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FREQUENCY RESPONSE OF A TIDE WELL

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1. Introduction.

With the advent of digital tide recording it becomes necessary to give careful thought to the frequency and duration of sampling the height of tide in a tide well if ambiguous results are to be avoided.

A simple example of the sort of problem involved is given in Figure 1. If a short period oscillation is transmitted with the tide from the open sea into the well, the frequency of sampling can easily give rise to a spurious oscillation in the recorded data with a period in the tidal range of periods.

It is therefore desirable to know how oscillations of different periods are transmitted through a tide well orifice.

2. Existing Theory.

2.1 The theory for the steady state response of a well has been given by A.M. Shipley (1963). This work gives the response as a function of

(a) the ratio of the diameters of the well (D) and the inlet pipe (d)

(b) the frequency of oscillation (f)

and (c) the amplitude of oscillation outside the well (A_o).

The response takes two forms

(1) a reduction in amplitude given by A/A_o, where A is the amplitude inside the well corresponding to A_o outside

(2) a phase lag between the two oscillations.

2.2 Examples of these response functions are given for typical well dimensions as found in the Mersey system of gauges. Figures 2 and 3 are for the Tranmere gauge, and Figures 4 and 5 for the Gladstone gauge.

2.3 In the broadest of terms, the tidal range of frequencies can be said to go from 0 c.p.h. to 1 c.p.h. Waves of tidal period will therefore be transmitted to both the Tranmere and Gladstone wells without alternation and without phase lag, though the critical point has been reached at Gladstone. A further reduction in the orifice at Gladstone would mean that tides and surges were not being properly recorded.

On the other hand, short period waves are less attenuated at Tranmere than at Gladstone. At the latter gauge, the cut off point for waves of amplitude 1 ft. is 34 c.p.h., i.e. just less than 2 minute period. This should have the effect of appreciably reducing the short wave "embroidery" on the gauge records.

One of the most troublesome gauge phenomena in tidal work is the existence of seiches. If these are not required for investigation it would be well if they could be appreciably reduced by suitable choice of the ratio d/D. But as the normal harbour seiche period is of the order of 10-45 minutes, i.e. frequencies of 6-1·3 c.p.h., it will be seen that even at Gladstone, 1 ft. seiches will not be appreciably attenuated. Reduction of d/D to introduce seiche attenuation would unfortunately bring the tidal responses below unity, i.e. introduce distortion into the tidal recording.

3. Extension of Theory.

3.1 Shipley's work does not allow for the attenuation of short waves due to the depth of the orifice below the water surface. Since the waves are transmitted to the well via pressures, and since the pressure disturbances associated with short period surface waves decrease exponentially with depth, we may expect the cut off frequencies in the diagrams to be smaller than those given. Figures 2 to 5, in other words, are only valid for low water conditions.

Extension of the theory to include this effect is given in the appendix. The effect on the amplitude response is approximately given by multiplying Shipley's response by the factor $\exp(-1{\cdot}11\, f\sqrt{h})$, where f is in c.p.s. and h is the depth of the orifice below the water level in feet.

The above factor is unity for tidal frequencies, and decreases as the period decreases. Examples of the corrected responses for the two gauges discussed are indicated by dotted lines in Figure 2 to 5 in the case of h = 20 ft.

4. Conclusions.

Although the response function of the tide gauge itself has not been taken into account in the foregoing, enough has been done to indicate that the steady state response of wells in present day use can be quite easily checked, and that data exists for the design of wells to be used with digital recording gauges.

As a general rule in British waters, to ensure a 1:1 response for tidal periods, the area of the orifice should not be less than 1/500 of the area of the well.

Appendix.

