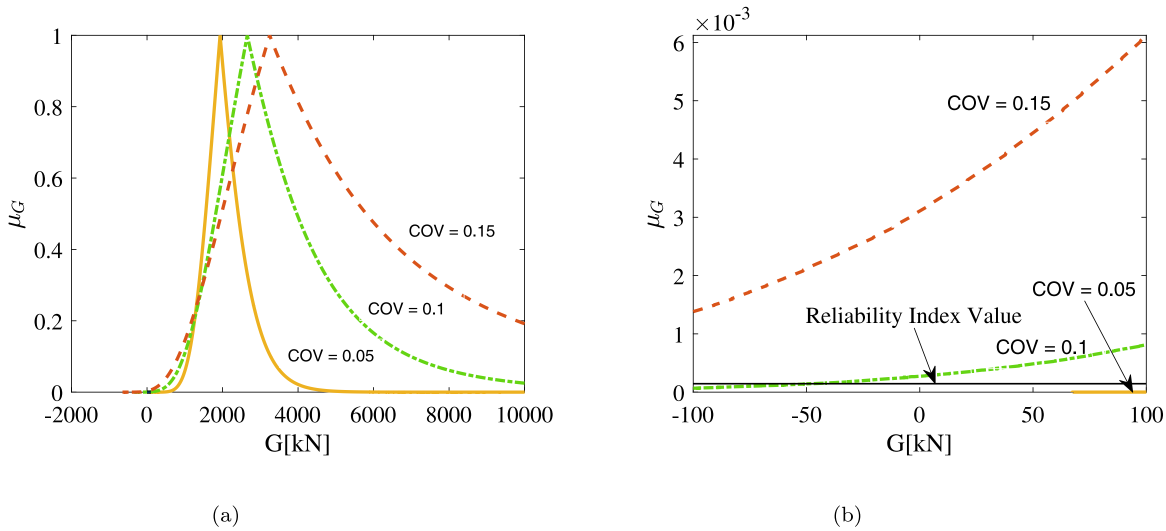


Fig. 7. Fuzzy Performance functions for 3 different COVs (a) truncated function around G = 0, (b) close-up and comparison with proposed Target Reliability Value.

Table 4
Angle of the internal angle of friction obtained through direct shear testing.

Technician	Number of tests	Type	Mean value [°]	Interval [°]	COV [%]
T1	22	ϕ_{cv}	29.95	29.14–30.77	1.65
		ϕ_{peak}	38.47	36.81–40.25	2.9
T2	10	ϕ_{cv}	29.57	27.52–31.23	3.84
		ϕ_{peak}	33.67	33.06–34.31	1.39
T3	4	ϕ_{cv}	32.53	31.77–33.62	2.76
		ϕ_{peak}	43.11	41.83–44.38	2.67

stress as follows:

$$\tau_f = c + \sigma_v \tan(\phi) \tag{29}$$

where τ_f is the shear strength, c is the cohesion, σ_v is the vertical stress normal to the failure place, and ϕ is the angle of internal friction. Since the sand is dry, the cohesion c is equal to 0. Eq. (29) can be used to obtain the critical or peak angle of friction, ϕ_{cv} and ϕ_{crit}, respectively. Statistics of the results from each technician are reported in Table 4. The dispersion of the results are shown in Fig. 8 where it can be observed the higher dispersion of the peak angle of friction compared with the critical value.

3.3.2. Foundation design and probabilistic reliability

The experimental results are used to design the pad foundation of Section 3.1 by means of the Level I semi-probabilistic approach. The nominal value for the angle of internal friction used for the design of the foundation is taken as the 5% fractile of the normal distribution interfered with the data obtained by each technician in Table 4.

It is worth mentioning that given the relatively low number of executed tests, the adoption of the fractile value could be unreliable and the minimum or average value of the interval of data could be used instead (e.g., see Länsivaara et al., 2022). Nevertheless, no significant differences in the foundation size have been obtained.

The final designed sizes of the pad foundation are $B = L =$ 1.24 m, 1.52 m and 0.95 m for the soil values obtained by technicians T1, T2, T3, respectively. The level of safety is then assessed through a Level II approach by computing the failure probability; the reliability requirement is met for both approaches because of the low dispersion of the data (see Table 4), in accordance with the analysis conducted in Fig. 6 of Section 3.2.1.

