STUDIES OF STANDING-WAVE GEOMAGNETIC PULSATIONS IN THE MAGNETOSPHERE

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ABSTRACT. An analysis has been made of the power spectra of Pc3 and Pc4 geomagnetic pulsations recorded at Halley Bay, Antarctica, St Anthony, Newfoundland, and at South Georgia. Halley Bay and St Anthony are geomagnetically conjugate stations on the $L = 4.2$ magnetic shell and South Georgia is situated on the same geomagnetic longitude as these two stations, but at a lower L-value of 1.8. The power spectra of the pulsations recorded at the conjugate stations show that at any one time there may be several standing hydromagnetic waves on a particular shell, and that the frequencies of the oscillations differ from the expected natural oscillation frequency of the field line. On many occasions similar maxima are also observed in the power spectra of the pulsations recorded at the lower-latitude station, South Georgia. It is suggested that, on most occasions, the characteristics of the pulsations observed on the ground are mainly due to the travelling fast-mode hydromagnetic waves in the magnetosphere rather than standing waves on individual field shells.

INTRODUCTION

Geomagnetic pulsations (quasi-sinusoidal oscillations of the Earth's magnetic field) have been observed at the Earth's surface for many years, and recently such pulsations have also been observed in magnetic-field and energetic charged particle data from satellites. These pulsations, or waves, are believed to be hydromagnetic waves in the Earth's magnetosphere. Geomagnetic pulsations as seen on the ground may in general be divided into two classes: those of a regular and mainly continuous character, Pc, and those with an irregular pattern or wave form, Pi. In this paper we are concerned only with Pc in the subdivisions Pc4 and Pc3, with frequency ranges 6.7–22 and 22–67 mHz respectively, at the two Antarctic stations Halley Bay and South Georgia, and at St Anthony, Newfoundland, which is geomagnetically conjugate to Halley Bay.

By the 1960s ground-based measurements of magnetic pulsations showed certain similarities between pulsations recorded at the intersection of a geomagnetic field line with the Earth in the northern and the southern hemispheres. It was suggested that one hydromagnetic wave mode, the Alfvén mode, is guided along the magnetic field line. A wave travelling in this mode could be reflected from conjugate ionospheres to give oppositely directed travelling waves and thus a standing wave. This is illustrated in Fig. 1. The period of the oscillation of the standing wave will be twice the time the Alfvén wave takes to travel from hemisphere to hemisphere. The period of oscillation will thus depend on the length of the field line. As the length of the field line increases with latitude so will the frequency of the natural oscillation decrease with latitude.

In a 3-dimensional dipole field the Alfvén mode gives torsional oscillations of complete magnetic shells, the set of field lines connected to a particular latitude. This oscillation is termed the guided toroidal mode. The fundamental frequency of oscillation and its variation with latitude will be similar to that expected for the Alfvén wave. The motion of the plasma and the associated twisting of the magnetic shell results in east–west oscillations of the magnetic field and north–south oscillations

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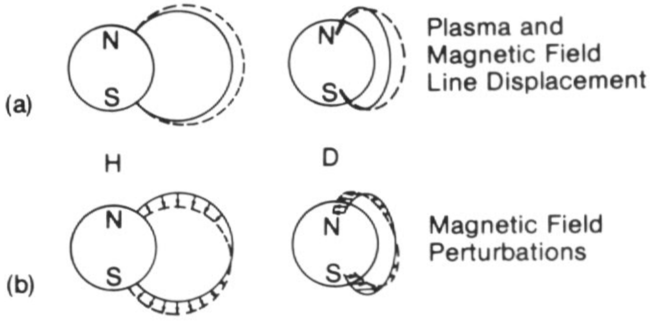

Fig. 1. A schematic illustration of a field-line resonance.

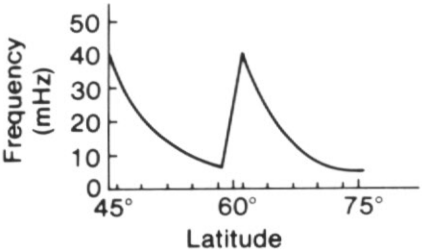

Fig. 2. A schematic illustration of the variation of natural field-line resonance with latitude (Orr and Hanson, 1981).

of the electric field at the top of the ionosphere. The effect of the currents driven in the ionosphere by the electric field is to screen the magnetic-field variations at the top of the ionosphere from the ground. However, the same currents in the ionosphere produce further magnetic-field variations on the ground. The direction of these variations is rotated by 90° with respect to the variations at the top of the ionosphere (Hughes, 1974). Thus, for the fundamental harmonic, the north–south oscillation on the ground at opposite ends of the field line should be in phase while the east–west oscillations should be out of phase. Hughes and Southwood (1976) have also shown that, as well as the rotation of the horizontal polarization, the ionosphere causes a dramatic reduction in ground amplitude relative to the magnetospheric amplitude. This can be by an order of magnitude when the horizontal scale length of the region of the oscillation is about 50 km.