In the case of a tide well, the pressure outside at the depth of orifice is given by (cf Proudman 1953)

$$p_{out} = p_a + \rho g \left(z_0 + \exp[-2\pi h/L] A_0 \sin \varpi t \right) \qquad (1)$$

and the pressure inside the tide well at the orifice by

$$p_{in} = p_a + \rho g (z_0 + z) \qquad (2)$$

Here p_a is the atmospheric pressure, ρ the density of water, g acceleration due to gravity, z_0 the height of sea level above the orifice in the absence of a wave of amplitude A_0, angular frequency ϖ and wave length L, h the depth of the orifice below the free surface and z is the height of water inside the well with respect to the sea level outside. From (1) and (2) we have for the difference of pressure, p, outside and inside the tide well at the depth of orifice

$$p = \rho g h_t \qquad (3)$$

where

$$h_t = A_0 \exp[-2\pi h/L] \sin \varpi t - z \qquad (4)$$

According to Doodson and Warburg (1941) the rate of rise of water in the well is given by

$$0{\cdot}6 \frac{e}{E} \sqrt{2 g h_t} = \lambda h_t^{1/2} \qquad (5)$$

where

$$\lambda = 0{\cdot}6 \frac{e}{E} \sqrt{2g} = 4{\cdot}8 \frac{e}{E} \text{ (f.p.s. system)} \qquad (6)$$

Here E is the area of the tide well and e the area of the orifice. Denoting the rate of rise of water in the well by $\frac{dz}{dt}$ we have from (4) and (5)

$$\frac{dz}{dt} = +\lambda \sqrt{| A_0 \exp(-2\pi h/L) \sin \varpi t - z |} \qquad (7)$$

for z rising inside the well

$$\frac{dz}{dt} = -\lambda \sqrt{| A_0 \exp(-2\pi h/L) \sin \varpi t - z |} \qquad (8)$$

for z falling.

The equations (7) and (8) differ from equations (1) and (2) in Shipley's paper in that A_0 the amplitude of the wave outside is multiplied by $\exp(-2\pi h/L)$.

Now

$$\frac{h}{L} = \left(z_0 + A_0 \sin \varpi t\right)\frac{1}{L} \tag{9}$$

For the relation between ϖ and λ we have (cf Jeffreys and Jeffreys (1950)).

$$\varpi^2 = \left(\frac{2\pi g}{L} + T'\frac{8\pi^3}{L^3}\right)\tanh 2\pi h/L \tag{10}$$

where $T'\rho$ is the surface tension.

Neglecting the term proportional to T' and regarding $2\pi h/L$ as small (this is especially true in shallow water areas) we have

$$\varpi^2 = \frac{4\pi^2 gh}{L^2} \tag{11}$$

or

$$f = \sqrt{gh}\,\frac{1}{L} \tag{12}$$

f, the frequency, is given by $\frac{\varpi}{2\pi}$.

Neglecting the term proportional to A_0 in (9) and (12) we obtain

$$\frac{h}{L} = \sqrt{\frac{z_0}{g}}\,f \tag{13}$$

Substituting from (13) in (7) and (8) we have

$$\frac{dz}{dt} = +\lambda\sqrt{|A_0 \exp(-1{\cdot}11\sqrt{z_0}\,f)\sin 2\pi ft - z|} \tag{14}$$

for z rising

$$\frac{dz}{dt} = -\lambda\sqrt{|A_0 \exp(-1{\cdot}11\sqrt{z_0}\,f)\sin 2\pi ft - z|} \tag{15}$$

for z falling.

For $f=0$ equations (14) and (15) reduce to equations (1) and (2) in Shipley's paper. The numerical solutions obtained in Shipley's paper still apply but the definition of A_0 the amplitude is changed to $A_0 \exp(-1{\cdot}11 f\sqrt{z_0})$.

For the interpretation of figures (2) and (3) in Shipley's paper we have

$$\frac{1}{\varepsilon} = \frac{\text{amplitude of steady state response}}{\text{amplitude of input} \times \exp(-1{\cdot}11 f \sqrt{z_0})} \tag{16}$$

or

$$\frac{A}{A_0} = \frac{1}{\varepsilon} \exp(-1{\cdot}11 f \sqrt{z_0}) \tag{17}$$

where A is the amplitude of the steady state response in the tide well

and

$$\mu = \frac{\lambda}{\sqrt{A_0 \exp(-1{\cdot}11 f \sqrt{z_0})}} \cdot \frac{1}{\varpi} \tag{18}$$

or

$$f = \frac{\lambda}{2\pi \sqrt{A_0 \exp(-1{\cdot}11 f \sqrt{z_0})}} \cdot \frac{1}{\mu} \tag{19}$$

For a given μ it will be difficult to calculate f from equation (19). We take values of f, A_0 and z_0, calculate μ from equation (18) and then from figures (2) and (3) from Shipley find the corresponding values of ε and phase lag. The value of ε for the given value of f and z_0 is used to find $\frac{A}{A_0}$ from (17).

