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The analysis of tidal streams — Laeso Rende

by

J. R. Rossiter

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institute of coastal oceanography and tides

NATURAL ENVIRONMENT RESEARCH COUNCIL

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This report was produced before the Tidal Institute became the Institute of Coastal Oceanography and Tides.

Background

At the request of Dr. Svansson of the Fisheries Board of Sweden, in 1967 the Institute undertook the analysis of the long Danish series of current measurements taken from the Laeso Rende Lightship during the years 1912-1913.

Laeso Rende (Lat. 57° 13' N, Long. 10° 42' E) is situated in the Kattegat between the Danish north-east coast and the island of Laeso. The observations, together with a description of the techniques (a "Libelle" current meter was used) will be found in the publication by Jacobsen (see ref.). Velocities and directions were tabulated at intervals of approximately 4 hours (0, 4, 8, 12, 16 and 20 hours each day) at depths (2½ 5, 10, 15 and 20 m). The total depth of water at the station was 22 m. Apart from a few breaks, the records span the period 5 September, 1912 to 18 November, 1913.

Although the quality of the basic observations may be suspect, and the sampling interval is large and inconvenient for tidal analysis, it was considered that such remarkably long time series at 5 depths merited analysis.

Analysis Procedure

Values of velocity (V cm/sec) and direction (θ° clockwise from north) were punched onto cards, and north-going (N) and east-going (E) components of velocity computed.

The ten series so generated were edited, by inspection, for large errors, and after correction were submitted to harmonic analysis using the Tidal Institute Flexible Analysis program TIFA.

A sampling time of 4 hours restricts the analysis to the tidal bands 0, 1, 2 and 3 cycles per day. All higher species present in the data will be "folded" into these lower frequencies; it is clear from the results (Table 1), however, that such contributions must be negligible.

TIFA will seek any selection of the orthodox tidal lines, plus any other frequencies (called "ODD") stipulated, assuming the sampling time to be 1 hour. When the sampling time is p hours, the analysis can be performed by defining lines in terms of $p\sigma$ instead of σ (σ = angular speed in degrees per hour). Care must be taken that $p\sigma$ for an ODD line does not coincide with σ for an orthodox line. It is not possible to treat the astronomical variables N, f and u required to represent nodal variations in a similar way; for these analyses the simplifications

$$N = f = u = u = 0,$$
$$f = 1$$

have been used.

The version of TIFA used produces the modulus of Zo; since the data were effectively to a zero datum, it was necessary to determine the sign of Zo independently. This was done first by computing daily mean values of N and E (centered on 1400 hours each day), then the overall mean. The tabulated daily means at each depth are available for study.

Analysis Results

Table 1 lists the harmonic constants of N and E for the 21 lines selected. There is a gratifying agreement between the major constituents at different depths, as well as indications of systematic variations in depth. The tidal regime is clearly semidiurnal, and virtually rectilinear in the N-S direction. Nevertheless, in order to understand the depth variations more fully, it was decided to determine the basic parameters of tidal current motion in two co-ordinates.

From Figure 1, the total velocity U is given by

$$U^2 = N^2\cos^2(\sigma t - n) + E^2\cos^2(\sigma t - e) \qquad (1)$$

where N, E are the component amplitudes and n, e their associated phase lags. The time origin used here is arbitrary. For maximum U, it follows from (1) that

$$R^2 \sin^2 2(\sigma t - r) = 0 \qquad (2)$$

where

$$\left.\begin{aligned} R^2\cos 2r &= N^2\cos 2n + E^2\cos 2e \\ R^2\sin 2r &= N^2\sin 2n + E^2\sin 2e \end{aligned}\right\} \qquad (3)$$

at time t_{max} given by $t_{max} = r/\sigma$ (4)

From (1) and (4),

$$U^2_{max} = \tfrac{1}{2} N^2\left\{1 + \cos(2r - 2n)\right\} + \tfrac{1}{2} E^2\left\{1 + \cos(2r - 2e)\right\} \qquad (5)$$

The direction of U_{max} is given by $\phi_{max} = \arctan \dfrac{N\cos(r-n)}{E\cos(r-e)}$

where ϕ is measured anticlockwise from east.

Table 2 lists values of r/σ, U_{max} and ϕ_{max} for M_2, S_2, N_2, O_1 and S_a, and these are plotted against depth in Figure 2. The vectors of mean flow (Z_o) are shown in Figure (3).

