

Statistical forecasting techniques applied to observatory data for core field modelling

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Why consider statistical forecasting?

Modelling of the geomagnetic field is hindered by noisy and incomplete observations, and the extent to which we can model and separate the contributions of the various field sources in these data. Forecasting of the core field and its **secular variation, SV**, is further complicated by an incomplete knowledge of the physics controlling magnetic field generation.

Core field forecasts are often produced by extrapolating modelled field values^[1] or alternatively, by methods such as core flow advection^[2], or inverse geodynamo modelling^[3]. Those approaches which use no physical knowledge of the field or flow behaviour cannot predict changes such as jerks, those that are based on physical processes generally lack the knowledge to resolve events on a timescale of a few years, and all will be subject to the accuracy of the simplifying model assumptions made.

Given that the behaviour of temporal splines near the model end points is dependent on regularisation choices, extrapolating a model reliably is not straight forward. We take a preliminary look at statistical time series forecasting of observatory data, with a view to including forecast values and their uncertainties in a field model inversion, to improve the accuracy of short-range model predictions.

Data and model considerations

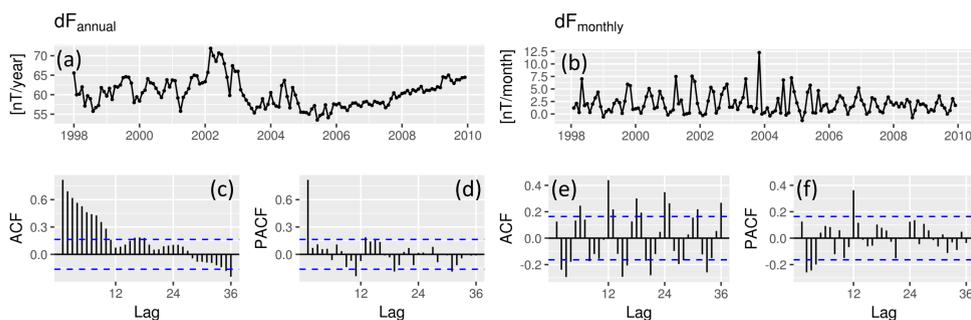


Fig.1 Annual differences of monthly mean (a) and first differences of monthly mean (b) scalar field at HAD, UK, with autocorrelation (ACF) (c,e) and partial autocorrelation (PACF) (d,f) functions. Blue dashed lines indicate 5% significance.

- Annual differences of monthly means generally aren't stationary (**Fig.1a**)
- Monthly **first difference (FD)** SV generally are approximately stationary (**Fig.1b**), with monthly data averaging out some external signal and periodicities seen in hourly data
- Monthly FDSV can be non-zero mean (**Fig.1b**), models must account for this
- Observatory data "cleaned" of external signals allows more robust forecasts of the core field
- We can't expect to predict jerks, but we expect to avoid influence from end-of-model effects by working directly from data
- Vector components at a single site have differing signal content, due to orientation relative to external sources, so are difficult to model jointly, scalar data provides a simpler approach
- Seasonal (annual and semi-annual) components and further harmonics are present in monthly FDSV data (**Fig.1e,f**)

ARIMA modelling

AutoRegressive Integrated Moving Average is a type of statistical time series model, which can be used to forecast future trends based on patterns in existing observations. The technique is naïve of any physical property underlying the data, but approximates the lag of the time series (**AR order**), and lag of the modelled time series residuals (**MA order**), with which the historic patterns in the data can be represented. ARIMA models require a stationary time series, accommodated via successive time differencing (**I order**). ARIMA models can also include a **seasonal component** with associated (AR, I, MA) orders, over a particular fixed period, and a constant term to accommodate series with a non-zero mean.

Here we model FDSV series, with a seasonal period of 12 months rather than seasonally differencing the data, to retain stationarity. We follow the **Box-Jenkins method**^[4], using ACF and PACF functions to assess likely model parameters, and optimal parameterisations determined by **Akaike Information Criterion**.