The natural frequency of the guided toroidal mode standing wave will depend on the length of the geomagnetic field line, and also on the local Alfvén velocity. The Alfvén velocity depends on the local plasma density and magnetic field strength. A schematic illustration of the approximate variation of the natural frequency with latitude, after Orr and Hanson (1981), is shown in Fig. 2. As the length of the field line linking northern and southern conjugate points increases with latitude, the natural standing-wave frequency decreases with latitude. This uniform variation is disrupted at a latitude of about 60°. There, the geomagnetic field line on the noon side extends out to a distance of about four Earth radii at the geomagnetic equator. This field line often coincides with the position of the plasmapause, the boundary between the plasmasphere containing the dense cold plasma close to the Earth, which co-rotates with it, and the plasmatrough containing the much more tenuous non-co-rotating plasma. At the plasmaplause the sharp negative gradient in the plasma density produces an abrupt increase in the natural standing-wave frequency. With increasing latitude in the plasmatrough, the frequency decreases again as the field line lengthens. Thus Fig. 2 may be regarded as showing the expected frequency of a standing wave recorded at a ground station at the latitude shown on the abscissa.

With data from the conjugate stations, Halley Bay and St Anthony at $L = 4.2$, we were thus able to investigate magnetic pulsations in the vicinity of the plasmapause. We would expect that as the boundary changes its position in response to changing solar wind conditions the stations would record predominantly low-frequency or high-frequency waves, depending on whether the stations were inside or outside the plasmasphere respectively.

Another hydromagnetic wave mode present in a dipole field is the fast mode, which travels across the magnetic field lines and throughout the magnetosphere. Due to the coupling between the fast mode and the guided mode in the non-uniform field, this mode is capable of transferring wave energy from a wave source in the magnetosphere to individual field lines or shells where guided-mode standing waves may be generated.

With additional simultaneous data from a low-latitude station, South Georgia ($L = 1.8$), which has a high natural standing-wave frequency, and is not susceptible to plasmapause movement, we have been able to study the propagation of wave energy in the magnetosphere.

DATA AND ANALYSIS

Digitally recording Rubidium Vapour Magnetometers (RVM) designed and constructed by the Geomagnetism Research Group (GRG) of the British Geological Survey (Stuart, 1982) as part of the International Magnetospheric Study (IMS) were installed at St Anthony, Halley Bay and South Georgia in July 1976 and operated respectively until 1979, 1987 and 1981. The RVM measures three orthogonal components of the geomagnetic field at intervals of 2.5 s and is suitable, therefore, for studying pulsations of periods greater than 10 s. The resolution of the horizontal components is about 0.1 nT.

The period chosen for this study is the midday interval of days 291–306 and 313 and 340 in the year 1976 (the magnetometer at SA was out of commission on days 307–312). At this time of the year, the southern summer, all these stations are in sunshine at local noon, which is taken to be 1400 UT. This was a rather quiet period, with some very quiet days, but with a few fairly disturbed days when the solar wind velocity (SWV) exceeded 500 km/s.

The geographic and eccentric geomagnetic coordinates of the three stations are shown in Table I.

Table I. Coordinates of three stations

	Geographic	*Geomagnetic**	*L-value**
St Anthony (SA)	51.4° 304.4°	60.1° 28.5°	4.1
South Georgia (SG)	−54.3° 323.5°	−42.0° 24.6°	1.8
Halley Bay (HY)	−75.5° 333.1°	−60.8° 28.1°	4.6

* Calculated from the IGRF at altitude 100 km for epoch 1976·9.

Spectral analyses using the Fast Fourier Transform (FFT) were carried out in two parts; (*a*) an 80-minute interval, with a decimation of 8, gives the power at frequency intervals at 0.8 mHz approximately over the Pc4 range (6.7–22 mHz) with 6.8 degrees of freedom and a normalized standard error of 0.54, and (*b*) a 40-minute interval, with a decimation of 2, gives the power at frequency intervals of 3.1 mHz approximately over the Pc3 range (22–67 mHz) with 13.5 degrees of freedom and a normalized standard error of 0.39. Other intervals and decimations are used in some of the figures. The auto-spectral analysis gives the power spectral density (psd) in $(\mathrm{nT})^2/\mathrm{Hz}$ in the H- and D-components, ϕ_H and ϕ_D, the coherence between the components, the degree of polarization, ellipticity and azimuth (A). Another program sums the psd in the H- and D-components to give the total horizontal psd, T, in each frequency band.