Some conclusions

The direction of maximum velocity is probably not too well defined by this analysis, and the results for O_1, are probably marred by the effect of background noise on its rather small amplitude. With these reservations, the following conclusions are worth noting :-

(1) Tidal currents are reasonably uniform from surface to bottom. In all cases (M_2, S_2, N_2 and O_1) the maximum velocity occurs at approximately mid depth.

(2) The phase of the tidal streams follows a pattern similar to that of the velocity; the maximum velocity, at mid depth, is also associated with maximum time lag. It seems possible that a linear time lag from surface to bottom (or vice versa) could have been introduced into the data as a result of the fact that only one current meter was used for all depths. Quote from Jacobsen's introduction:- "The measurements were taken nearly simultaneously at all depths, as the entire series could generally be carried out in fifteen minutes". Note that this is of the same order of magnitude as the relative time lags observed.

(3) Tidal currents rotate anticlockwise downwards, indicating inverse Ekman spiral due to bottom friction.

(4) The seasonal variation (Sa) and mean flow (Z_o) are significant in the upper layers, very small near the bed. They both rotate clockwise downwards, indicating the normal Ekman spiral arising from wind drift, and perhaps also from fresh water flow at the surface.

Reference

Current measurements from Danish Lightships. Medd. Komm. Havund., Serie Hydrografi, 11(8), Kobenhavn, 1923.

Table 1:- Analysis of Laeso Rende current observations at various depths, covering period 5 September 1912 to 14 November 1913

Time Zone -1. Units are cm/sec.

	N		E		N		E		N		E		N		E		N		E	
	H	g	H	g	H	g	H	g	H	g	H	g	H	g	H	g	H	g	H	g
Depth	$2\frac{1}{2}$ m				5 m				10 m				15 m				20 m			
Zo	26.0	-	2.5	-	24.7	-	4.6	-	9.7	-	3.3	-	-0.2	-	0.4	-	-2.2	-	-1.7	-
Sa	7.4	303	1.5	349	8.5	295	3.1	287	8.6	295	1.8	258	3.0	285	1.5	239	1.2	217	1.0	210
Ssa	6.9	76	0.9	323	5.1	76	1.7	339	1.5	269	1.0	324	2.4	256	0.7	240	2.4	263	0.5	150
$2Q_1$	1.7	44	0.4	64	1.6	32	0.5	329	1.3	45	0.6	340	0.2	41	0.6	341	0.9	330	0.2	284
Q_1	1.2	295	0.7	269	1.6	300	0.5	222	1.4	326	0.4	111	1.0	300	0.5	42	0.4	280	0.3	56
O_1	3.5	73	0.4	73	3.9	78	0.7	44	5.1	83	0.5	9	4.6	77	0.4	300	3.5	64	0.4	302
P_1	0.5	269	1.0	317	0.8	276	0.6	273	0.8	271	0.5	269	1.8	243	0.2	347	1.3	229	0.9	78
K_1	1.1	140	0.2	336	1.8	167	0.7	154	2.3	146	0.5	230	1.5	119	0.3	125	2.1	110	0.4	113
J_1	0.3	316	0.3	146	0.8	233	0.3	88	0.7	276	0.2	298	1.1	327	0.6	64	1.0	289	0.1	42
$2N_2$	0.5	46	0.5	121	0.9	103	0.3	338	1.5	135	0.2	90	0.9	139	0.2	169	0.5	62	0.4	90
μ_2	4.4	175	0.8	146	4.7	179	1.2	107	5.4	176	0.2	106	5.0	168	0.7	66	5.4	162	0.9	25
N_2	5.2	305	1.0	259	6.5	304	0.9	240	7.9	311	0.4	346	7.5	306	0.8	127	7.0	299	1.3	120
ν_2	1.9	9	0.9	36	1.9	335	1.7	21	2.5	31	0.1	224	2.2	359	0.4	301	1.6	29	0.7	190
M_2	20.4	350	1.7	306	22.6	350	3.5	333	26.0	359	1.1	340	25.9	355	1.3	189	23.2	347	2.0	181
L_2	1.5	115	0.4	253	1.6	131	0.2	51	1.2	114	0.5	319	1.8	104	0.2	50	1.9	84	0.1	159
S_2	6.1	318	0.6	280	7.0	321	0.5	333	9.3	331	0.6	330	8.9	329	0.6	235	7.3	320	0.7	146
K_2	1.1	288	0.1	230	2.1	293	0.3	270	1.7	289	0.4	294	2.0	293	0.1	173	1.7	305	0.4	121
$2SM_2$	2.4	29	0.7	297	2.3	14	0.3	59	2.2	26	0.4	325	2.0	18	0.4	145	2.4	13	0.3	103
MO_3	0.5	286	1.0	334	0.8	301	0.7	333	0.4	238	0.6	182	0.2	114	0.2	156	0.4	230	0.4	104
M_3	0.8	350	0.6	136	0.9	355	0.2	90	0.6	49	0.5	280	0.6	138	0.8	323	0.4	80	0.9	351
MK_3	0.8	101	1.1	110	0.6	75	0.9	119	0.6	162	0.1	19	0.5	71	0.6	223	0.4	211	0.3	205

