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Providing a Levelling Datum to a Tide Gauge Sea Level Record

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Abstract

A method is described for providing a levelling datum to a sea level record containing hourly heights (or similar) with the use of a second record from a nearby location consisting of high waters only but measured to a known datum. The method is tested using data from a pair of stations in the Thames estuary where there is a predominantly semidiurnal tide. It is then applied to the determination of a datum for an important historical sea level record at Liverpool. The historical background to that important record is explained. The limitations of the method are investigated using data from a pair of stations approximately 50 km apart on the north coast of Wales. This latter case study provides insight into which aspects of the tide contribute to inaccuracies in the method when the stations are some distance apart.

Keywords: datum determination; high water level variations; ocean tidal constituents; historical sea level change at Liverpool

Word count: 7300

1. Introduction

This paper discusses a method of providing a levelling datum for a sea level record containing hourly or similar sea levels. Many such records exist which were obtained either without a well-defined datum, or with a datum that cannot now be related to presently-known benchmarks or land survey levels. The method requires the existence of a separate record of high waters for which the datum is known, this second record having been obtained at a nearby location at the same time as the first one. The method is shown to work well for nearby stations with predominantly semidiurnal tides.

The following Section 2 explains the general idea behind the method. The case study in Section 3 then demonstrates that the method works with centimetric accuracy for a pair of stations only 8 km apart on opposite banks of the estuary of the River Thames in the southeast of England. The tide there is predominantly semidiurnal. This leads to Section 4 in which the method is used to determine a datum for an important historical sea level record from Liverpool. That record has not so far been included in regional studies of sea level rise because of its lack of a datum. Some of the historical background to that important record is explained. The datum is provided by using a simultaneous record of high waters from a station approximately 5 km away on the opposite bank of the River Mersey. These stations are also so close together that there are only small differences in their semidiurnal tides. As a result, a reliable new data point is added at the start of the existing long-term sea level time series for Liverpool spanning the 19^{th}-21^{st} centuries.

As a more general test of the method, Section 5 considers its application to a pair of stations 50 km apart on the North Wales coast. At first sight, the method appears to work as well as in the examples from the Thames and Mersey. However, a set of simulation studies, taking into account the roles of individual tidal constituents, shows that once the distance between a pair of stations has become so large that there are significant differences in the tide between them, then the accuracy becomes closer to decimetric than centimetric. Nevertheless, such an accuracy may still be useful for some geodetic applications. Discussion and conclusions are given in Section 6.

2. The General Idea

Imagine a predominantly semidiurnal tide such as at Liverpool (Figure 1a,b) where the M2 constituent has an amplitude of approximately 3 metres (Amin 1985) and the tidal form factor has a value of 0.0567. [1] High water (HW) levels above an arbitrary datum can be parameterised as:

$$HW = HMTR + VHW + SL$$

where HTMR is the half mean tidal range. This is the average level of the high tide above whatever is the non-tidal sea level (SL) at that time, and it will have a value that is a little larger than the amplitude of the main M2 constituent (Woodworth et al. 1991). VHW represents tidal variability in HW around that average high tide level and will have a zero mean when averaged over a long period. This component is largely due to the spring-neap cycle with a period of 14.77 days. This cycle is primarily due to the beating of the M2 and S2 constituents, with an amplitude given by that of S2. However, many other semidiurnal and diurnal constituents will also beat together to contribute to the 14.77 day cycle in HW. [2] VHW will also vary on other timescales, in particular monthly and half-yearly, due to the roles of the N2 and K2 constituents respectively. In addition, there will be ‘diurnal inequality’ due to the diurnal constituents. The annual and semiannual components of sea level variability (the tidal constituents SA and SSA) will also contribute to HW fluctuations. For present purposes, these are considered to be part of the astronomical tide. The non-tidal variability in sea level (SL) will also contribute to variations in HW due to processes such as storm surges. That variability averages over an extended period to provide the mean sea level (MSL) of the record relative to the arbitrary datum.

Therefore, consider two nearby stations as shown in Figure 1b for which measurements are available at the same time. We denote the Salthouse Dock location as station ‘H’, as that is where HW information only is available. These HW values are measured with respect to a known datum. Meanwhile, the New Brighton location is denoted as station ‘F’, as that is where there is a full sea level record (e.g. 15 or 30-minute or hourly values) but measured to an unknown datum. One can compute another set of HW values from this full record, but in this case relative to station F’s unknown datum. The object of the exercise is to compare the two sets of HW values and thereby to transfer the datum of H to the record of F so that a scientifically useful determination of MSL relative to a land levelling datum can be obtained.

One assumes that the amplitudes of all the main constituents (M2, S2 etc.) at the two locations (H/F) in this predominantly semidiurnal tidal regime will be in same ratio (α) across the semidiurnal band. This will obviously be the case for nearby stations such as in Figure 1b. Therefore, a linear regression can be performed of HW^H values in terms of HW^F values, a procedure in which one is inspecting primarily the S2 variation in HW. Once α is known, then one can also estimate $HMTR^H$ in terms of

[1] The tidal form factor is defined by the sum of the amplitudes of the O1 and K1 constituents divided by the sum of the amplitudes of M2 and S2. A location with a form factor lower than 0.25 is normally considered to have a semidiurnal tidal regime (Pugh and Woodworth 2014).

[2] Using the set of constituents discussed in Section 4, one can identify no less than 8 diurnal and 10 semidiurnal pairs of constituents which contribute to the 14.77 day cycle in HW.

$HMTR^F$, by assuming that the M2 constituents at the two locations are in the same proportion.[3] One then has:

$$HW^H = HMTR^H + VHW^H + SL^H$$

$$HW^F = HMTR^F + VHW^F + SL^F$$

which results in:

$$[MHW^H – MHW^F] = [MSL^H – MSL^F] + (\alpha\text{-}1)\ HMTR^F$$

or

$$[MSL^H – MSL^F] = [MHW^H – MHW^F] - (\alpha\text{-}1)\ HMTR^F \qquad [1]$$

where MHW denotes Mean High Water over an extended measurement period. Each VHW will average to zero over that period, and $[SL^H – SL^F]$ will average to give $[MSL^H – MSL^F]$. $MHTR^F$ can be calculated from the full record for station F, being a quantity that does not require knowledge of any datum.

The viability of the method depends on how well $[MSL^H – MSL^F]$, calculated from the information on the right-hand side of Equation [1], estimates the real $[MSL^H – MSL^F]$ difference between the stations. If there is no sea level gradient (relative to the geoid) between them (i.e. the Mean Dynamic Topography, MDT, at the two locations is the same), then $[MSL^H – MSL^F]$ will be determined only by any differences in height datum at the two locations. Specifically, $[MSL^H – MSL^F]$ will be the datum of the data of station H below that of station F.

3. Case Study at Thames Stations

Figure 1a,c shows the locations of the tide gauges at Southend-on-Sea and Sheerness, on the north and south banks of the Thames estuary respectively. The record available from Southend spans 1928-1984 and consists of hourly values of sea level obtained from a gauge at the end of a 2 km long pier. The record from Sheerness, in the period of its overlap with Southend, was obtained from a gauge approximately 8 km away and also contains hourly values. Both sea level records are expressed relative to the national levelling system (Ordnance Datum Newlyn, ODN).

The tide in the Thames estuary is predominantly semidiurnal, with an amplitude of M2 over 2 metres at these two locations and a form factor of 0.094. Many details of the tide at Southend can be found in Amin (1983). He showed that the tidal range is slightly larger at Southend than Sheerness and peaks about 5 minutes earlier. A further comparison of the tide at the two stations is provided in Figure S1 and Table S1 in the Supplementary Information of the present paper.

For the purpose of this case study, we are going to assume that we have a continuous record of hourly values at Southend (F) but with an unknown datum, while we use only the computed high waters at Sheerness (H) measured relative to ODN. We can then determine how well $[MSL^H – MSL^F]$ computed using Equation [1] compares to the real $[MSL^H – MSL^F]$ difference.

[3] HMTR will be slightly larger than the amplitude of M2. The 'excess' amount includes contributions from every other tidal constituent, as demonstrated in Doodson and Warburg (1941) and Appendix A of Woodworth et al. (1991). To the extent that about half of this excess arises from S2, it can be shown that the total HMTR at the two locations will be in the proportion α, just as the S2 and M2 amplitudes are assumed to be.