In addition, cross-spectral analysis gives the phase differences and the coherence γ^2 between the components at the two stations.

STANDING WAVES AT HALLEY BAY AND ST ANTHONY

As almost geomagnetically conjugate stations, Halley Bay and St Anthony could be expected to be at opposite ends of an oscillating field line. If this field line resonates at its perfect fundamental odd-mode resonance, certain relations between the relative phase and ellipticity parameters would exist between the ends of the field lines due to the geometry of the oscillations. The H-components at each end of the field line should be in phase, $\phi_H(\mathrm{SA}) - \phi_H(\mathrm{HY}) = 0$,whereas the D-components should be in antiphase, $\phi_D(\mathrm{SA}) - \phi_D(\mathrm{HY}) = \pm 180°$. The horizontal ellipticity at each end should be the same but with opposite sense of rotation, $e(\mathrm{SA}) + e(\mathrm{HY}) = 0$, and the angle of the major axis will be shifted with respect to the H-component, $A(\mathrm{SA}) + A(\mathrm{HY}) = 0$.

Selection of hydromagnetic standing waves

The initial selection of possible cases of standing waves is based on the coherence between the H- and D-components at the two conjugate stations, SA and HY. The values of γ^2 which give a 95% confidence that the error, $\Delta\phi$, of the estimates of ϕ_H and ϕ_D is less than 45° are 0.82 for the Pc4 analysis and 0.56 for the Pc3 analysis. All cases when γ^2 exceeded these values were listed with the values of ϕ_H, ϕ_D, e and A, with a note as to whether there is a peak in the power spectrum at one or both stations. We have also listed a few cases when there were good power peaks but γ^2 was less than the given value. About a quarter of this initial list were eliminated because either $\phi(\mathrm{HY}) - \phi(\mathrm{SA})$ was far from 0° or 180° or the expected conjugacy [illegible] e or A was poor. Our final list (not reproduced) contained 92 cases in the range 6.7–25 mHz and 81 in the range 20–67 mHz (the Pc4 and Pc3 ranges were extended slightly to include an overlap, 20–25 mHz). The means and standard deviations (SD) of the values of the phase differences, ellipticity and azimuth sums are shown in Table II; these values are close to those obtained by Green and Hamilton (1981) in their study of Pi2 pulsations at the same two conjugate stations.

Examples of standing waves

Fig. 3 shows the power spectra of the T-component recorded at SA and HY (and also SG) on the rather disturbed day 292; it can be seen that the correspondence over the range 7–50 mHz is extremely good. Figs 4a and b show high coherence between the two stations over the whole range 10–40 mHz, with phase differences near 0° (H) and 180° (D). Fig. 5 shows the pulsating nature of the waves and how the variations in amplitude occur simultaneously at the conjugate stations.

There can be little doubt but that on this occasion the conjugate stations recorded

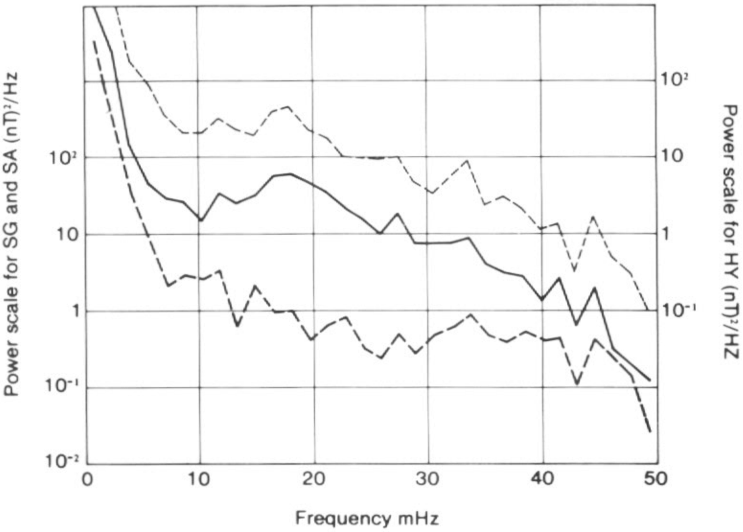

Fig. 3. Power spectra of pulsations recorded at Halley Bay, St Anthony and South Georgia on day 292, 1976, 1330–1430 UT. The upper trace is for HY, the middle for SA and the lower for SG.