3.3.3. Design membership functions

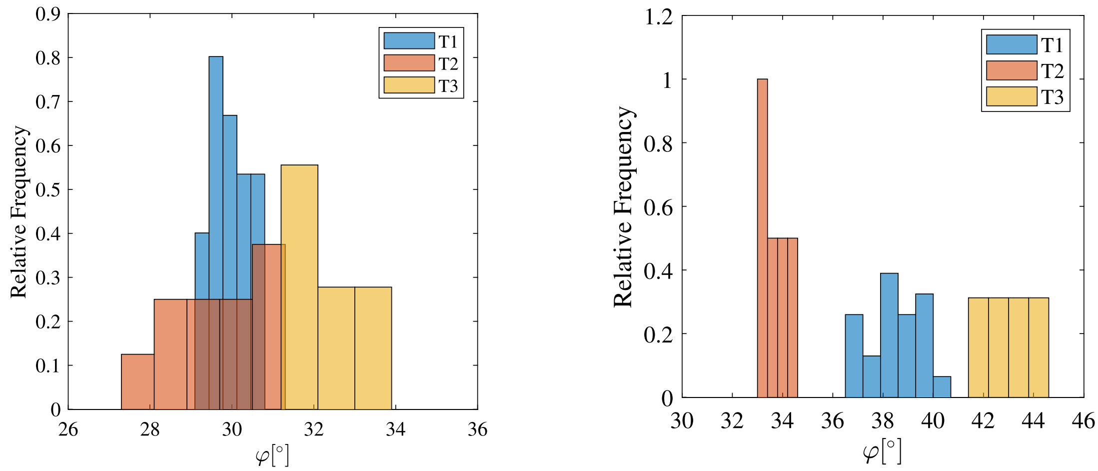
Because of the few data used to perform the probabilistic reliability analysis, the proposed possibilistic reliability assessment of Section 2 is hence conducted. The indirect database labelled as “SAND/7/2794” (Ching et al., 2017), collecting experimental investigation of reconstituted soils such as Erksak, Hokksund, Monterey, Ottawa, Sacramento River, Ticino, and Tonegawa sands, is adopted for supporting the reliability analysis of the investigated geotechnical problem of Section 3.1. Remarkably, more and different databases could be also used; the choice is subjective to the practitioner who can opt for local or global datasets. As stated in the BS EN1997 (British Standards Institution et al., 2007), values should be compared to local experience, large scale field trials and published data (e.g., Phoon and Kulhawy, 1999; Phoon et al., 2022a) so to effectively consider the dispersion and variability of the data.

The dataset of 1257 samples, illustrated as histograms in Fig. 9, is characterized by a median of 39.90° and $COV = 13\%$; the black curves represent the Ruspini partitions, obtained in *Step 1* of the proposed procedure (Section 2.2.1) whilst the red dots are used to indicate the centres of the prototypes. Three different numbers of partitions or subclasses, e.g., $n_{cl} = 3, 6, 9$ are used to define three levels of knowledge and local experience defined as Low, Typical, and High Degree of Understanding, in analogy with the prescriptions of the Canadian Highway Bridge Design Code - CSA S6:19 (CSA, 2019). Each degree is consistent with a certain degree of knowledge embedding local experience and engineering judgement, as an example. The higher the degree of knowledge, the higher the number of fuzzy partitions and hence, the less uncertain and disperse are the values used for the assessment of the reliability.

The Ruspini partitions are then associated to possibilistic (empirical) membership functions, through the ACF-transformation described in *Step 2*; Fig. 10 shows the partitions for each degree of understanding represented as continuous black curves.

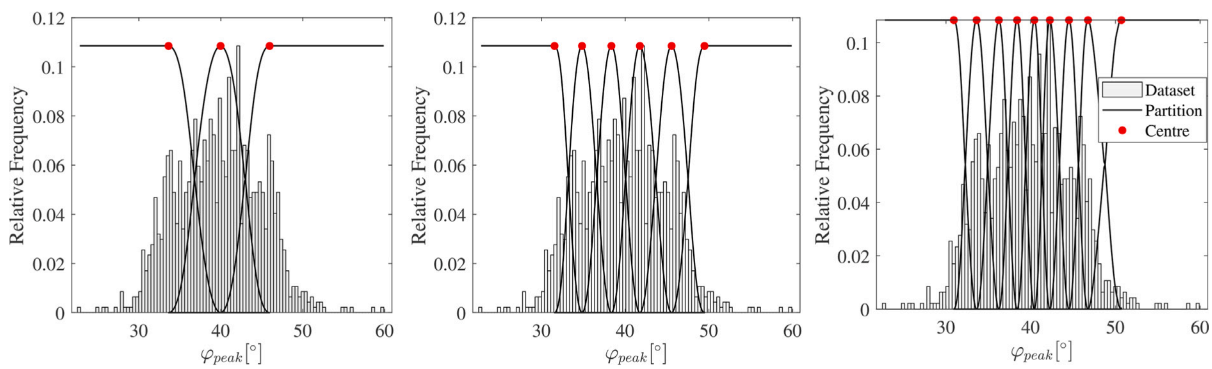
Therefore, the design membership functions for each degree of understanding are established by allocating a nominal value, e.g. the 5% fractile value of the angle of internal friction, and performing the convex combination described in *Step 3*.