Table 2:- Direction, time and magnitude of maxima of some tidal stream components at Laeso Rende

	Depth (m)	$r/\sigma = t_{max}$* (mins)	U max (cm/sec)	ϕ^o max
M_2	2.5	0	20.4	87
	5	1	22.8	82
	10	18	26.0	88
	15	10	25.9	93
	20	-6	23.3	95
S_2	2.5	0	6.1	85
	5	6	7.0	86
	10	26	9.3	86
	15	22	8.9	90
	20	4	7.3	96
N_2	2.5	0	5.2	82
	5	-1	6.5	86
	10	14	7.9	88
	15	4	7.5	96
	20	-6	7.1	101
O_1	2.5	0	3.5	83
	5	8	3.9	81
	10	21	5.1	89
	15	9	4.6	94
	20	-17	3.5	93
S_a	2.5	0	7.5	82
	5	-21	9.0	70
	10	-21	8.7	79
	15	-53	3.2	68
	20	-180	1.6	50

* relative to t_{max} at 2.5 m depth.

FIGURE 1

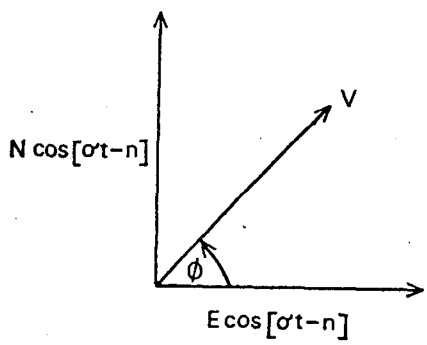


FIGURE 3

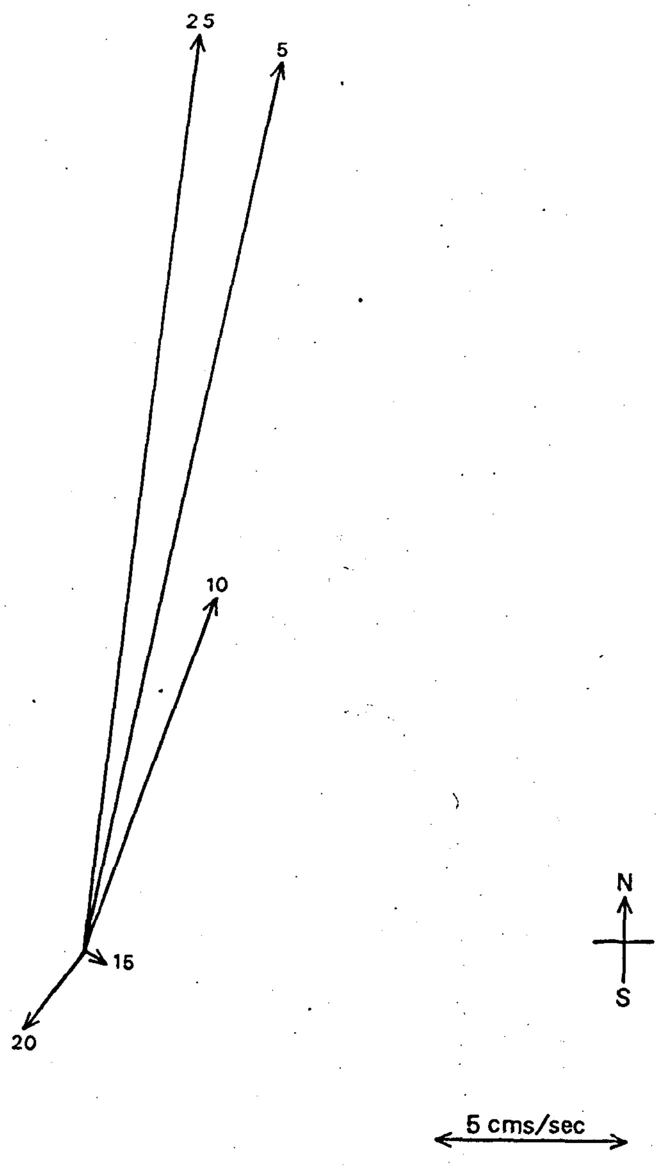
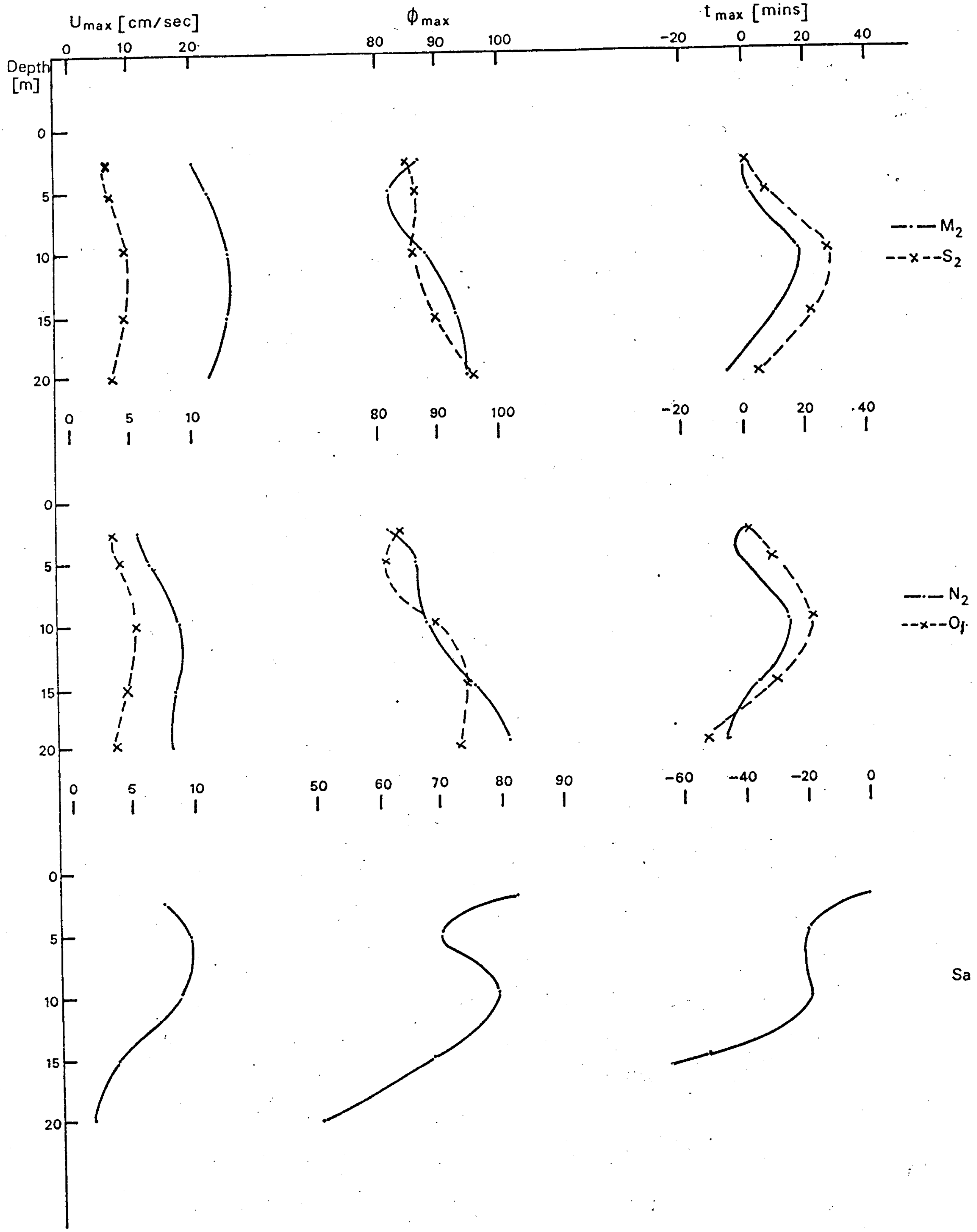


FIGURE 2



Umax [cm/sec]
φmax
t max [mins]
Depth [m]
0
5
10
15
20
0
10
20
80
90
100
-20
0
20
40
M2
S2
0
5
10
80
90
100
-20
0
20
40
N2
O1
0
5
10
50
60
70
80
90
-60
-40
-20
0
Sa