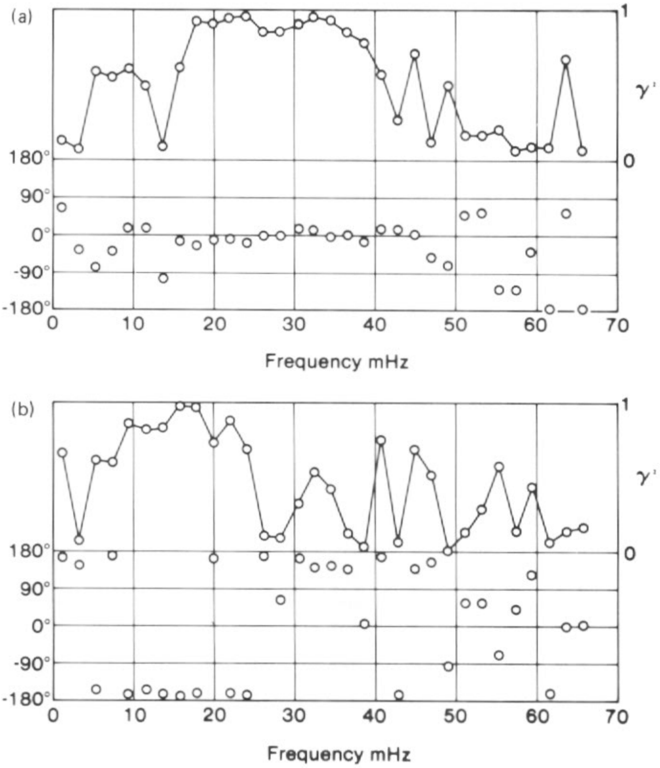

Fig. 4. a. The H components coherence and phase difference between SA and HY during the interval 1330–1430 UT on day 292. b. As a, but for the D components.

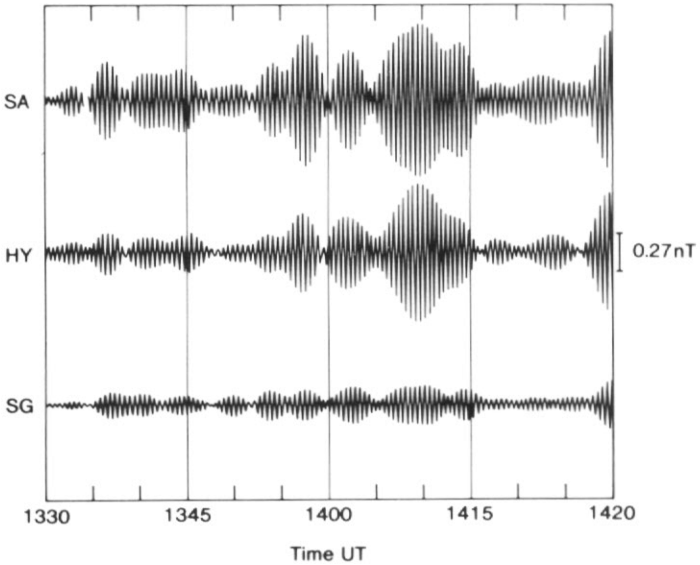

Fig. 5. The H component signals seen at the stations SA, HY and SG on day 292, filtered with a passband of 30–36 mHz.

signals from hydromagnetic waves over a range of frequencies: this day was somewhat remarkable, but on all days the power spectra show good correspondence over the whole range of frequencies.

Table II. Summary of comparisons between SA and HY of phases, ellipticities and azimuths

	Pc4			Pc3		
	No.	*Mean*	*SD*	*No.*	*Mean*	*SD*
$\phi_H(SA)-\phi_H(HY)$	62	29°	17°	55	27°	15°
$\phi_D(SA)-\phi_D(HY)$	65	−166°	20°	58	−158°	18°
$e(SA)+e(HY)$	92	0.01	0.24	81	0.03	0.28
$A(SA)+A(HY)$	92	3°	23°	81	1°	23°

ϕ, phase; e, ellipticity; A, azimuth.

Discussion of standing-wave results

As the plasmapause at local noon is normally close to the geomagnetic latitude of SA and HY, we might expect from Fig. 2 that the frequencies of standing waves would be around 10 mHz on days of lower geomagnetic activity and around 40 mHz on days of higher activity, when the plasmapause tends to move radially inwards. The variation of the morning values of the geomagnetic latitude of the plasmapause, L_{pp}, with the geomagnetic activity index K_p (Orr and Webb, 1975) is given by

$$L_{pp} = 0.18\,(K_p-4)^2+3.64,$$

where K_p has been taken as the mean values of K_p in the period 21–06 UT. We have plotted the values of the frequency, f, of all the listed standing waves against the calculated value of L_{pp} for that day: the plot is not reproduced as the points appear to be almost randomly distributed, with no resemblance to Fig. 2, and with no change of frequency values as L_{pp} increases through 4.2, the L-value for HY and SA. This confirms the findings of an earlier (unpublished) study in which Pc3 events were identified on analogue charts throughout the day during the southern summer of 1978–9 over a wide range of K_p values: there was no evidence of any difference between values of frequency, ellipticity or azimuth when HY was inside or outside the presumed position of the plasmapause as determined by L_{pp}.