We used data from two individual years (1971 and 1973) that had very few gaps at both locations. However, to ensure completeness for each year and to simplify the case study by having records that were purely tidal and without measurement anomalies, a tidal analysis was made containing 62 constituents (5 long-period, 18 diurnal, 20 semidiurnal and 19 higher-frequency constituents) plus an MSL value, from which 1-minute predictions were generated, preserving their MSL values from the original records. HW values were then computed for each station-year, and these HW values analysed as described in Section 2. It will be seen later in Section 5 that the use of predictions in this way, instead of the original data itself, provides a way of understanding which aspects of the tide have an impact on the performance of the method.

Figure 2 shows that HW values at the two locations are highly correlated (correlation coefficient 0.998), as expected for two stations so close together and with a similar tide. A linear regression fit using Southend high water as the independent variable provides a slope (α) of 0.9911 using 1971 data, while [MHW^H – MHW^F] has a value of 2.6 cm.[4] Equation [1] then provides an estimate of [MSL^H – MSL^F] of 4.5 cm which may be compared to the real MSL-difference [MSL^H – MSL^F] of 5.2 cm computed from the full set of hourly values. Table 1 summarises findings and shows that using data from 1973 provides similar results. One concludes that the method works in this case with satisfactory centimetric accuracy. For the purpose of the present study, this is as far as the method is required to go.

However, one may make two further enquiries. One is to ask why [MSL^H – MSL^F] is not exactly zero, given that the sea levels at both stations are measured with respect to ODN. Modelling of different meteorological effects on sea levels at the two stations and a small MDT-difference between them might provide an oceanographic answer. Small errors in the national (ODN) and local levelling at the two locations could provide an additional geodetic one. A second question is to ask why the individual MSL values at both stations are higher than ODN, which is a datum based on an estimate of MSL at Newlyn at around 1920 (Bradshaw et al. 2016). The answer comes from sea level rise around the UK during the past century, MSL having risen by more than a decimetre since then. Figure 3 (thin black curve) shows that MSL at Sheerness was at about ODN in 1920 but now lies well above that level.

4. A Datum for the Liverpool Sea Level Record of Denham

4.1 The 1834 Denham record

Lieutenant Henry Mangles Denham spent seven years from 1833 undertaking the most comprehensive survey up to that time of the River Mersey and its approaches to Liverpool. Accounts of his work can be found in Mountfield (1953) and Ritchie-Noakes (1984). He was elected Fellow of the Royal Society in 1839 and later undertook extensive hydrographical surveys in the South Pacific. He retired with a knighthood and with the rank of Admiral (Royal Society 1888).

During March-October 1834, Denham made tidal recordings at Liverpool with what seems to have been a float and stilling well gauge. It is even possible that it was a self-registering gauge, the first such gauges having been installed in the Thames only a couple of years earlier (Palmer 1831). He recorded values of sea level every half hour, on the hour and half hour, together with measurements of the times and heights of high and low waters and of half-tide level. In addition, he noted the time of the turn of the tidal stream and a set of meteorological parameters including wind speed and direction, air pressure and temperature, and gave a general statement of weather conditions. The data were entered into a fine leather-bound book and presented to John Lubbock, the Vice-President of the

[4] Sea levels will be expressed to one millimetre and slopes to four decimals. Equation [1] involves the term (α-1) $HMTR^F$ and $HMTR^F$ is ~200 cm at the Thames ports or ~300 cm at Liverpool, so four decimals are needed to be within one millimetre.

Royal Society (Royal Society Archive MA/156, https://catalogues.royalsociety.org/CalmView/). Denham and Lubbock do not appear to have made use of the measurements, although a subset was used by Whewell (1840).

It is strange that Denham did not record in the book the exact location of the measurements, their datum or the time zone used. However, from Denham (1840) one infers that the location was the Rock Lighthouse, New Brighton (Figure 1b). Mountfield (1953) provides some confirmation of this by noting that in October 1833 the Dock Trustees had upheld a complaint from Denham that the 'lighthouse keepers at the Rock Lighthouse had wantonly neglected to keep accurate tide gauge records'.[5] This also suggests that data also existed from 1833 which has not survived. In fact, as will be shown below, the exact location of Denham's measurements is not important for the method, as long as they were obtained somewhere in the Mersey.

Denham had his own theory, implied in the book and as he described at the Dublin meeting of the British Association for the Advancement of Science (Denham 1835), that the tidal curve went through the same mid-tide point each cycle. This opinion was also held by other researchers in the 1830s (e.g. William Walker at Plymouth, see Deacon 1971). However, the theory did not stand the test of time as a more complete analysis of his data would have shown (e.g. see the criticisms of Shoolbred 1875).

The half-hourly sea level data were entered into computer files, quality controlled and analysed by Woodworth (1999), from which more details of the Denham data set and its analysis may be obtained. In brief, the data appear to be of good quality, except for a period spanning 10-23 June when there was a 'tape slipping'. This 'tape' is almost certainly the wire to which a float and counterweight were connected in a stilling well tide gauge.

A tidal analysis by Woodworth (1999), and a comparison to analyses of later tidal data in the area, showed that the time zone used by Denham must have been local mean time. The MSL of the 6-month record was computed to be approximately 16.25 feet (495.3 cm) above datum. This allows a calculation of Mean Tide Level (MTL) as about 16.5 feet above datum, given that MTL is known to be several cm above MSL at Liverpool (Woodworth 2017). From a map opposite page 135 of Denham (1840), one reads that his working datum was 'Low Water Level of Equinoctial Spring Tides' (LWES), and he states that Half Tide Level was 16 feet 6 inches above that datum. Therefore, the MTL computed by Woodworth (1999) is consistent with being Denham's reported Half Tide Level. This sensible choice of datum lies between what is nowadays defined to be Mean Low Water Springs (MLWS), approximately 13.8 feet (4.2 m) below MSL, and Lowest Astronomical Tide (LAT), approximately 17.1 feet (5.2 m) below MSL.

Consequently, it seems that Woodworth (1999) interpreted Denham's data set in essentially the same way as he did himself. However, Denham did not document precisely what his working datum (LWES) meant in terms of nearby benchmarks on land or of the datums used at the nearby port of Liverpool. Therefore, the MSL of his data set appeared back in 1999 to have little value in any study of long-term sea level change at Liverpool. Section 4.3 will show that situation is now more satisfactory.

4.2 Salthouse Dock HW measurements

[5] One puzzle is that, if the tidal measurements had been made at the lighthouse itself, which dries out at low tide, then it would have been impossible to record the lower part of the tidal curve. However, Denham's data show no evidence for such bottoming out. One possibility is that the measurements were in fact made by the lighthouse keepers at a wooden pier approximately 600 m from the lighthouse. Both locations would have been known locally as 'the Rock Lighthouse' as far as anyone was then concerned.

Woodworth (1999) also discussed the contents of Royal Society Archive MA/331-333. This archive contains tables of measured heights and predicted times (i.e. predicted from the tide tables then in use) of high waters from July 1827 to August 1835, together with meteorological data (wind direction, air pressure and temperature and general remarks), at Prince's Dock, Salthouse Dock (Figure 1b) and Queen's Half Tide Dock (QHTD) in the north, centre and south respectively of the then Liverpool dock estate. These data sets were sent to Lubbock by Jesse Hartley, the Liverpool dock engineer, in 1835. All of the Salthouse high waters were computerised by Woodworth (1999) together with a month at the beginning and end of the records for the other two docks.

The datums of the high water measurements were not explicitly noted by Hartley. However, from analysis of the data from all three sites, it is clear that measurements must have been made by the individual dockmasters with respect to the sills of their own docks. The median of differences of the reported high waters for July 1827 and August 1835 were found to be:

QHTD – Salthouse: 23 and 21 inches for 1827 and 1835 respectively
Prince's – Salthouse: 71 and 70 inches respectively

These findings are consistent with the information in a handwritten page enclosed within archive MA/331-333, also sent to Lubbock in 1835 by Hartley. The page includes a table of the height of the sill of each Liverpool dock relative to the local surveying datum (Old Dock Sill, ODS). Those for Prince's Dock, Salthouse Dock and QHTD are given as 6 ft, 1 inch and 1 ft 9.5 inches below ODS respectively, from which one concludes that Salthouse's sill was 71 and 20.5 inches above those of Prince's and QHTD respectively. (These reported sill levels are also consistent with later tables given in Ritchie-Noakes (1984). See Woodworth (1999) concerning whether QHTD was actually Brunswick Dock; this does not affect the present discussion). These differences in sill levels are consistent with those in the high waters.