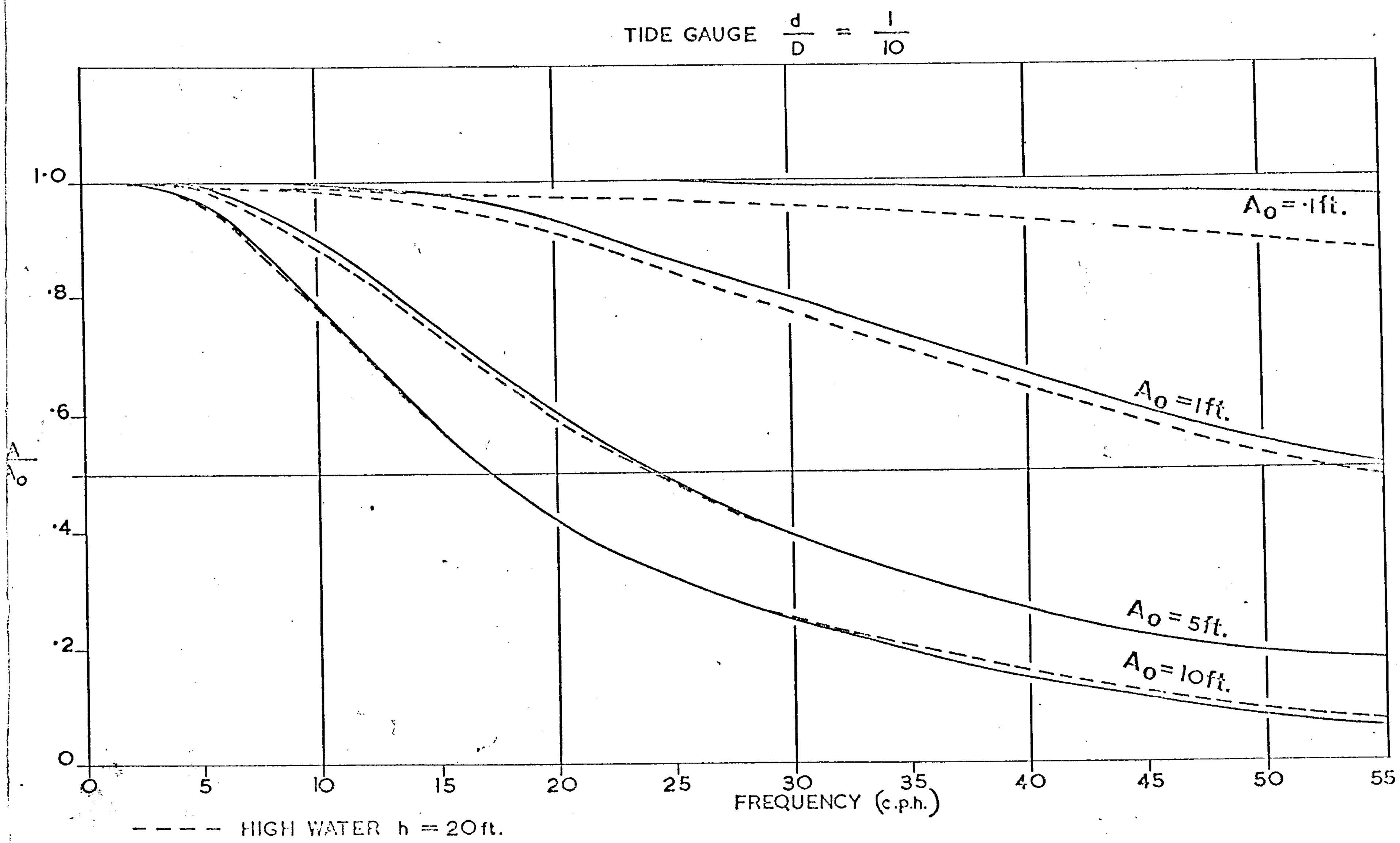
References.