We have determined the frequency of occurrence of the frequency of the listed standing waves in bandwidths of 5 mHz, and the results are shown in Table III. The power of pulsations decreases with increasing frequency, being typically two orders of magnitude lower at 67 mHz than at 6.7 mHz, so it is likely that our method of selecting standing waves is not unbiased. However, the fact that it is the 20–25 mHz band, which is at the lower end of the Pc3 range and at the higher end of the Pc4 range, that is a maximum makes it clear that there is a real maximum frequency of occurrence in this range, above which it decreases steadily to 67 mHz, and below [illegible]ch it is almost steady, though there may be a small maximum around 7 mHz, in agreement with Orr and Hanson (1981). The frequency of occurrence of standing waves will, of course, depend on the frequency of occurrence of the forcing waves.

Table III. Frequency of occurrence of SA/HY standing waves in various frequency bands

Pc4		*Pc3*	
mHz	%	*mHz*	%
6.3–10	25*	20–25	23
10–15	20	25–30	15
15–20	21	30–35	13
20–25	34	35–40	11
		40–45	9
		45–50	8
		50–55	7
		55–60	7
		60–65	6

* Corrected for smaller band size.

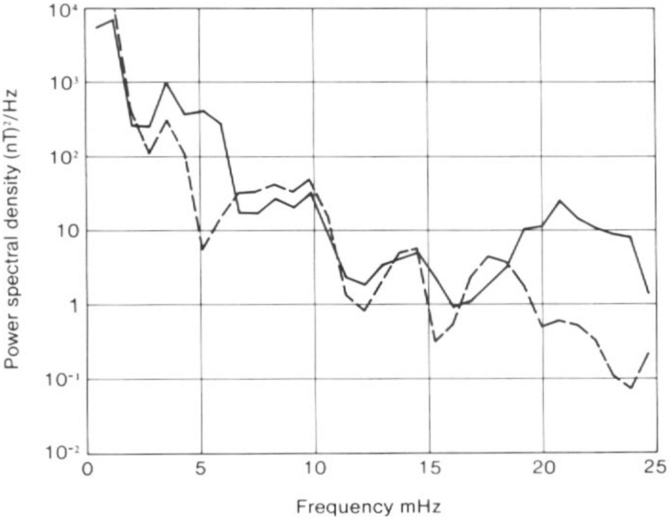

Fig. 6. The total horizontal component power spectra for 1340–1420 UT on day 324. The solid line is for HY and the dashed line is for SG.

HALLEY BAY AND SOUTH GEORGIA RESULTS

In addition to the conjugate station pair, the British Antarctic Survey also operated a magnetometer at South Georgia. SG is on a similar longitude to the HY/SA pair but is on a field line 'inside' the $L = 4.2$ line at $L = 1.8$. With this station it is possible to examine some of the waves seen on the $L = 4.2$ conjugate pair as seen at a lower-latitude station at the same longitude.