It is not clear who provided the information on this page. It also includes a statement of the levels of mean spring and neap high waters above ODS for 1816 and 1819-21 i.e. two values, each averaged over four years. Values for spring and neap low waters are given averaged over only 1819-21. These years are at approximately the same time as an earlier survey of the Mersey around 1820 by the canal engineer Francis Giles, and they precede Hartley's appointment. Therefore, it is possible that Hartley's information came from Giles.

Consequently, it is believed that the Salthouse HW values were measured with respect to the Salthouse sill level rather than to ODS. Fortunately, this issue is not an important one, given that Salthouse's sill was only 1 inch lower than that of the Old Dock. Measurements at Salthouse are known to have continued beyond this period. For example, Lubbock (1837) refers to data from May 1836. However, the records have not survived.

4.3 Application of the method to the Denham datum

Figure 4 shows that the high waters from the Denham data set (omitting the short period with a slipping tape) and from Salthouse are highly correlated (correlation coefficient 0.991), with the exception of a small number of outliers which appear to be due primarily to incorrect recording of Salthouse high waters in the Royal Society records or their transcription into computer files (although the latter was checked twice). A linear regression fit between them, using the Denham data as the independent variable, provides a slope (α) of 1.0119 +/- 0.0074.

A slope slightly larger than 1.0 is expected from previous knowledge of the Mersey tide. The compilations of harmonic constituents at various Mersey locations given in Woodworth (1999) and

Lane (2004) suggest that the M2 amplitude at Salthouse in the Mersey Narrows is approximately 1% larger than at the mouth of the river at New Brighton. However, individual tide gauge data sets obtained at different times and at different locations provide too noisy a data set for estimating small tidal differences reliably. In addition, while the modern port has tide gauges at various locations in the Mersey providing simultaneous data, they use different technologies which could well render any small observed tidal differences unreliable. A numerical tidal model could provide a better idea of tidal gradient along the river. However, while such models have been constructed for various applications (e.g. Luo et al. 2013), none of them have proved capable of providing accurate information on the tidal-differences required here.

In order to use Equation [1], one needs to know $HMTR^F$, which can be approximated to adequate accuracy for present purposes by the amplitude of M2 determined from the Denham data (294 cm). One obtains a value of -353.7 cm for $[MHW^H – MHW^F]$ directly from the two sets of high waters themselves. Equation 1 then shows that the datum of Salthouse data must have been 357.2 cm above the datum of the Denham data.

It can be appreciated that the important term in the $[MSL^H – MSL^F]$ calculation is the $[MHW^H – MHW^F]$ difference. The tide-related term $(\alpha\text{-}1)$ $HMTR^F$ contributes only about 3.5 cm, which is important to know, but is small because of the two sites being so close. (Hence the case study in the next section.)

The remainder of the analysis is just arithmetic. As mentioned above, we know that the Salthouse datum (its sill level) was one inch below ODS. Therefore, the datum of the Denham data must have been 359.7 cm below ODS. This results in an MSL calculation for the Denham data of 495.3 – 359.7 = 135.6 cm above ODS. And, as we know from levelling that ODS is 4.54 ft (138.4 cm) below the national levelling datum (ODN), so MSL during April-October 1834 at Liverpool must have been 2.8 cm below ODN.

In order to use an estimate of MSL such as this in a scientific study, it is usual to apply various adjustments (Woodworth 2018; Hogarth et al. 2021) for (1) the seasonal cycle in MSL, and (2) air pressure variability. Furthermore, one has to consider uncertainties due to (3) interannual MSL variability, (4) accuracy of local levelling from ODS to the sill of Salthouse Dock, and (5) the linear regression fitting of Figure 4.

In the case of (1), we can make use of 15, 21 and 13 complete years of later data from Liverpool obtained at Alfred Dock, George's/Prince's Pier and Gladstone Dock respectively, to see whether measuring in only April-October biases the estimate compared to measuring over a full year. These later data are obtained from the Permanent Service for Mean Sea Level (https://www.psmsl.org). Of course, the procedure assumes that the seasonal cycle in 1834 was similar to that in later years. It suggests that that the short-term MSL estimate is under-estimated by 15, 19 and 19 mm respectively.

Similarly for (2), one can make use of 7 years (1827-1834) of air pressure measurements from Prince's Dock (Woodworth 2006) to see whether air pressure was unusual during April-October 1834 compared to that over the full 7 years. In fact, air pressure during the 7 months in 1834 (1011.84 mbar) was little different to the 1010.88 mbar of the longer-term average. If one assumes an IB-response of sea level to air pressure at Liverpool (Thompson 1980), then that implies the short-term MSL estimate is under-estimated by approximately 1.0 cm.

For (3), one can take the 4.6 cm in Table 2 of Woodworth (2018). This is calculated from the standard deviation of IB-corrected annual MSL values at Gladstone Dock during 1955–2014, with a linear trend over this period subtracted from the annual values. This is probably a conservative estimate; the table shows a lower value using data from Birkenhead.

Then for (4), one might assume one inch (rounded to 3 cm), given that Liverpool dock sill depths were expressed in units of inches in the handwritten note mentioned above. This is probably a conservative estimate as any local levelling between the Old Dock Sill and the sill of Salthouse Dock (a distance of only ~100 metres) should have been more accurate than this.

Finally, for (5) one might include the contribution of the formal error in α to the estimated [MSL^H – MSL^F] in Equation [1] i.e. 0.0074 $HMTR^F$ = 2.1 cm.

As a result of the under-estimates of 2.9 cm in all suggested by (1) and (2), one calculates an adjusted MSL for Denham's data set of +0.1 cm relative to ODN. If we consider conservatively that the under-estimates of (1) and (2) are 100% uncertain, then contributions (1-5) to the uncertainty in MSL can be estimated as 1.9, 1.0, 4.6, 3.0 and 2.1 cm respectively, which results in an overall standard error in MSL of 6.3 cm when contributions are combined in quadrature.

Figure 3 shows the final estimate of MSL at Liverpool in 1834 derived from Denham's data i.e. 0.1 ± 6.3 cm ODN. This is superimposed on a plot of Liverpool MSL obtained as part of a recent analysis of UK data by Hogarth et al. (2021). The values given for 1816 and 1819-21 were calculated by Hogarth et al. (2021) from a table of levels published by Webster (1848). However, Webster's table simply reproduces the information in the handwritten note from Hartley a decade earlier. There is no explanation of how the high and low water spring and neap levels in that note were calculated, and there is no surviving data to support them. The point for 1844 is derived from only 10 days of measurements at Liverpool in order to define the Ordnance Datum Liverpool (ODL) national datum which preceded ODN (Jolly and Wolff 1922). Therefore, in spite of the Denham 1834 MSL and its uncertainty being similar to the early 19th century values and uncertainties estimated by Hogarth et al. (2021), it is clear that the Denham MSL estimate, supported as it is by surviving half-hourly data and for which a datum has been provided by a reliable method, is a more respectable one.

5. North Wales Case Study

Equation [1] appears to work well for nearby stations, so it is of interest to ask how well it might work for stations further apart and, if it does not, to investigate the reasons why not. Consequently, this section considers a pair of stations 50 km apart on the north coast of Wales, to the west of Liverpool (Figure 1a,d). The tide at Holyhead and Llandudno is also predominantly semidiurnal (form factors 0.0878 and 0.0658 respectively), much the same as for the Thames ports and Liverpool, the amplitude of the main M2 tide increasing from west to east. Figure 5, taken from Robinson (1979), indicates the standing wave character of the semidiurnal tide in the central Irish Sea. The amplitude of M2 at Holyhead is about 0.67 of that at Llandudno (approximately 180 and 269 cm respectively), while the other main semidiurnal constituents are similarly in proportion (0.70, 0.68 and 0.68 for N2, S2 and K2 respectively) (Figure S2 and Table S2 in the Supplementary Information).

Holyhead and Llandudno have tide gauges that are part of the UK National Network, providing continuous 15-minute averages of sea level. Each record is measured with respect to its local Admiralty Chart Datum (ACD). However, we are going to assume that we have a continuous record at Holyhead with an unknown datum (as for Southend in Section 3), while we use only the computed high waters at Llandudno measured relative to ACD Llandudno. In turn, ACD Llandudno is known in terms of ODN (similar to the use of ODN at Sheerness in Section 3). We use data from two individual years (2013 and 2014) that have very few gaps, and follow Section 3 in using tidal predictions to compute high waters at each station. This provides a way of understanding which aspects of the tide have an impact on the performance of the method.