Doodson, A.T., and Warburg, H.W., 1941 : Admiralty Manual of Tides, H.M. Stat. Off., 81 pp.

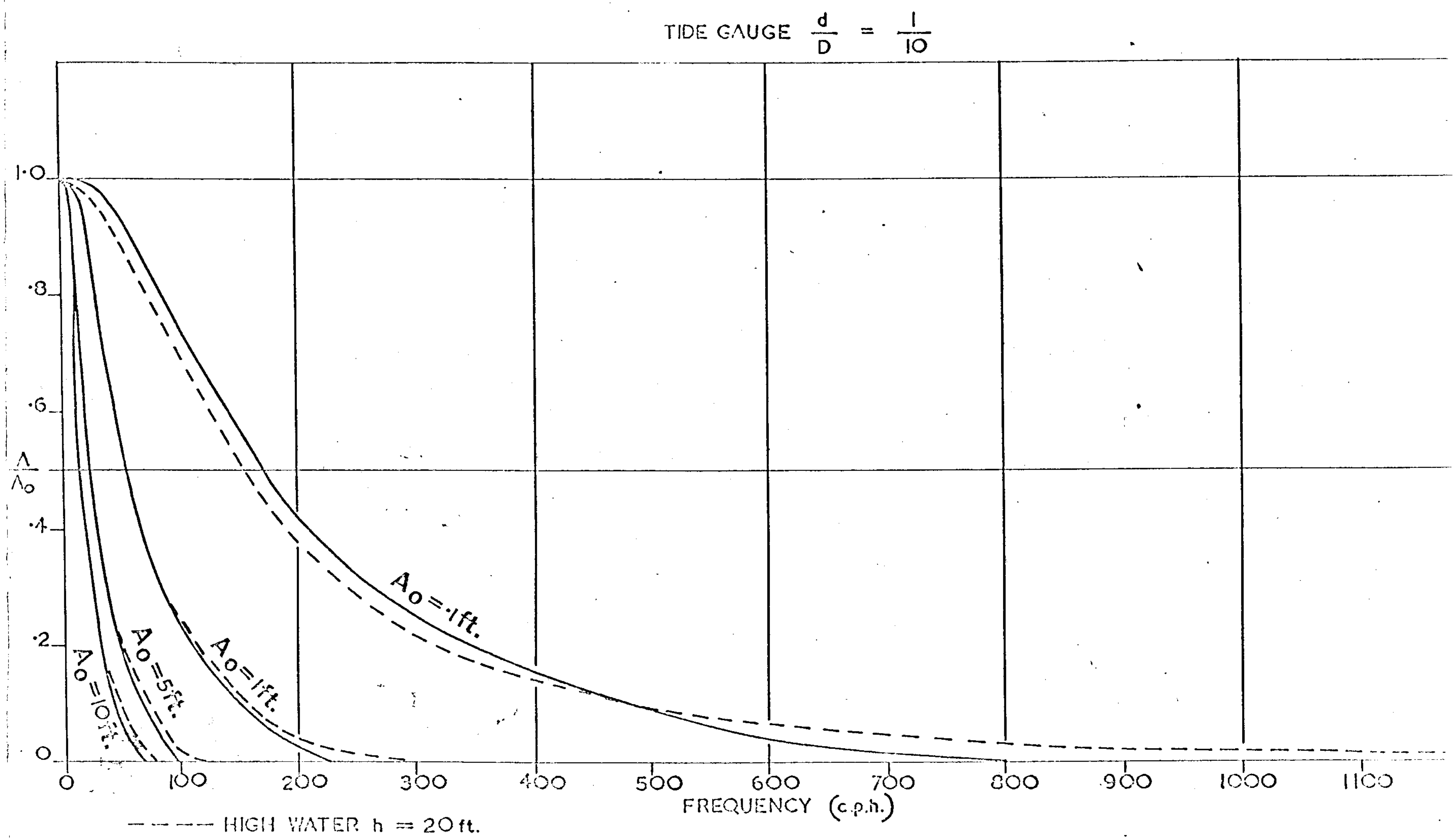
Jeffreys, H., and Jeffreys, B.S., 1950 : Methods of Mathematical Physics, Camb. Univ. Press Cambridge, 515 pp.