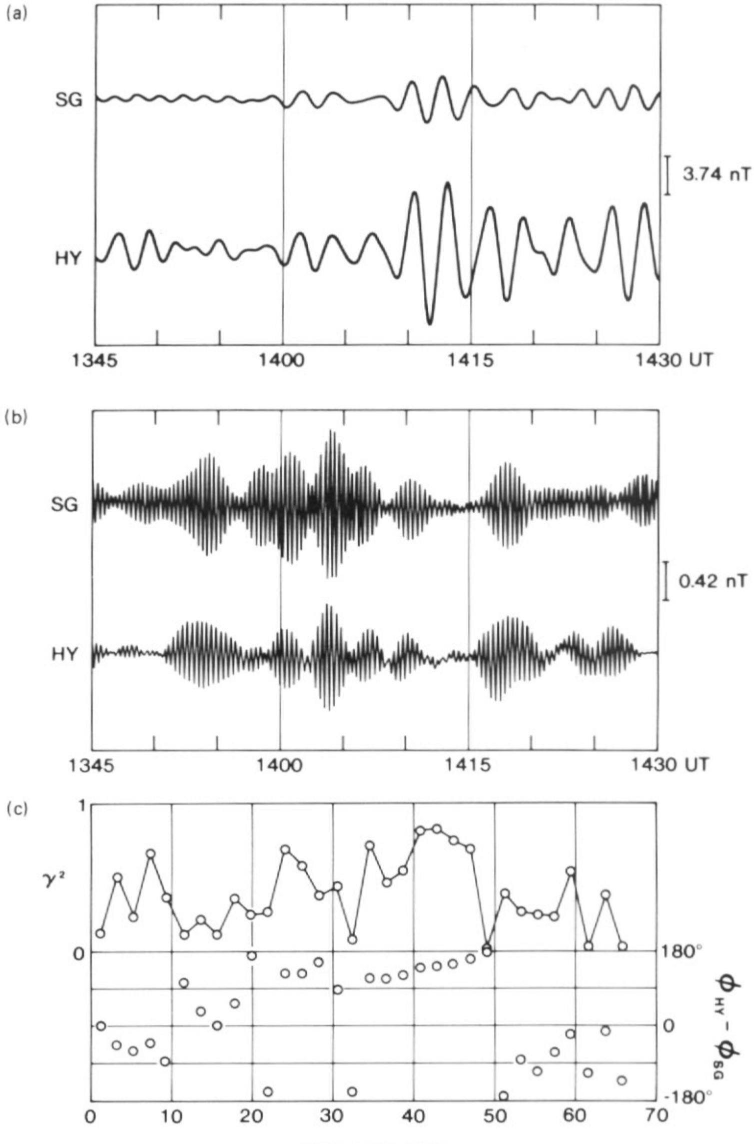

Fig. 7. a. The H-component signals seen at SG and HY on day 317, filtered with a passband of 5–10 mHz. b. As a, but filtered with a passband of 41–50 mHz. c. The coherences and phase differences between the H-component signals recorded at SG and HY from 1345 to 1430 UT on day 317.

Selection of events

There is no geomagnetic station conjugate to SG to enable us to pick out standing hydromagnetic waves on the SG records, so the events which have been identified are (*a*) those which show a high coherence (with the same criteria as used above) with HY, and (*b*) those which show a marked peak, some three times the background level, on the power spectrum.

Amplitude and frequency comparison

Although the geomagnetic latitude of SG is some 19° less than that of HY, there are a number of occasions when low-frequency pulsations are recorded simultaneously at the two stations. The SG power spectrum plotted in Fig. 3 shows several peaks corresponding to the standing waves observed at SA and HY, and Fig. 5 shows a very similar pattern of pulses: there are many occasions when the same pulsations were seen at both the higher-latitude and lower-latitude stations. Usually the power of the SG pulsation is about an order of magnitude lower than that at HY, but there are occasions when it is as great or greater. Fig. 6 shows the power spectra of day 324, with slightly larger peaks at 9.8 and 14.5 mHz at SG than at HY. Figs 7a and b show filtered spectra for day 317 in two passbands; 5–10 mHz and 41–50 mHz. The correspondence of the pulses in both bands is very good, the amplitude being higher at HY at the lower frequencies, but at 45 mHz the amplitude at SG is the greater. It seems that both stations see forcing waves at both the higher and lower frequencies. However, the standing-wave response has a larger amplitude at HY at the lower frequency and a larger response at SG at the higher frequency. This is as predicted by the natural standing-wave frequencies shown in Fig. 2. Fig. 7c shows the coherences and phase differences between the H components recorded at HY and SG during this interval.

One source of the forcing wave is generally accepted to be an instability caused by the solar wind at the magnetopause. Therefore, one might expect that the variations in amplitude in these records could be related to the parameters of the solar wind. It is clear, however, that the pulses do not occur at the same time for the three frequency bands, indicating that the simultaneous generation of waves over a wide range of frequencies does not occur, and we infer that the observed variations in amplitude are not simply due to fluctuations in the solar wind.

Phase difference between HY and SG

It is possible that, if the frequencies of the waves observed at HY and SG are sufficiently different from any resonant phenomena at both stations, the phase difference between the two stations is due to the time taken for the forcing wave to travel through the magnetosphere between the L-shells of the stations. We can calculate an estimated value of the time taken for a wave to travel through the plasmasphere, for the situation when both stations are in the plasmasphere, from the $L = 4.2$ to $L = 1.8$ field lines (and it makes little difference if L_{pp} is somewhat less than 4.2). At the equator this is a distance of 2.4 R_E, or 15300 km. From values of field strength (B) and plasma density n_0 given by Cowley (1972), the calculated travel time is about 30 s and the mean Alfvén velocity is 500 km s^{-1}.