Figure 6 compares HW values from Llandudno and Holyhead for the 2013 data from which finds a correlation coefficient of 0.993 and a slope (α) of 1.5065. This value can be input to Equation [1], together with the value of $HMTR^F$ obtained from the complete Holyhead record (194.1 cm), from which one obtains using Equation [1] that the datum of the Holyhead sea level record must be 80.6 cm above that of the Llandudno HW data (ACD Llandudno). This can be compared to [MSL^H – MSL^F] of 79.8 cm calculated from the entire set of 15-minute values for 2013 at the two stations. Table 2 summaries these findings. In fact, it is known from national levelling that ACD Llandudno is 3.85 m below ODN, while ACD Holyhead (the 'unknown' datum of our Holyhead data) is 3.05 m below ODN i.e. an 80 cm difference in the ACD datums at the two locations.

Therefore, one might conclude that the method provides an accurate (centimetric) determination of the datum of the Holyhead record relative to ACD Llandudno, some 50km away. In addition, one notes that the real [MSL^H – MSL^F] estimates for the two separate years, and those obtained via Equation [1] using the 2013 data, are close to the 80 cm known from ODN levelling. This suggests that there is little difference in MDT between the two stations, consistent with maps of MDT around the UK in Figure 6 of Hipkin et al. (2004), derived from altimeter and geoid data and from ocean models.

At this point one might end the investigation by concluding that the method works to a centimetre or so. However, a further set of tests is possible, thanks to the HW time series being computed using tidal predictions. This enables us to investigate the extent to which the various components of the tide influence findings from the method, and so whether the result obtained above was fortuitous. In other words, we investigate whether a hypothetical 'Holyhead' and 'Llandudno', with different selections of tidal constituents from the total actually observed, provides the same results for the Holyhead datum.

Each test provides a scatter plot of high waters at Llandudno versus those at Holyhead, similar to that in Figure 6; these are shown in the corresponding sub-plots of Figure S3. These plots are educational in their own right. Each plot provides an estimate of slope (α), while the different selections of constituents also result in different values of [MHW^H – MHW^F]. The [MSL^H – MSL^F] estimated from Equation [1] changes as a consequence of both. Table 3 summaries the findings of each test (a-i), showing values of α and derived Holyhead datum.

Test (a) represents an idealised situation, using only the observed amplitudes for M2 and S2 at each location, and fixing the phase lags at Llandudno to be the same as at Holyhead. That results in a perfect straight line with a slope of 1.4665 (Figure S3a), almost identical to the ratio of S2 amplitudes at Llandudno and Holyhead. However, Table 3 shows that the datum derived from Equation [1] is 83.6 cm and not the 79.8 cm one has from the real [MSL^H – MSL^F] in Table 2. That difference of ~4 cm arises from the ratio of M2 amplitudes at the two places being 1.4875, slightly larger than that of S2, which, when multiplied by the $HMTR^F$ of 189.7 cm using this subset of constituents, explains the difference. In other words, the very small spatial difference in semidiurnal admittance between the two locations already results in an inaccuracy using Equation [1] of several cm.

Test (b) uses M2 and S2 only, but now with the actual observed phase lags at Llandudno. In this case, almost the same slope and datum are obtained, but Figure S3b now has a stretched ellipse instead of a straight line. The ellipse is simply a reflection of how near-coincident pairs of high waters at the two locations vary through the spring-neap cycle. However, the approximately 20° larger phase lags at Llandudno have little impact on derived datum. The use of M2, S2 and K2 results in a similar ellipse but with larger ranges between maximum and minimum high waters, K2 providing a seasonal modulation of S2 (not shown). Test (c) includes M2, S2 and N2 for which there is again little difference in slope, as there is also for M2, S2, K2 and N2 (not shown). The ellipse in Figure S3c appears to have more gaps than the earlier ones due to the stretching of the ranges of high waters with an additional constituent. There is also little change of slope in test (d) using an additional 16 semidiurnal terms, but not including MU2. Figure S3d still has much of the character of an ellipse, although with more scatter. The inclusion of the small MU2 constituent in test (e) has important effects. From Table 3 one sees that the slope of test (e) differs from that of test (d) by more than that of test (d) from test (c). In addition, an interesting feature of Figure S3e is that it demonstrates a tighter distribution than Figure S3d; one might have imagined that adding constituents results in more scatter. This feature is explained by a test using M2, S2 and MU2 only, which results in a distribution more like a straight line than the ellipse of Figure S3b for M2 and S2. The reason can only come from MU2 beating with M2 to contribute to the spring-neap cycle in high waters in an opposite way to S2 beating with M2. As a result, given the actual amplitudes and phases at these stations, a narrowing occurs of some of the ±5 cm width of the ellipse in the earlier plots.

Test (f) shows that adding diurnals reduces the slope further, as might be expected given that diurnals have similar amplitudes and phase lags at the two locations, and results in the wider distribution in Figure S3f. However, including the higher-frequency constituents more than compensates for the decrease of slope due to the diurnals (test g), while resulting in a wider distribution (Figure S3g). That the higher-frequency terms play some part is as anticipated, given that M4 and MS4 are phase-locked with the main tidal constituents. Finally, the long-period tides are added; first by adding SA, SSA, Mm and Mf (test h), and second by adding MSf also, to make up the full set of 62 constituents (test i). MSf can be seen to have by far the largest effect on the slope and datum. The frequency of this long-period tide is the same as that of the spring-neap cycle, and so it is not surprising that it also modifies the assumed role of S2 within that cycle. Somewhat fortuitously, it restores the derived datum value to approximately the real [MSL^H – MSL^F]. However, it could have been appreciated well before this point that the now wider, and somewhat non-linear, distributions in the plots do not really lend themselves to parameterisation by a simple slope.

In summary, the method initially appears to work well when using all constituents. However, Table 3 shows that even for the idealised case involving only the main semidiurnal constituents, the accuracy is not better than ~4 cm. The inclusion of smaller terms such as MU2 and MSf can have a big impact. Therefore, these examples of hypothetical 'Holyhead' and 'Llandudno' stations show that the method is not reliable when using locations several 10s km apart, when there can be important differences in semidiurnal admittance and other tidal components. The same conclusion comes from the use of 2014 data. In this case, findings are almost identical to those for 2013 in Table 3 for tests (a-g), but the long-period components are larger at both stations in 2014 (MSf being 50% larger) and result in the derived datum using all components in Table 2 of 85.1 cm.

We considered whether a variation on the method might be to undertake a harmonic analysis of the two HW time series and focus only on their 14.77 day spring-neap components. Several authors have considered how harmonic analysis might be applied to the study of high and low waters (Doodson 1951; Foreman and Henry 1979; Zetler et al. 1965). However, to our knowledge, there has never been detailed consideration given to the analysis of high waters alone. Ideally, one would like to assume that the amplitude of the spring-neap 14.77 day cycle (HSN) was due to S2 only, in which case α might be estimated from HSN^H/HSN^F. Once again, the problem is that the observed 14.77 day cycle contains contributions from a large number of other diurnal and semidiurnal constituents and also from the MSf long-period tide.

A second variation might be to consider time series of HW-difference between the two (HW^H-HW^F), the two sets of high waters occurring within a few minutes of each other. As a result, (α-1) in Equation [1] can be replaced by $HSN^{DIFF}/H^{S2(F)}$, where HSN^{DIFF} is the amplitude of the 14.77 day cycle in HW-difference and $H^{S2(F)}$ is the amplitude of S2 derived from the Holyhead (F) HW record. In this case, some of the diurnal and long-period contributions to the individual HW time series will cancel, although the observed 14.77 cycle will still contain contributions from the other constituents. Unfortunately, tests with both of these variations did not result in better findings for derived datum.

6. Discussion and Conclusions

The most straightforward example of using a second sea level record to provide a levelling datum to a first record, obtained at the same time and location, would be when both are in the form of hourly or similar values. In this case, the difference between the MSL of the two records provides a simple measure of the difference in datum. One might ask why one should care about the first record if the second was a perfectly good one with a datum. However, the first might be much longer than the second and, as long as its datum is considered to have been the same throughout, then there will have been a major benefit in transferring the datum. (This assumes that the two records are measured equally well and that measurements by different technologies do not introduce scale or other differences between them.) The same situation applies when the two records are obtained only a short distance apart, when there is no significant difference in MDT between them.