Proudman, J., 1953 : Dynamical Oceanography, Methuen, London, 374 pp.

Shipley, A.M., 1963 : Dtsch. Hydrogr. Z., 16, 136-140.



TIDE GAUGE d/D = 1/10
1·0
·8
·6
·4
·2
0
A/A₀
0
5
10
15
20
25
30
35
40
45
50
55
FREQUENCY (c.p.h.)
A₀ = ·1ft.
A₀ = 1ft.
A₀ = 5ft.
A₀ = 10ft.
– – – – HIGH WATER h = 20ft.



TIDE GAUGE $\frac{d}{D} = \frac{1}{10}$
$\frac{A}{A_o}$
1·0
·8
·6
·4
·2
0
A_o=·1ft.
A_o=1ft.
A_o=5ft.
A_o=10ft.
0
100
200
300
400
500
600
700
800
900
1000
1100
FREQUENCY (c.p.h.)
— — — HIGH WATER h = 20 ft.

Figure 1. Example of "Aliassing"

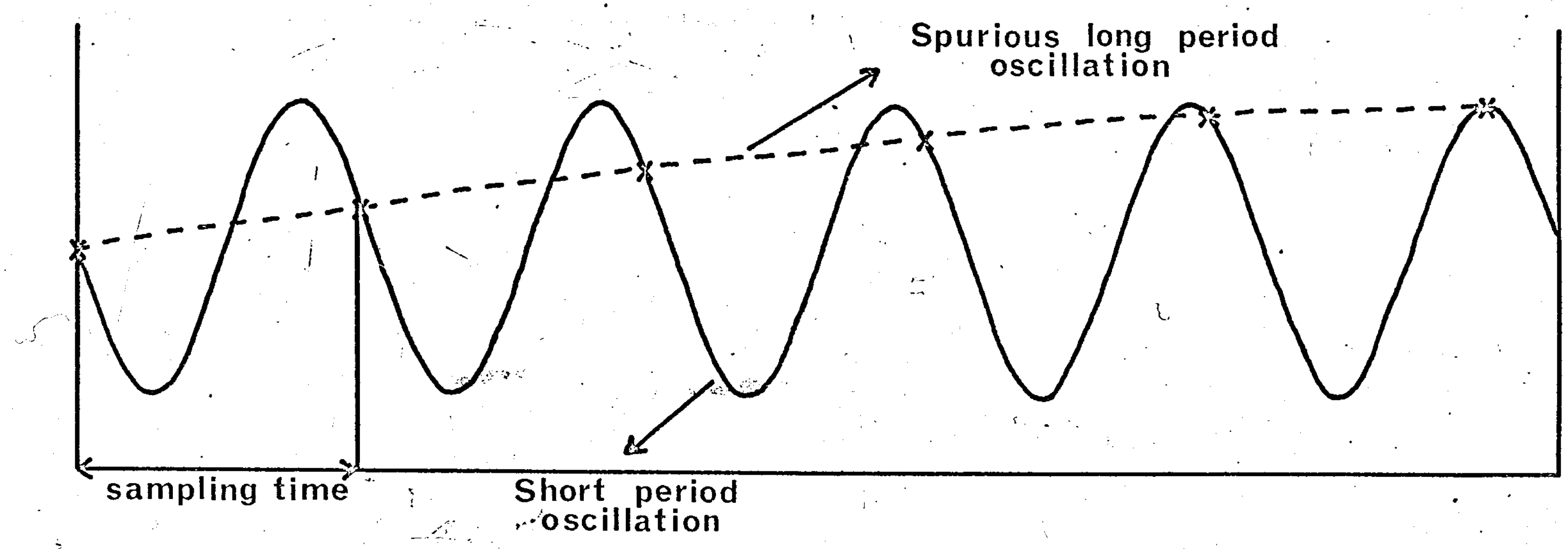


Figure 2. TRANMERE GAUGE

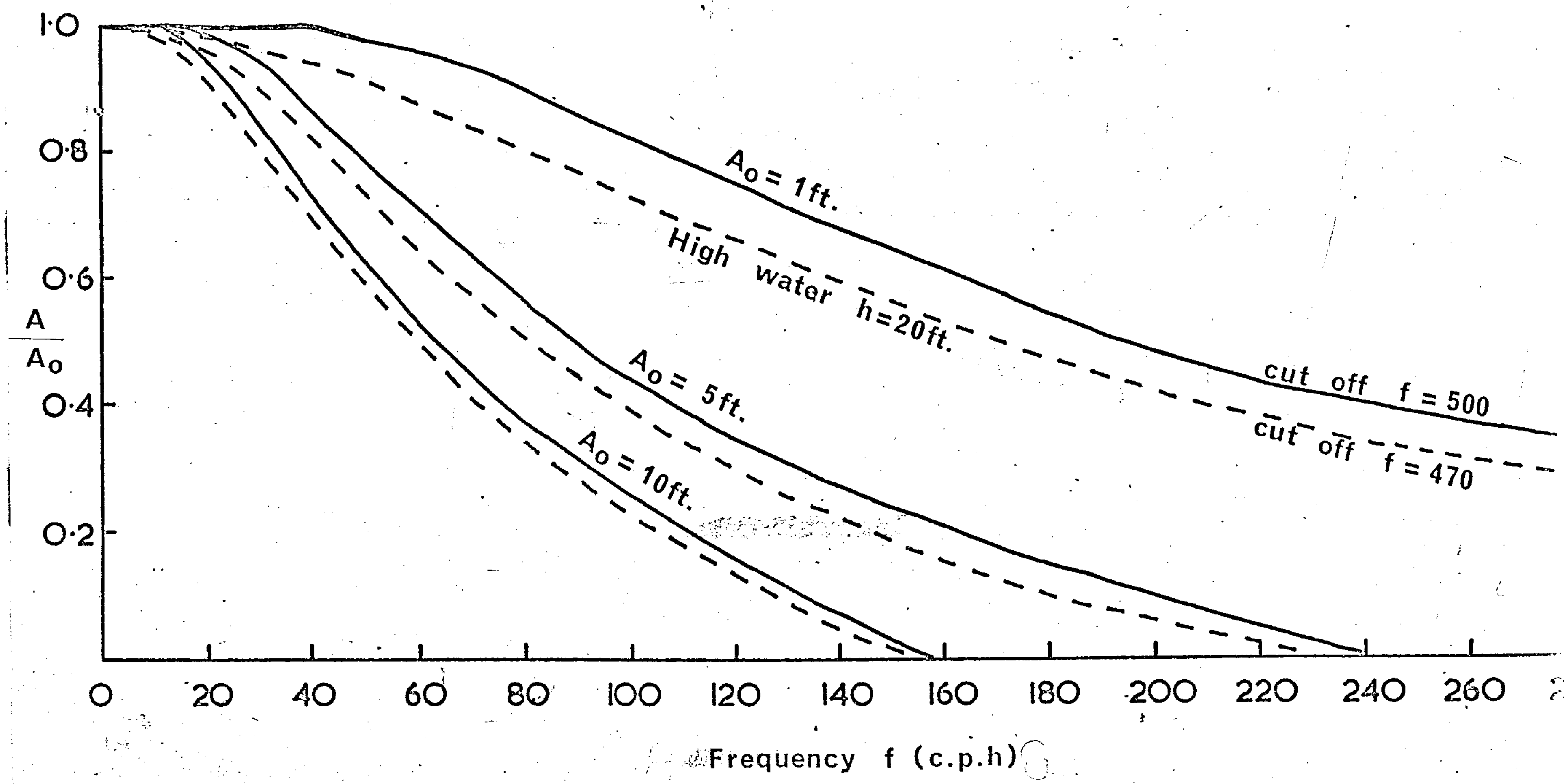


Figure 3