The design membership functions for each level of understanding are shown in Fig. 10. It can be observed that in the cases in which the nominal value corresponds or it is nearby to the prototype centre (the core), the design membership function is exactly or very similar to the possibilistic partition as happens for instance to Technician 3 in Fig. 10a. Otherwise, the design MF derives from the merging of two adjacent partitions as described in Step 3 (Section 2.2.3).



(a) ϕ_{cv} (b) ϕ_{peak}

Fig. 8. PDFs of the angle of internal friction obtained by the 3 technicians: (a) critical and (b) peak values.



(a) Low DoU (b) Typical DoU (c) High DoU

Fig. 9. Dataset with fuzzy partitions for (a) Low, (b) Medium, (c) High Degree of Understanding.

3.3.4. Assessment of the possibilistic reliability

After the clustering and positioning, each Technician is equipped with three Design MFs for each Degree of Understanding. These will be used for the fuzzification of the performance function G and the assessment of the possibilistic reliability. The Design MF is a fuzzy number described as in Eq. (3); the left- and right- sides are essentially a 2-tuple listing an ordered pair of values, namely an angle of internal friction φ and its corresponding value of membership, $u_\varphi(\varphi)$, sorted in ascending order of φ.

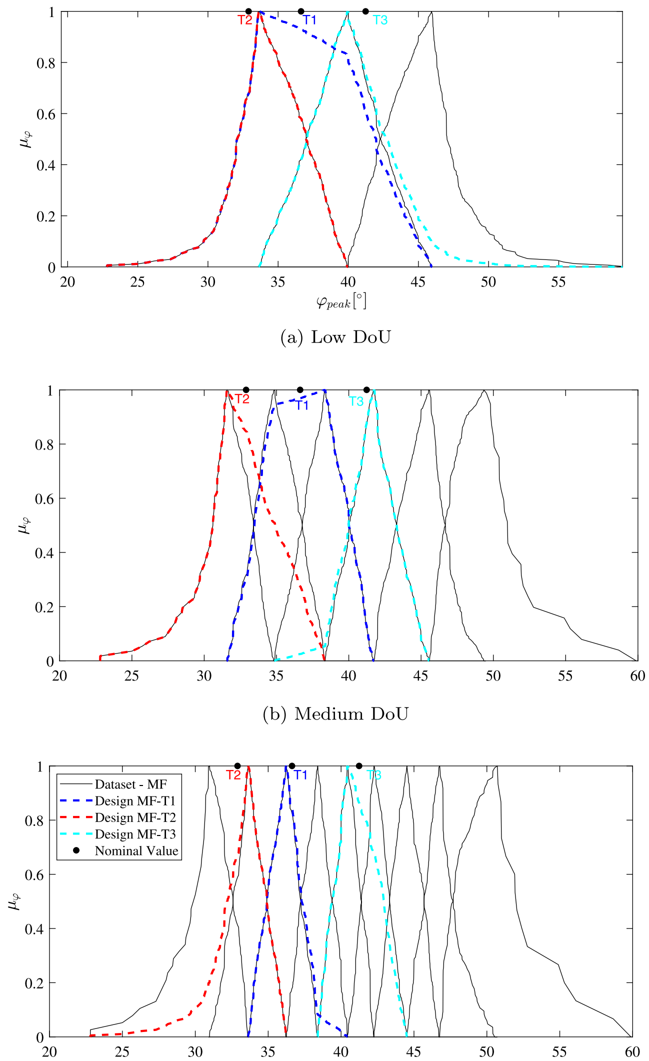
If the performance function as the one investigated for this problem is monotonic, the possibilistic reliability analysis consists in computing the deterministic value of the performance function G for each value of φ as ordered in the Design MF. Therefore, in this case, the analysis is similar to the approach used for the Monte Carlo Simulation where a stochastic problem is converted into multiple deterministic problems.