Another simple situation occurs when both records are from the same location or very close, and when the second record consists of either high water values, or both high and low waters, with a known datum. In this case, a comparison of MHW (or MHW and MLW) values in the two data sets enables the first sea level record and its MSL to be expressed relative to the known datum i.e. [$MHW^H - MHW^F$] = [$MSL^H - MSL^F$]. The situation is more complicated when the two records are obtained some distance apart, as then one has to consider how differences in the tide affect a comparison of the two time series of HW, as demonstrated in Section 2. The suggested procedure, represented by Equation [1], is a straightforward, some might say trivial, one and yet, to my knowledge, it has never been applied to a sea level record up until now. [6]

In the first case study discussed in Section 3, use was made of sea level data from a pair of nearby stations in the Thames estuary where the tide is predominantly semidiurnal. As a result, it is shown that it is possible to use a record of high waters with a known datum to provide a datum to a nearby full sea level record which does not have a datum of its own, when the tide at the two locations is similar (i.e. when $\alpha \sim 1.0$). As a result, the method has been applied successfully to provide a datum to an important record consisting of six months of half-hourly sea level measurements made by Denham at Liverpool in 1834. The resulting estimate of MSL at that time, 0.1 ± 6.3 cm above ODN, will be of interest in studies of long-term regional sea level change.

In a second case study, an investigation was made of how well the method works when the two stations might be some distance apart, with differences in the magnitudes of their semidiurnal tides (i.e. when α is somewhat different from 1.0). The method is shown to apparently work well at first sight (i.e. to within ~1 cm) using data from the Holyhead and Llandudno pair of test stations for 2013. However, simulation studies have shown how subtle differences in the semidiurnal tides, and contributions from diurnal, shallow-water and long-period constituents, must impact on the effectiveness of the method. The range of biases due to differences in the tide at the two locations is of the order of a decimetre. One concludes that Holyhead and Llandudno are just a little too far apart for the method to work well.

As a consequence, it is believed that the true accuracy of the method in semidiurnal regions such as around the UK might be taken as centimetric when the stations are nearby (say 5-10 km apart), when semidiurnal admittances are almost the same, and there are little differences in the longer wavelength

[6] The only similar exercise that I know of is an unpublished study of inferring the datums of temporary tide gauge records in the Bristol Channel by using simultaneous high water levels at Avonmouth (David Pugh, private communication).

diurnal and long-period tides. In this case, small differences in shorter-wavelength shallow water tides might be expected but should not be large enough to impact on the method. For larger distances apart, such as between Holyhead and Llandudno, the accuracy will be much lower. Of course, the reasons for inaccuracy discussed for those two stations in Section 5 will be specific to them. It is suggested that anyone intending to apply the method to their own data sets should make similar studies in order to assess how well the method might work in their own tidal regime.

The knowledge of how measurements of sea level relate to the heights of local benchmarks and land survey datums is essential in many studies in marine geodesy and climate change. Therefore, it is hoped that the method described above, or variants of it, will enable additional tide gauge sea level records to be employed in such studies.

Acknowledgements

Peter Hogarth is thanked for the information on Liverpool MSL in Figure 3. Some figures were generated using the Generic Mapping Tools (Wessel and Smith 1998).

Data availability statement

All tide gauge data discussed in this paper can be obtained from the British Oceanographic Data Centre (https://www.bodc.ac.uk).

Disclosure statement

No potential conflict of interest was reported by the author.

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Table 1. Quantities involved in the calculation of the datum of Southend data using HW information from Sheerness (cm).

	1971	1973
Southend (F) MSL above ODN	8.9	5.4
Sheerness (H) MSL above ODN	14.2	8.4
Real [MSL^H – MSL^F]	5.2	3.0
[MHW^H – MHW^F]	2.6	0.0
Slope (α)	0.9911	0.9858
$HMTR^F$	217.3	219.6
[MSL^H – MSL^F] from Equation [1]	4.5	3.1

Table 2. Quantities involved in the calculation of the datum of Holyhead data using HW information from Llandudno (cm).

	2013	2014
Holyhead (F) MSL above ACD Holyhead	328.1	333.6
Llandudno (H) MSL above ACD Llandudno	407.9	413.2
Real [MSL^H – MSL^F]	79.8	79.6
[MHW^H – MHW^F]	178.9	180.3
Slope (α)	1.5065	1.4831
$HMTR^F$	194.1	197.1
[MSL^H – MSL^F] from Equation [1]	80.6	85.1

Table 3. Slope (α) of the distribution of high water levels at Llandudno versus those at Holyhead, and inferred datum of the Holyhead sea level record relative to ACD Llandudno datum, for each test using different subsets of the 2013 tidal constituents from the two stations. Plots of the distributions for tests (a-h) are shown in Figure S3 while that for test (i) is given in Figure 6.

Test	Slope (α)	Datum (cm)	Selection of Constituents
(a)	1.4665	83.6	M2 and S2 only with Llandudno phase lags set to be the same as at Holyhead
(b)	1.4628	84.3	M2 and S2 only with observed phase lags at both locations
	1.4637	84.2	M2, S2 and K2
(c)	1.4521	86.3	M2, S2 and N2, which introduces a monthly component
	1.4538	86.0	MS, S2, K2 and N2
(d)	1.4386	89.0	19 semidiurnal constituents i.e. all 20 but not MU2
(e)	1.4138	94.2	All 20 semidiurnal constituents
(f)	1.3987	97.0	All 18 diurnal and 20 semidiurnal constituents (38 total)
(g)	1.4275	95.9	All 18 diurnal, 20 semidiurnal and 19 higher-frequency constituents (57 total)
(h)	1.4439	92.7	As (g) with SA, SSA, MM, MF long-period constituents (61 total)
(i)	1.5065	80.6	All 62 constituents including MSf

Figure Captions

1. (a) Map of the British Isles with red, black and blue boxes indicating locations discussed in the text in (b) the Mersey estuary, (c) the Thames estuary, and (d) the north coast of Wales respectively. The red dots in (b-d) indicate stations for which high water values with a known datum are employed, while yellow dots indicate stations with full tide gauge records for which datums are determined by the method described in the text.

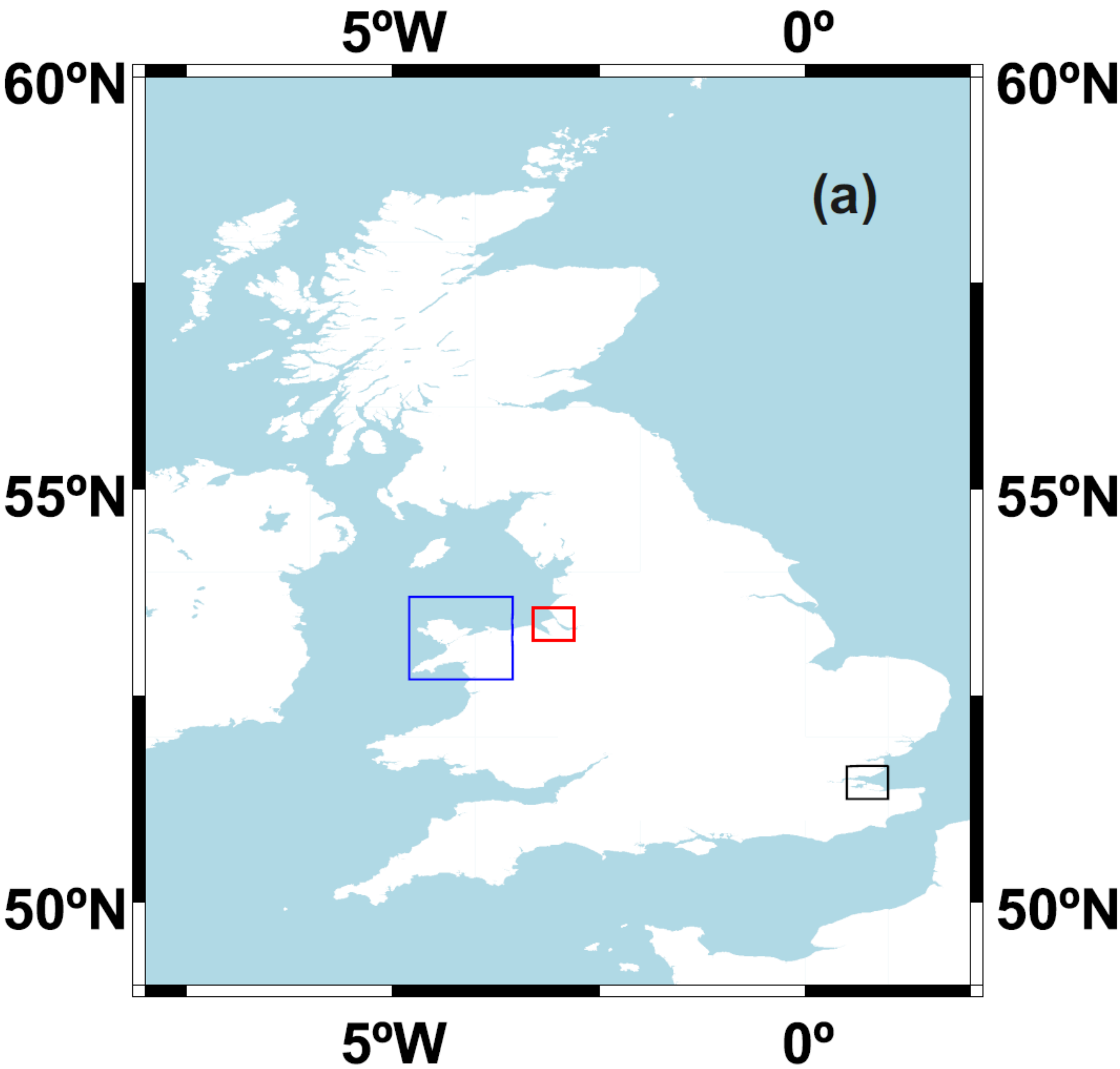
2. High water levels at Sheerness versus those at Southend during 1971. The red line shows a linear regression with the Southend values as the independent variable.