The calculated travel time, t_p, gives a predicted variation of phase with frequency

$$\frac{\mathrm{d}}{\mathrm{d}f}\Delta\phi = 0.36\, t_p$$

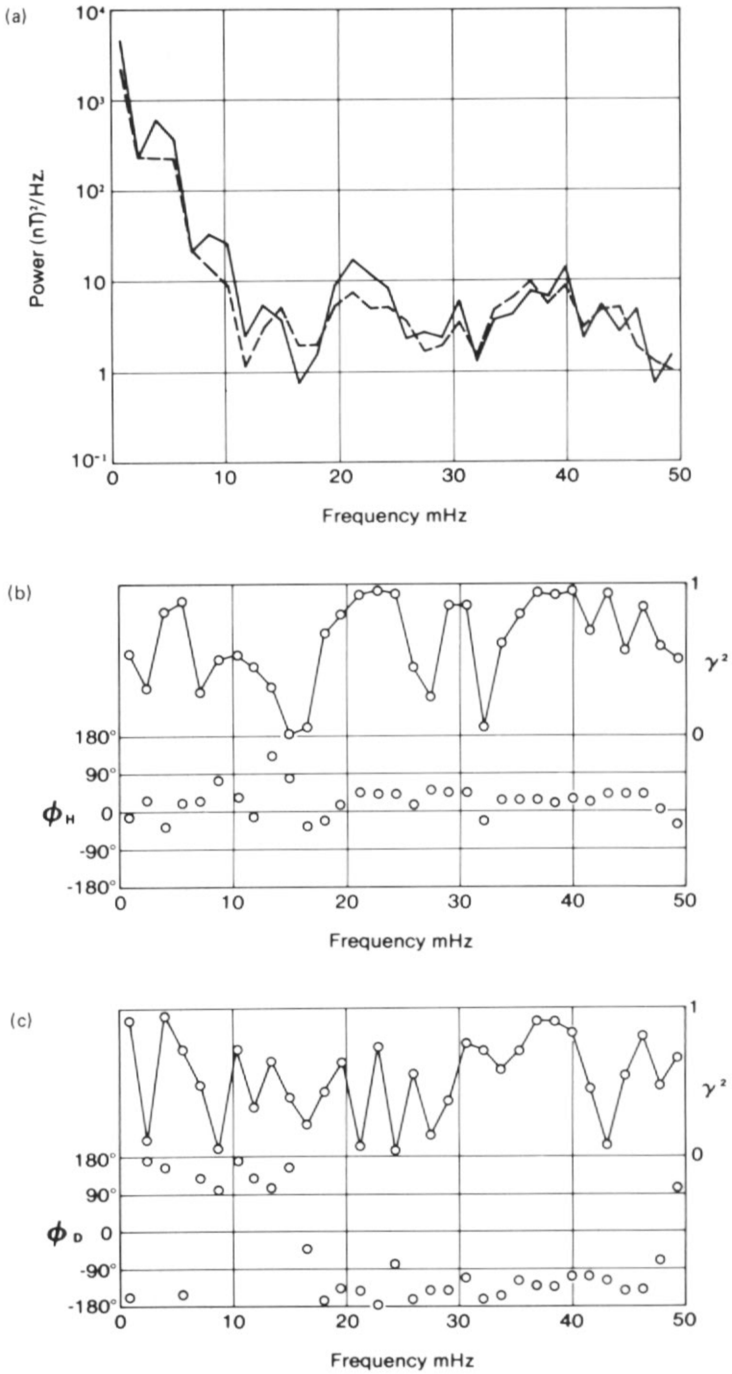

Fig. 8. a. The total horizontal component power spectra for HY and SA from 1340 to 1420 UT on day 324.
b. The coherence and phase difference for the H-components for the interval shown in Fig. 8a.
c. The coherence and phase difference for the D-components for the interval shown in Fig. 8a.

From all the observed values of $\Delta\phi_{\mathrm{H}}$ and $\Delta\phi_{\mathrm{D}}$ we have derived regression equations which give

$$\frac{\mathrm{d}}{\mathrm{d}f}\Delta\phi_{\mathrm{H}} = 13.6 \quad \sigma = 0.89 \text{ for 49 degrees of freedom}$$

$$\frac{\mathrm{d}}{\mathrm{d}f}\Delta\phi_{\mathrm{D}} = 12.6 \quad \sigma = 0.83 \text{ for 47 degrees of freedom}$$

giving a mean value of $t_{\mathrm{p}} = 36$ s. This is in good agreement with our estimated value, and suggests that on a number of occasions there is a fast-mode wave travelling radially inwards at the Alfvén velocity in the plasmasphere.