Whilst the Monte Carlo Simulation requires the evaluation of the performance function for each realization in order to estimate the statistics of the outcome, in the proposed approach, because the function is monotonically increasing, the analysis can be stopped as soon as the first positive G is achieved; this value is associated with a certain value of the membership function $u_\varphi(\varphi)$.

Therefore, the proposed approach of Eq. (18) consists of comparing the obtained $u_\varphi(\varphi)$ with a target value of reliability as proposed in Table 2; in this case, the target possibility value is consistent with the probability of failure of 7.2348 $\times$ 10^{-5} (class RC2 in the 50-year reference period) is 1.447×10^{-4}.

The outcome of the possibilistic reliability assessment is reported in Table 5. For low DoU, the safety criterion is not met by all the three technicians; for this level, the dispersion and variability of the whole database strongly affect the reliability assessment yielding a failed outcome. On the other hand, since the probabilistic reliability approach prescribed by the BS EN1997 (British Standards Institution et al., 2021) does not explicitly consider the local experience and the available data from literature or datasets, the safety is verified for all the three cases. For the medium and high degrees of understanding, the reliability assessment is verified for Technicians 1 and 3. Nevertheless, while Technician 1 had performed several testing (n = 22), Technician 3 should take the decision based on only a few (n = 10) samples; this means the outcome can be acceptable at this level only if some further aspects are considered, such as local experience or correlations with the results from different test fields, if available. On the other hand, Technician 2 requires a high degree of confidence or understanding to accept the safe outcome of the analysis.

If the test fails at the DoU, the practitioners can either, for instance, conduct more experimental tests to increase their DoU, or propose a different design of the foundation (e.g. larger size) to obtain a safe design even at lower DoU.



(a) Low DoU

(b) Medium DoU

(c) High DoU

Fig. 10. Design Membership functions for (a) Low, (b) Typical, (c) High Degree of Understanding for the 3 Technicians.

Table 5
Possibilistic reliability assessment.

Degree of understanding	T1	T2	T3
Low	Fail	Fail	Fail
Typical	Safe	Fail	Safe
High	Safe	Safe	Safe

4. Possibility reliability assessment of wind turbine pile foundations

In this section, a numerical application of the proposed procedure is applied to a bivariate data problem. The three-step methodology of Section 2 is applied.

4.1. Problem description

The investigated problem, illustrated in Fig. 11(a), regards a 65-kW onshore wind turbine founded on an Oxford Clay. Wind turbine data are re-elaborated from the application in Austin and Jerath (2017). The tower is 23 m high, and its mass is 10700 kg. In this case study, a 2×2 pile group has been designed with pile diameter, D, equal to 0.25 m and

Table 6
Possibilistic reliability assessment.

DoU	Low	Typical	High
Test result	Fail	Safe	Safe

pile length, L, of 4.5 m. An equivalent vertical static load of about 54.73 kN is applied on each pile of the pile group foundation accounting for gravity and wind loads. The performance function G as in Eq. (1) is evaluated for assessing the structural safety of the onshore wind turbine at the Ultimate Limit State.

The pile bearing capacity, R of Eq. (1), is derived from the results of a Cone Penetration Test (CPT) sounding by adopting the direct method proposed by Bustamante and Gianeselli (1982) where the cone resistance, qc, is used to derive 2 variables, the pile unit end bearing, qp as well as the pile unit side friction, fp through a discontinuous transformation.

4.2. BGS database clusterization

To assess the reliability of the designed pile group, the result of an experimental CPT sounding is compared with published data, and also local experience is accounted for, as recommended by EN 1990 (British Standards Institution et al., 2021). To apply the proposed methodology of Section 2, the dataset of CPT soundings stored in the National Geotechnical Properties Database (Self et al., 2012) managed by the British Geological Survey (BGS), is considered. Experimental results conducted on 105 boreholes in the Oxford Clay formation, from 3 different members (i.e., Stewartby, Peterborough and Weymouth), are considered.