TRANMERE GAUGE $D = 8''$, $d = 1\frac{1}{2}''$

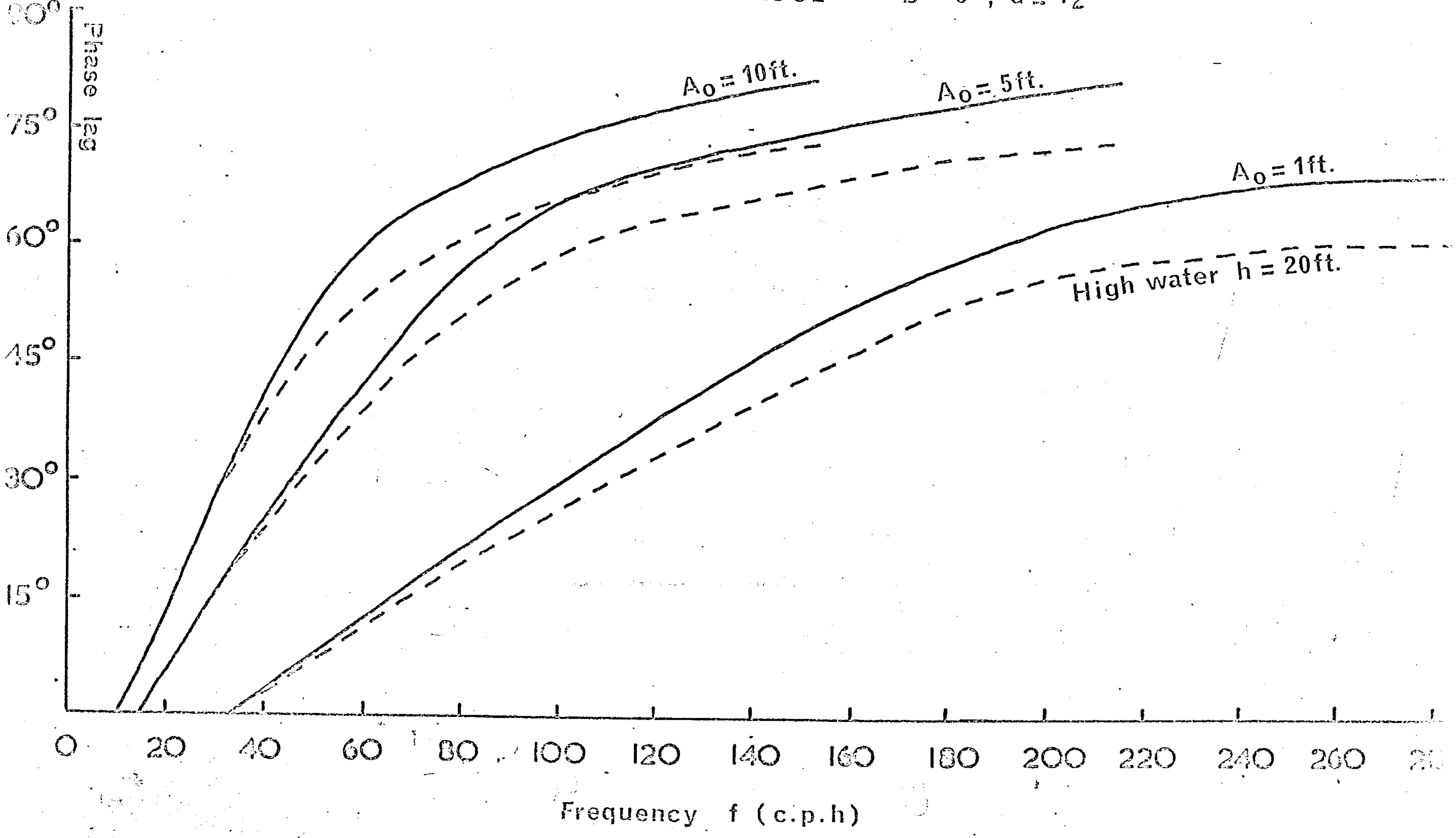


Figure 4

GLADSTONE GAUGE $D = 5'$, $d = 8 \times 1''$

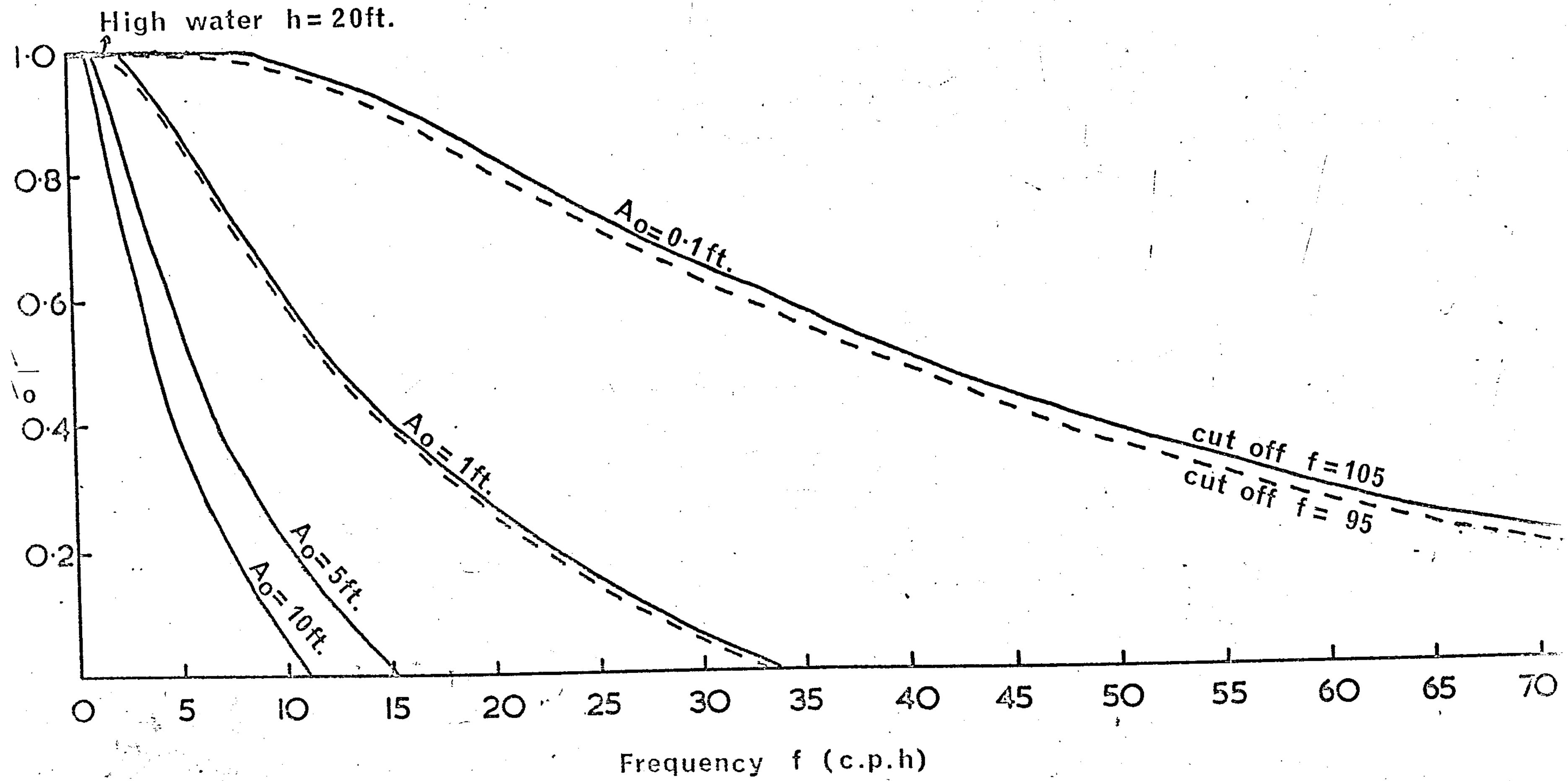


Figure 5 GLA STONE GAUGE D= 5′, d = 8 x 1″