HARMONICS

It is difficult to identify harmonic series from the listed events, even if we assume that the lower-frequency waves are themselves harmonics of a yet lower fundamental. However, on one occasion, the power spectra for SA and HY for day 324, Fig. 8a, shows marked maxima at around 5, 10, 20 and 40 mHz, with weaker peaks around 15 and 30 mHz. Figs 8b and c show generally high coherence for the H- and D-components at these frequencies with phase differences around 0° (H) and 180° (D) as expected for resonant conjugacy. It appears that on this occasion the SA–HY field line was resonating powerfully with a fundamental frequency of about 5 mHz with higher harmonics. For another day, day 294, Fig. 9 shows a series of power maxima at SG which appear to be harmonics of an 8.3 mHz fundamental, which is itself not visible. The spectrum appears somewhat artificial, but it is difficult to imagine any instrumental fault with a period of 120 s. The HY spectrum also shows a marked maximum at 24 mHz. There seems little doubt that SG is observing a real series of harmonics.

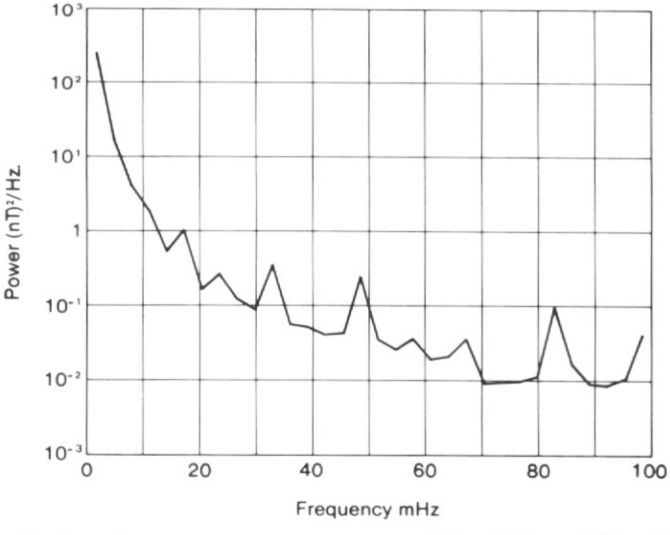

Fig. 9. The total horizontal-component power spectra recorded at SG from 1740 to 1820 on day 294.

Discussion

The power spectra show standing waves over a range of frequencies on a particular field line. Those frequencies are often quite far removed from the natural resonant frequency of a particular field line, as for example the lower-frequency waves observed at the lower-latitude station SG ($L = 1.8$). This suggests that many of the waves observed at a particular station are due to forcing fast-mode waves travelling throughout the magnetosphere, and there are relatively few instances of resonance at a particular station. However, the most common range of frequencies observed at all stations was the 20–25 mHz band, with frequency of occurrence falling off rapidly at higher frequencies. Now Fig. 2 would indicate that at the conjugate pair of stations HY and SA we would expect to see a predominance of waves around either 7 mHz or 40 mHz, depending on whether the station pair is inside or outside the plasmasphere. However, it could be that the plasmapause is not the sharp bounda[illegible] depicted in Fig. 2, and a range of frequencies, possibly peaking in the 20–25 mHz range, could be excited over this region and detected at HY and SA.

Another possible explanation of the similarity of the power spectra recorded at SA and HY could be that the fast-mode forcing waves are capable of generating their own standing waves in the magnetosphere. This could happen if waves were reflected between the natural boundaries in the magnetosphere – the magnetopause, the plasmapause and the ionosphere. These so-called cavity modes would produce power spectra that would be seen throughout the cavity in question. Evidence of fast-mode waves travelling radially in the magnetosphere was shown in the section as the phase differences between HY and SG.

However, the higher-frequency Pc3 pulsations seen at SG show that standing waves on field lines or shells do account for a number of waves observed at lower latitudes.

Conclusions

We have presented observations of magnetic pulsations which suggest that, for many of the waves observed, it is the hydromagnetic fast-mode wave, which is propagating throughout the magnetosphere, which causes many of the pulsation signals seen on the ground. On some occasions, standing hydromagnetic wave modes and resonances on geomagnetic field lines are generated, and these waves contribu[illegible] to the pulsations seen on the ground. However, these standing waves and resonances on individual shells seem only to account for relatively few observed geomagnetic pulsations.

We have not been able to identify events when these fast-mode waves generate standing cavity-mode waves, but the similarity of the power spectra at SA and HY and also at SG (see, for example, Fig. 3) could suggest that the cavity is influencing the power spectra over a range of L-values.

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