Fig. 11(b) shows with a red-colour curve the cone resistance, qc, derived from the experimental CPT sounding used for the pile design whilst the grey curves represent all the records collected in the BGS database. To account for different sources of uncertainty, in this application, a measurement error is introduced; considering the actual recording as a mean trend, 100 samples for each record are generated by summing up a random, normally distributed, fluctuation (see e.g. Ching et al., 2018) with coefficient of variation (COV) equal to 0.15, selected as the lower bound of the COV values investigated in Phoon and Kulhawy (1999) and Salgado et al. (2019) for CPT testing. The generated values are represented by a blue circle in Fig. 11(c).

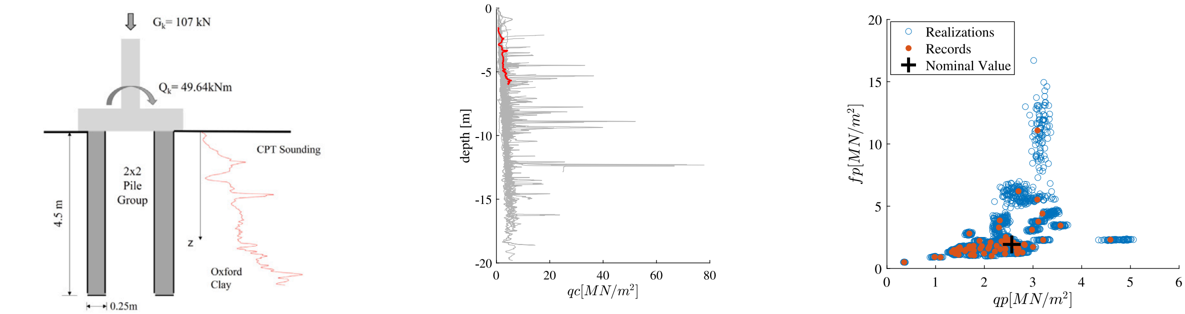
Figs. 12 and 13 show the distribution of the cone and frictional resistance and their Ruspini partitions shown as black-curves for 3 different number of subdivisions, i.e., $n_c = 3$, $n_c = 6$ and $n_c = 9$.

4.2.1. Assessment of the possibilistic reliability

Each Ruspini partition is then independently transformed into empirical membership functions through the ACF-transformation, as depicted with black curves in Figs. 14(a) and 14(b) for the specific *low* DoU case. The membership functions of the cone resistance, u_{qp}, and frictional resistance, u_{fp}, for the reliability assessment are obtained by positioning the nominal values used for the pile design (in this case, the cone and frictional resistance obtained by the CPT sounding) and then merged to derive the red-colour membership functions in Figs. 14(a)–14(b) for different DoU.

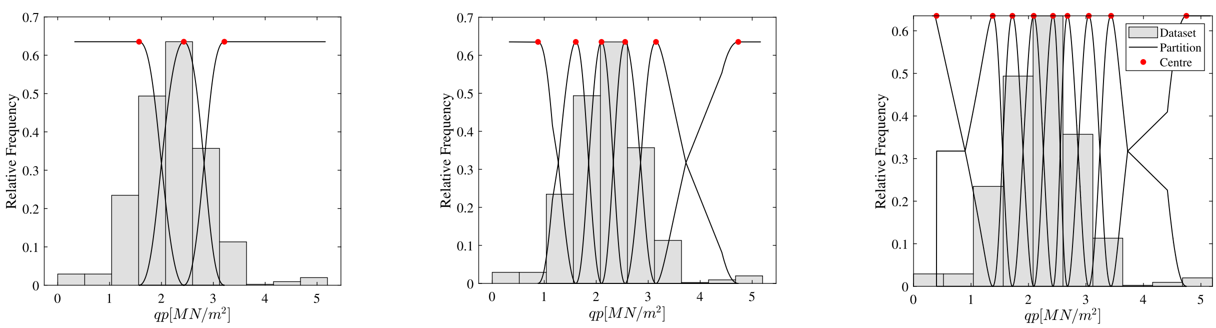
It is worth noting that the support of the design membership function is smaller when a higher DoU is considered, demonstrating that by increasing the local experience and engineering judgement, the method is able to capture the decrease of the uncertainty of the parameter. Finally, the possibilistic reliability assessment is conducted for each DoU by verifying Eq. (18).