Test scenario

We calculate monthly mean series from AUX_OBS_2 hourly mean observatory scalar data for the period 1999–2015, prepared after [5]. We use 8 example observatories, with as close to continuous hourly data series as possible, and split the data into a training set for 1999–2010, and a validation set for 2010–2015.

We fit ARIMA models for each observatory series in the training set, and forecast each monthly; for the period of the validation set.

We utilise a BGS field model, denoted MEME2010, to compare our ARIMA forecasts to field model extrapolation. MEME2010 is constructed after the procedure of [2], from the same observatory data set as for the ARIMA models, as well as CHAMP and Ørsted vector and scalar, data for 1999–2010. The main field Gauss coefficients are extrapolated linearly for the period of 2010–2015, and the extrapolation and forecasts compared to the validation data.

Results

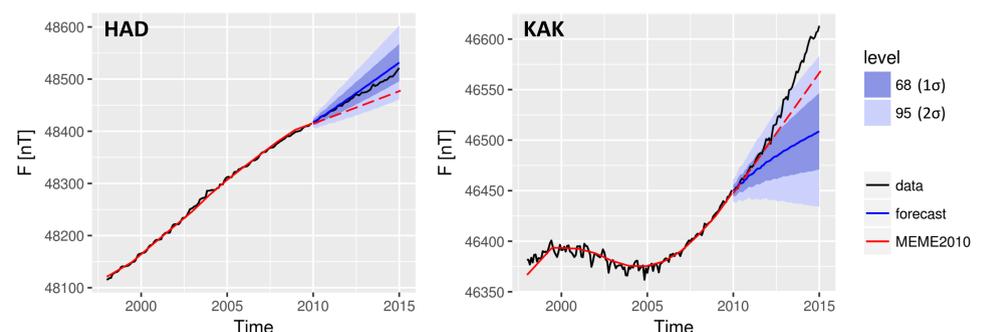


Fig.2 Comparison of monthly mean data (black), MEME2010 model (red), and forecast (blue). Forecast and model extrapolation performance is variable at each observatory. At HAD, the forecast performs better than the extrapolation; at KAK, the forecast is misled by accommodating the strong long period variation in the ARIMA model.

Obs.	Training RMSE [nT]	% data within 1σ of forecast		Prediction-data RMSE [nT]	
		ARIMA	MEME2010	ARIMA	MEME2010
ASP	5.4	66	27.4	12.9	
BOU	4.8	46	44.0	22.8	
CLF	4.2	100	7.8	23.3	
HAD	4.4	100	8.7	31.9	
HER	3.7	100	12.1	3.3	
HON	6.5	43	17.1	17.2	
KAK	5.4	38	53.5	16.5	
NGK	4.4	100	2.0	35.2	
Mean	4.8	74	21.6	20.4	
Median	4.6	83	14.6	20.0	



Table 1 Mean **root-mean-square error (RMSE)** is similar between forecasts and extrapolation, but forecasts show a smaller median RMSE. Regarding constraining a field model beyond the available data span, it is promising to note that the mean of over 70% of the forecast samples are within 1σ of the validation data over 5 years.

Summary and further work

ARIMA models can represent observatory time series within a mean RMSE here of 5nT/decade, and produce forecasts on the same order of accuracy as core field model linear extrapolation, over 5 years. The ARIMA model parameters can certainly be tailored further, especially in cases such as KAK (**Fig.2**), and the applicability to the global data set of observatories must be assessed.

A more extensive analysis of the statistical properties of the data, and in vector components, is needed, as well as consideration of other suitable forecasting techniques, such as machine learning algorithms^[6].

Given the preliminary results here, we will look to include forecast data and their uncertainties in a core field model inversion, to directly compare performance versus field model extrapolation on short timescales. This work could also be extended to include "virtual-observatory" time series, greatly increasing spatial coverage.

References

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