3. An estimate of MSL at Liverpool in 1834 from the Denham data in the present work (blue diamond) superimposed on a series of estimates of MSL at different times at Liverpool from a recent analysis of UK MSL by Hogarth et al. (2021). See the Supplementary Information of Hogarth et al. (2021) for details of the exact locations in the Liverpool area of each MSL value as indicated by the different colours. The lower thin black time series shows the record of monthly mean sea levels from Sheerness. All MSL values are expressed relative to Ordnance Datum Newlyn (ODN).

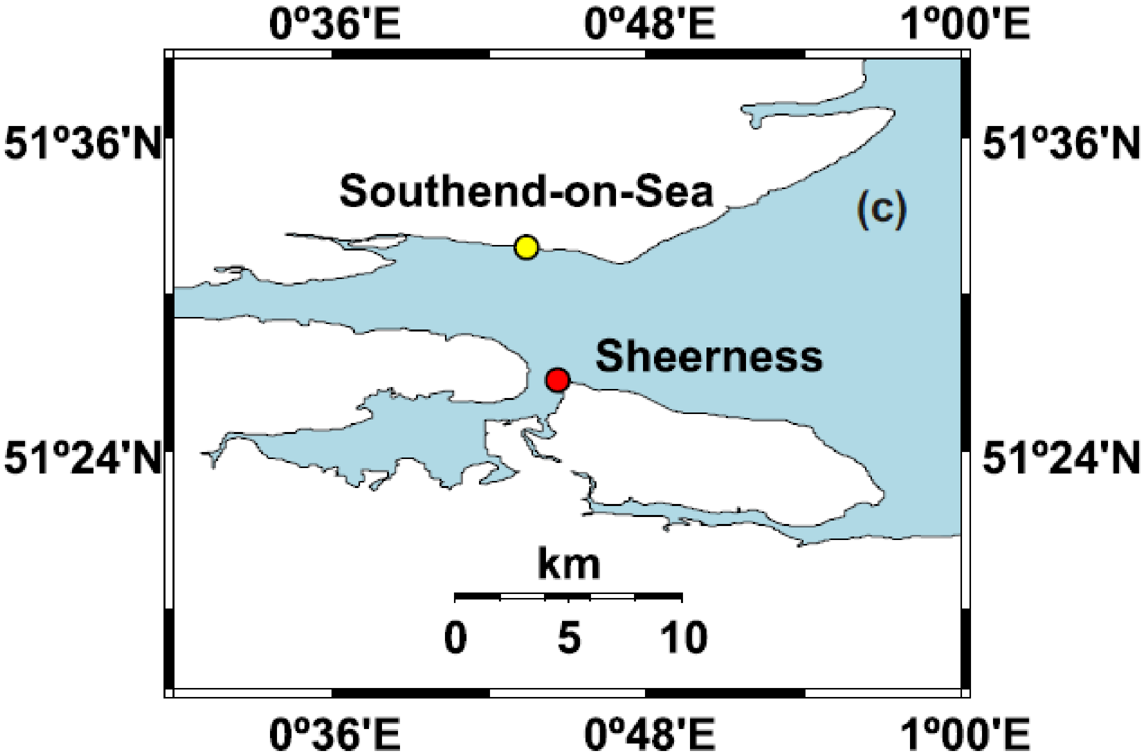
4. High water levels in the Salthouse Dock data set versus those obtained from the Denham sea level record at Liverpool during April-October 1834 omitting the short period with a slipping tape. The red line shows a linear regression with the Denham values as the independent variable.

5. The amplitude (dashed lines in metres) and Greenwich phase lag (solid lines in degrees) of the M2 constituent of the ocean tide in the central part of the Irish Sea from Robinson (1979).

6. High water levels at Llandudno versus those at Holyhead during 2013. The red line shows a linear regression with the Holyhead values as the independent variable.



5ºW
0º
60ºN
55ºN
50ºN
(a)


0°36'E
0°48'E
1°00'E
51°36'N
51°24'N
Southend-on-Sea
(c)
Sheerness
km
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5
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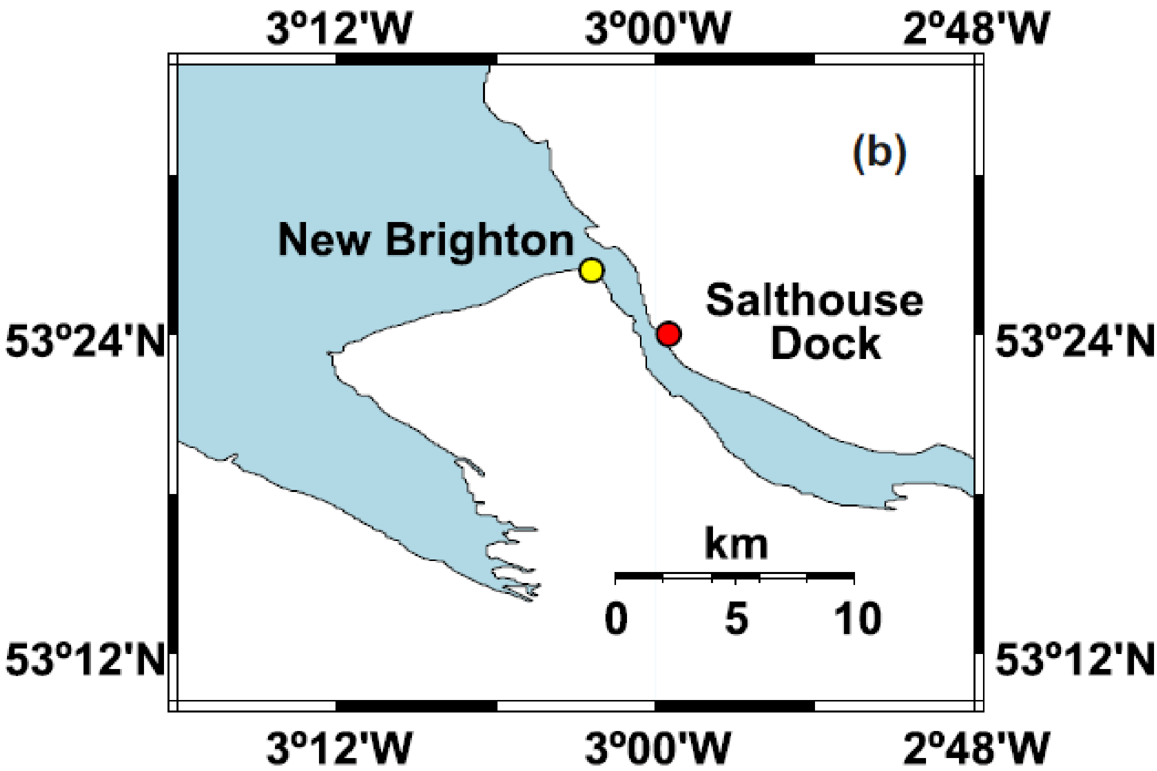
3°12'W
3°00'W
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53°24'N
53°12'N
(b)
New Brighton
Salthouse
Dock
km
0
5
10