Practically, the input to obtain the value of the membership function of the performance function, u_G, is described by a tuple of 3 elements, namely the value of the end-bearing resistance, the value of the side friction and the associated level of membership in $[0, 1]$. The number of



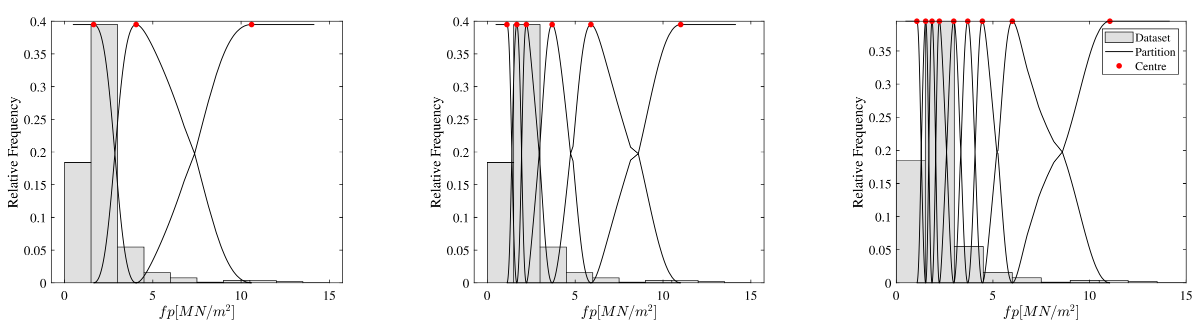
(a) Wind Turbine Foundation problem (b) CPT soundings (c) Dataset

Fig. 11. (a) CPT soundings obtained from the BGS database (Self et al., 2012) in grey colour and reference CPT test (red colour curve), (b) Distribution of the Cone and Frictional Resistances.



(a) *Low* DoU (b) *Typical* DoU (c) *High* DoU

Fig. 12. (a) Fuzzy Clustering of the cone resistance for *low* DoU ($n_c = 3$), (b) *Typical* DoU ($n_c = 6$) (c) *High* ($n_c = 9$) DoU.



(a) *Low* DoU (b) *Typical* DoU (c) *High* DoU

Fig. 13. (a) Fuzzy Clustering of the side friction for *low* DoU ($n_c = 3$), (b) *Typical* DoU ($n_c = 6$) (c) *High* ($n_c = 9$) DoU.

tuples is related to the discretization adopted to obtain the membership functions.

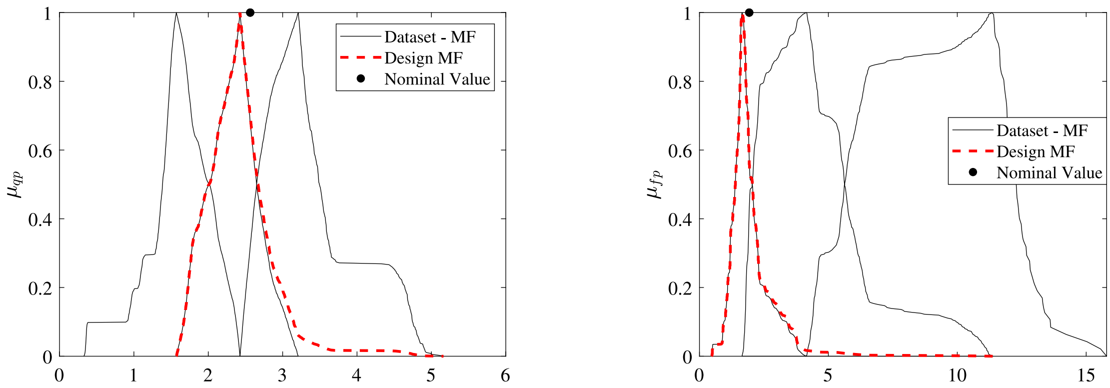
Because of the monotonicity of the performance function, G, the computation of the membership function, u_G, is simply obtained by calculating the deterministic function, G, for each tuple sorted in ascending order of the resistance parameters. For each tuple, the crisp/deterministic value of the performance function, G, is obtained and the corresponding degree of membership, α_{cut} is the value of the membership function defined in the same tuple. The sequential analysis stops when the first non-negative value of G is obtained.

If the membership function's value at $G = 0$ is greater than the Target Reliability Value ($\alpha^* = 1.447 \times 10^{-4}$), the criterion is satisfied (*Safe*), otherwise not and the structure is unsafe (*Fail*). As evidenced in Fig. 15, at low DoU, the value at $G = 0$ is equal to $\alpha = 0.078 > \alpha^* = 1.447 \times 10^{-4}$, therefore the reliability fails. Moreover, it can be observed that by increasing the DoU, the support $\left[a_G, b_G\right]$ of membership function u_G is reduced and, consequently, a lower impact of the uncertainties on the result is expected.

Table 6 summarizes the outcome of the proposed possibilistic reliability assessment; for *Low* DoU, the pile design is considered unsafe due to the large variation and weight of the database; for *Typical* and *High* DoU, if local experience and number of samples are sufficient, the design can be considered safe.

5. Discussion and further work

Despite the continuous advance of knowledge in geotechnical reliability with newly developed methods to better support decision-making and safety assessment under various sources of uncertainty (see e.g., Christian, 2004; Phoon, 2020), it is well recognized that there is quite a reluctance to apply new concepts in practice. A way to reduce the gap between "state-of-the-art" and "state-of-the-practice" (e.g., Chwała



(a) qp (b) fp

Fig. 14. Empirical and Design membership function for *low* DoU, (a) end-bearing resistance (b) side friction.

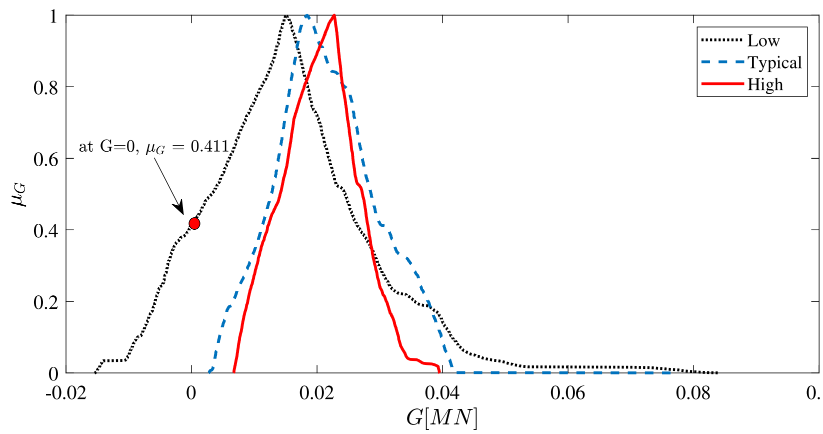


Fig. 15. Membership function u_G of the fuzzy performance function G for three DoU.

et al., 2023; Phoon, 2023) lies in the proper education and training of geotechnical engineers unfamiliar with techniques initially developed for different fields (see e.g., Baecher and Christian, 2003; Zhang, 2023).

By recognizing the importance of attracting more and more practitioners to the new developments, this paper aims to introduce a simplified and easy-to-use reliability method by clearly separating the unfamiliar theory of possibility from its use in practice. This proposed method traces the well-known Level II reliability approach in which the parameters are now sorted pairs, used to replace the list of random, usually assumed normally distributed, values. The distribution of the elements is provided by third-party experts through possibilistic analysis and transformation of Big Indirect Data from geotechnical databases. Practitioners' role is to perform a series of deterministic analyses and to verify if a certain threshold has been crossed according to Eq. (18). Therefore, the proposed method is formulated in a way that the assessment does not require expertise in data analysis, encouraging the engineering geotechnical community to embrace the approach.

It is worth emphasizing that the proposed method does not intend to replace or establish dominance over probabilistic approaches; the possibility and probability theories are well related, each offering advantages and disadvantages. On the other hand, whilst building codes (e.g., British Standards Institution et al., 2007) prescribed conventional approaches based on the classical frequentist interpretation, it is becoming more and more recognized the fundamental contribution of the information "hidden" in large and continuously growing databases. Therefore, geotechnical reliability is moving towards more data-informed decision support (Phoon, 2023) where the Bayesian interpretation plays an important role in developing better alternatives to the conventional ones; in particular, the recently proposed hierarchical Bayesian model (Ching et al., 2021) is the first data-driven method developed for dealing with big indirect data.

The method proposed in this study is a first attempt to follow the same challenges addressed by the hierarchical Bayesian model, through a different interpretation, the Possibility theory, to exploit the benefits of dealing with imprecise and vague information, offered by the Fuzzy Theory (Zadeh, 1965). Besides, the proposed method is computationally efficient compared to a Monte Carlo Simulation by requiring only a fine discretization around the zero of the limit state function to achieve accurate results. It is worth mentioning that the clusterization and the multivariate analysis of the database are not covered in this study; as proposed, the database analysis will be performed by third-party experts. Further studies are required to obtain the optimal number of clusters and the correlation coefficients for the various attributes.