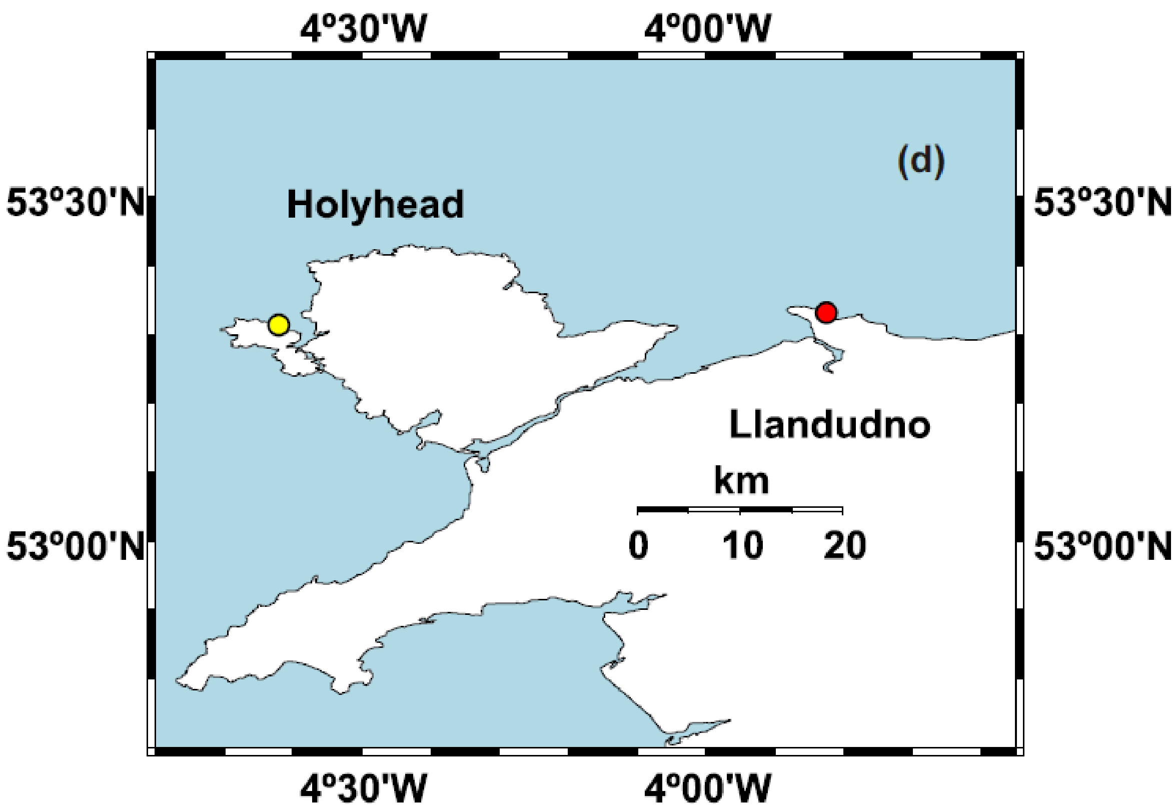

Figure 1. (a) Map of the British Isles with red, black and blue boxes indicating locations discussed in the text in (b) the Mersey estuary, (c) the Thames estuary, and (d) the north coast of Wales respectively. The red dots in (b-d) indicate stations for which high water values with a known datum are employed, while yellow dots indicate stations with full tide gauge records for which datums are determined by the method described in the text.

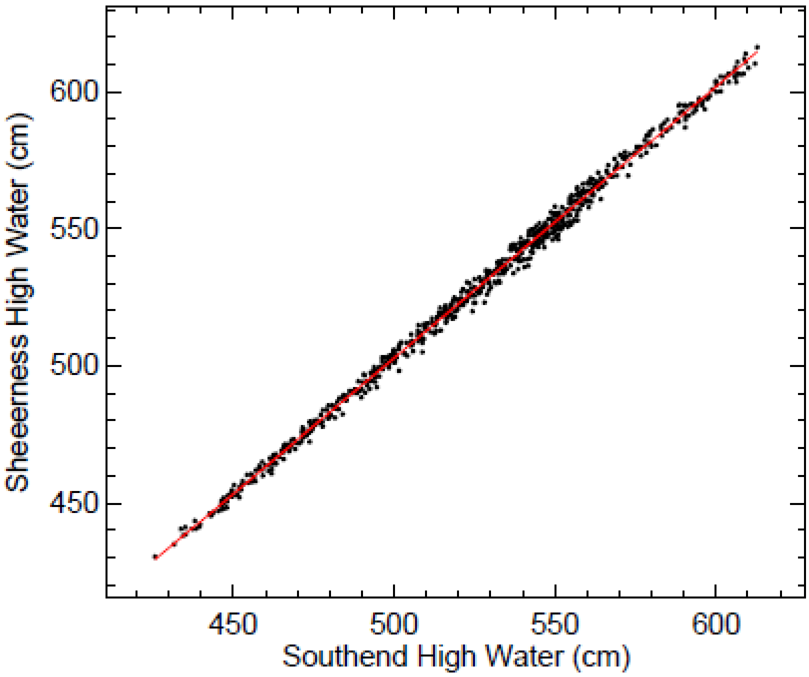

Figure 2. High water levels at Sheerness versus those at Southend during 1971. The red line shows a linear regression with the Southend values as the independent variable.

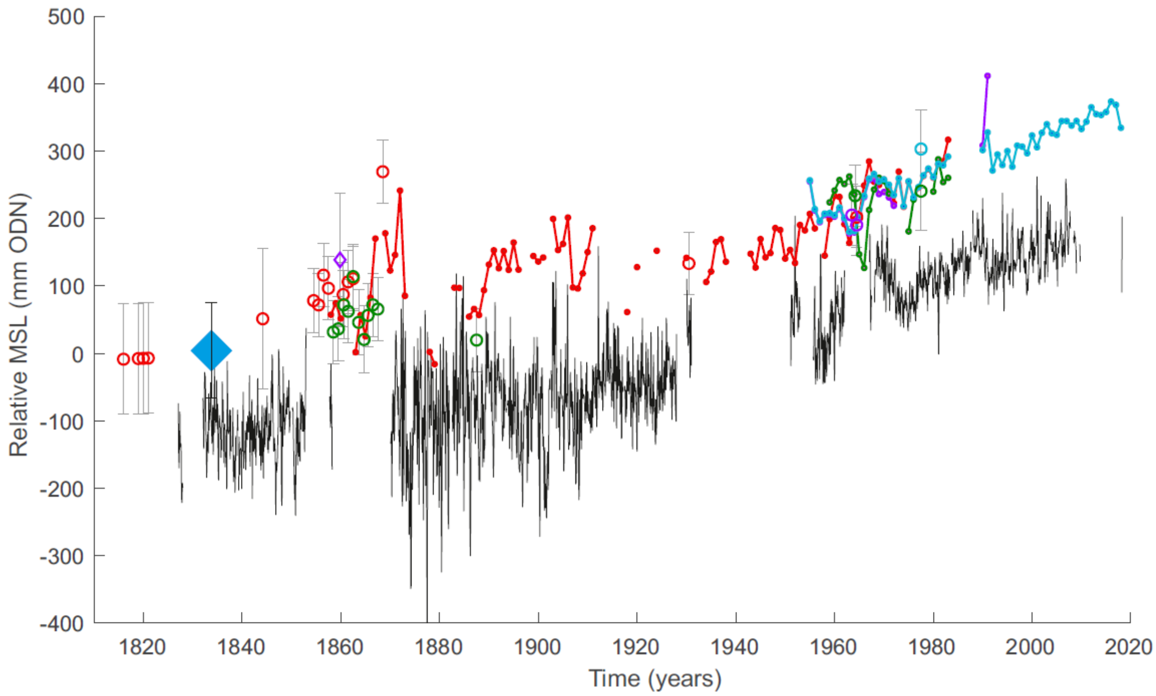

Figure 3. An estimate of MSL at Liverpool in 1834 from the Denham data in the present work (blue diamond) superimposed on a series of estimates of MSL at different times at Liverpool from a recent analysis of UK MSL by Hogarth et al. (2021). See the Supplementary Information of Hogarth et al. (2021) for details of the exact locations in the Liverpool area of each MSL value as indicated by the different colours. The lower thin black time series shows the record of monthly mean sea levels from Sheerness. All MSL values are expressed relative to Ordnance Datum Newlyn (ODN).