Nevertheless, since the multivariate possibility distributions intrinsically accounted for the concept of possibilistic correlation (Carlsson et al., 2005) as a measure of interactivity between various attributes (Fullér and Majlender, 2004), the mathematical formulation of the proposed method presented in Section 2 accounts for both univariate and multivariate parameters. On the other hand, the outcomes of the application in Section 4.1, can be different if further attributes of the database were considered; the reliability assessment is governed by the subjective and objective information provided by the data.

Moreover, in case the performance function cannot be expressed through an analytical function, numerical approaches should be used to simulate the behaviour of the geotechnical problem. In the case of spatial dispersion of the soil parameters, the possibility distributions of the parameters, obtained from the elaboration of the database, can be used to produce an interval of values by using the concept of α_{cut}. Therefore, at each level, numerical approaches such as the Interval Field Method (Feng et al., 2023) or the Fuzzy Finite-Element Method (Muhanna and Mullen, 1999) can be used in conjunction with the proposed possibilistic data-driven approach. Nevertheless, future studies are required to assess the efficiency and efficacy of the proposed method in complex engineering problems.

6. Concluding remarks

In recent research, the quantitative description of possibility distributions and data-based (frequentist and/or epistemic) elicitation and estimation of membership functions have received great attention (Dubois and Prade, 2016; Masson and Denœux, 2006; Ferson and Oberkampf, 2009; Hose and Hanss, 2020). In particular, several new results on data-driven analyses, connected to measure theoretical concepts, allow for possibility theory to provide "... a powerful framework for quantitative statistical inference, which is easily established and involves mostly computations based on probability–possibility transform and the Extension Principle" as stated by Hose in Hose (2022).

The properties of consistency and validity of the statistical information are clearly expressible in terms of possibility theory (e.g., specificity, consonance and efficiency of the obtained distributions) and their robustness can be calibrated quantitatively to allow possibilistic (statistical) inference.

The combined use of data-driven and quantitative information (based on probability and possibility measures/distributions) is now able to significantly reduce the effects of the overall uncertainty inherent with the reliability/inference computations and analyses.

This paper presents a novel non-probabilistic approach for the reliability assessment of geotechnical problems based on the Possibility Approach. The probability–possibility consistency is exploited to establish a target Reliability Index Value to provide a level of safety meeting the prescribed values of probability of failure provided by the Eurocode 1990 (British Standards Institution et al., 2021) when parameters are of ideal random nature. Moreover, when data are scarce, sparse or incomplete, the possibility theory allows the embedding of subjective information and aleatory uncertainties. Therefore, indirect datasets are used to infer fuzzy sets of parameters and local experience and engineering judgement are rigorously considered through a three-tier system based on the definition of the 'Degree of Understanding'. The worked applications of the reliability of shallow and pile group foundations show that:

- the semi-probabilistic approach does not provide a uniform level of safety and accordingly, due to the dispersion of the data, either an under- or over-conservative design can be achieved;
- because of the non-linearities involves in the evaluation of the bearing capacity of the shallow foundation, the distribution of the resistance is non-gaussian. Therefore, the Level II 'β' approach could lead to erroneous assessment of the reliability;
- the proposed approach provides a safe/unsafe assessment consistent with the prescribed probability of failure when the input data are of ideal random nature;
- the clusterization of an indirect database provides information used for the inference of the design membership function in order to assess the possibility reliability;
- the level of safety is associated with a defined degree of understanding, e.g., low - typical - high, that allows to rigorously embed the local experience into the reliability assessment of geotechnical problems.

CRediT authorship contribution statement

Alessandro Tombari: Conceptualization, Methodology, Software, Validation, Investigation, Resources, Writing – original draft, Writing – review & editing, Visualization, Project administration, Funding acquisition. **Marcus Dobbs:** Resources, Data curation. **Liam M.J. Holland:** Resources, Data curation. **Luciano Stefanini:** Conceptualization, Methodology, Software, Investigation, Writing – original draft, Writing – review & editing, Visualization.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Alessandro Tombari reports financial support was provided by Engineering and Physical Sciences Research Council.

Data availability

Data will be made available on request.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.compgeo.2023.105967.

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