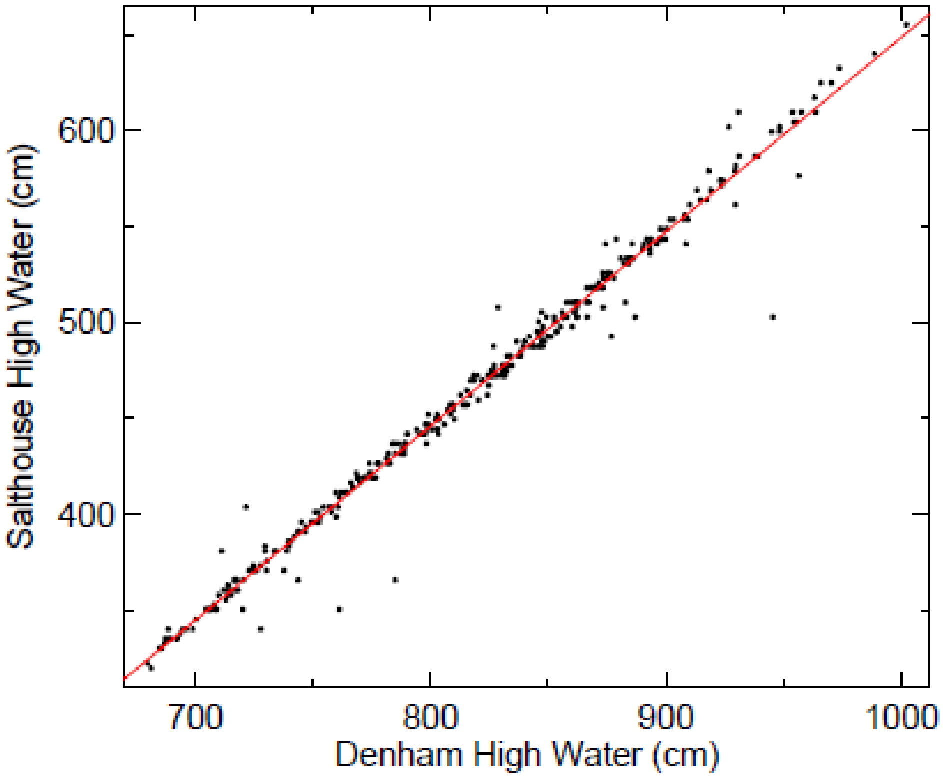

Figure 4. High water levels in the Salthouse Dock data set versus those obtained from the Denham sea level record at Liverpool during April-October 1834 omitting the short period with a slipping tape. The red line shows a linear regression with the Denham values as the independent variable.

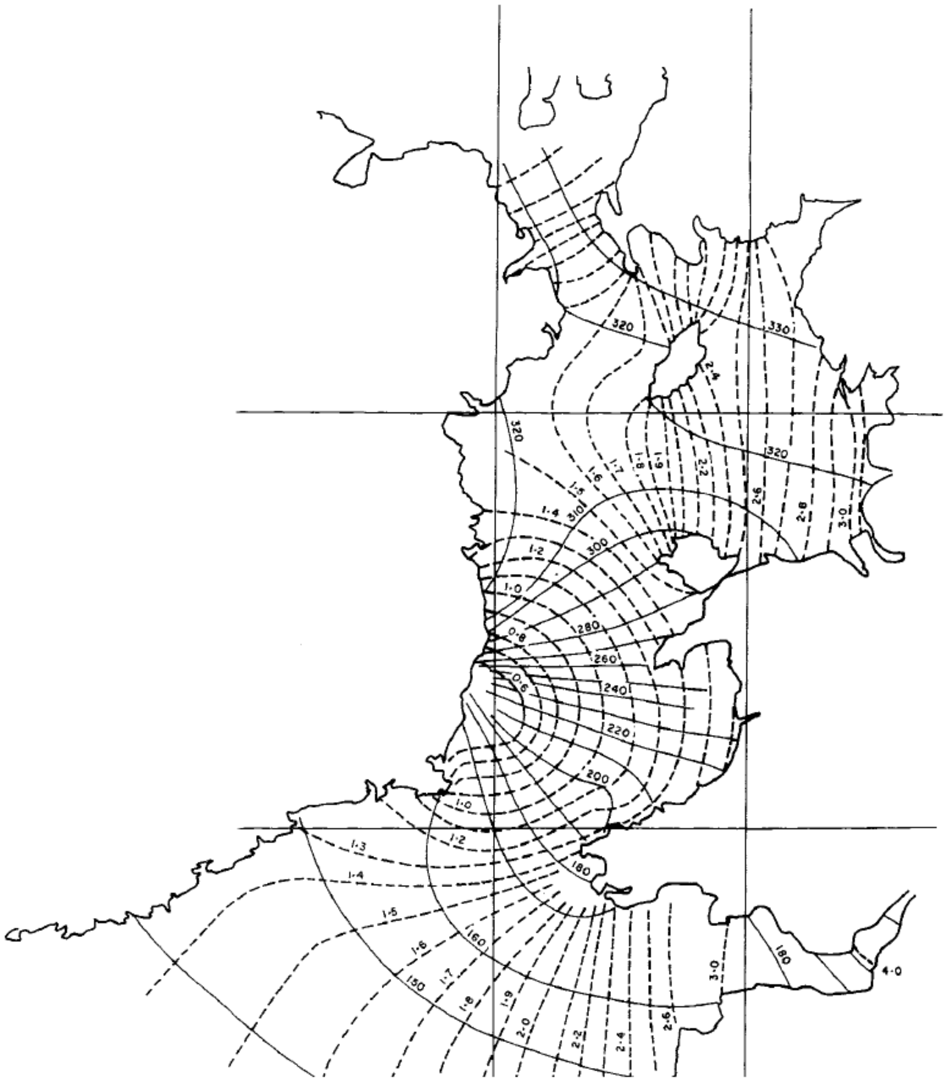

Figure 5. The amplitude (dashed lines in metres) and Greenwich phase lag (solid lines in degrees) of the M2 constituent of the ocean tide in the central part of the Irish Sea from Robinson (1979).

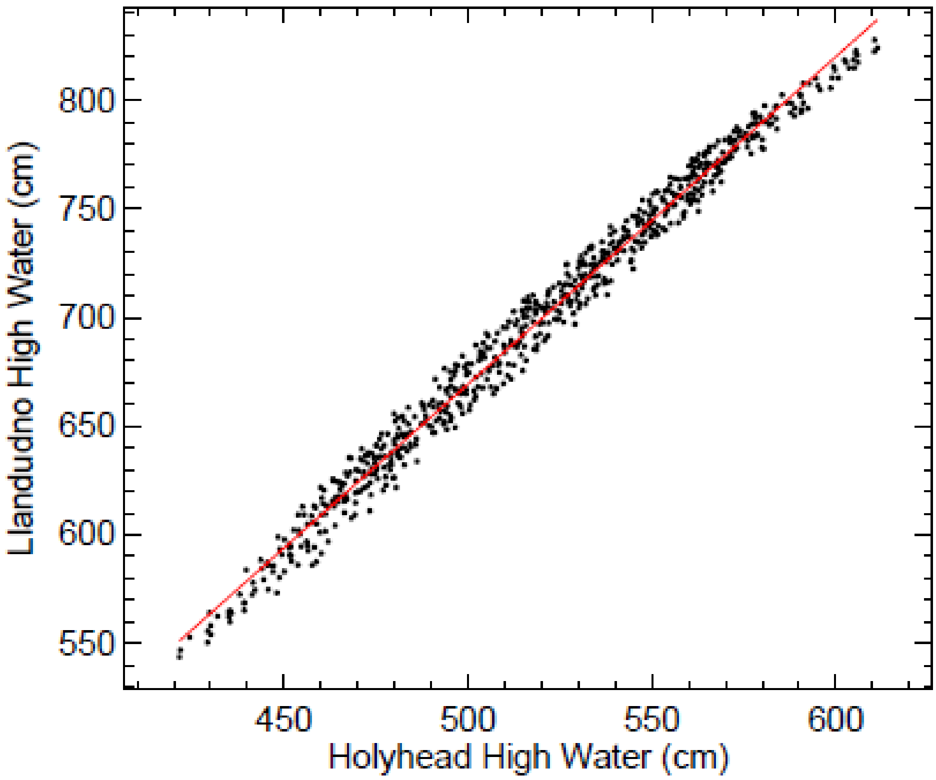

Figure 6. High water levels at Llandudno versus those at Holyhead during 2013. The red line shows a linear regression with the Holyhead values as the independent variable.