A Bayesian Approach for Predicting Risk of Autonomous Underwater Vehicle Loss during their Missions

Corresponding author address: Southampton Business School, University of Southampton, University Road, Southampton SO17 1BJ, UK

E-mail: m.p.brito@soton.ac.uk

Tel: +44 2380597583

Abstract: Autonomous Underwater Vehicles (AUVs) are effective platforms for science research and monitoring, and for military and commercial data-gathering purposes. However, there is an inevitable risk of loss during any mission. Quantifying the risk of loss is complex, due to the combination of vehicle reliability and environmental factors, and cannot be determined through analytical means alone. An alternative approach – formal expert judgment – is a time-consuming process; consequently a method is needed to broaden the applicability of judgments beyond the narrow confines of an elicitation for a defined environment. We propose and explore a solution founded on a Bayesian Belief Network (BBN), where the results of the expert judgment elicitation are taken as the initial prior probability of loss due to failure. The network topology captures the causal effects of the environment separately on the vehicle and on the support platform, and combines these to produce an updated probability of loss due to failure. An extended version of the Kaplan Meier estimator is then used to update the mission risk profile with travelled distance. Sensitivity analysis of the BBN is presented and a case study of Autosub3 AUV deployment in the Amundsen Sea is discussed in detail.

Keywords: Bayesian networks, survival statistics, expert judgment elicitation, autonomous vehicles.

1. Introduction

Autonomous Underwater Vehicles (AUVs) have a future as effective platforms for science research and monitoring, and for military and commercial data-gathering purposes. Increasingly they are being used in environments that are not benign[1, 2]. Environments such as under sea ice [3], under shelf ice [4], or along rocky coasts [5] intuitively give rise to a higher risk of loss should the vehicle malfunction. The risk of loss is real; for example, Australian and British AUVs have been lost under ice sheets [6] and one team maintained a lightweight tether to an AUV when operating under sea ice. The problem of predicting risk of loss is not only one of predicting the reliability of the vehicle as a whole, its sub-systems and its components, but also of how the operating environment, together with reliability, sets the probability of losing the vehicle. It is not obvious that an approach based on separate statistical analyses of vehicle reliability and the affects of the environment on probability of loss is either feasible or meaningful. Such an approach, when reduced to summary statistics such as mean time to failure, would ignore the interaction between individual faults or incidents and the environment, which we postulate to be at the centre of this problem.

One alternative would be to assess the probability of loss in various environments directly, by counting the frequency of occurrence. This frequentist approach is certainly appropriate for assessing the reliability of identical engineered systems, where probability of failure is derived from a long-run frequency of occurrence, usually from the study of many items in use. Such an approach is the foundation for general reliability handbooks [7]. This is also the approach taken for obtaining reliability statistics in the offshore industry, for example the OREDA database [8], first published in 1984 [9]. However, this methodology "does not give the designer or manufacturer any insight into, or control over, the actual causes of failure since the cause-and-effect relationships impacting reliability are not captured" [10]. It is precisely that cause-and-effect between vehicle fault or incident and the environment that we seek to establish.

In [11], the authors present a risk management process tailored to AUV deployment in extreme environments. The method was used to support the decision to deploy the Autosub 3 AUV underneath an ice shelf, the Pine Island Glacier, Amundsen Sea, Antarctica in 2009 and later in 2013 [12]. Expert judgment was sought to quantify the likelihood of loss given a fault, and the experts' supporting text provided insights into possible causes and effects. The expert judgments were aggregated using mathematical analytical methods In contrast to the simple, yet high risk, case of AUV operation under an ice shelf, operations in other environments pose more complex risk scenarios, examples include under sea ice and coastal operations. Furthermore, the risk is often modified by the characteristics of the support platform. There is a set of AUV mission circumstances, therefore, where the range of factors is sufficiently large that it would be impracticable to ask an expert panel to review and assess every possibility. A method is needed to estimate risk under different conditions that minimizes the call on external experts, yet is well founded on their judgments.

We propose a three-stage approach to predicting risk of loss of an AUV during a mission in an environment that is different from that agreed as the nominal conditions. The first stage uses the formal process of eliciting expert judgment to quantify the likelihood of each failure leading to loss under a set of nominal conditions [13-15].

The second stage generalizes the experts' judgments to a new operating environment. For this stage a solution founded on a Bayesian Belief Network (BBN) approach [16] is proposed as it is an accepted method for modelling complex probability problems where it is possible to establish a causal relationship between domain variables [17, 18]. The design of the network topology captures the causal effects of the environment separately on the vehicle and on the support platform (e.g. a ship), and combines these to produce the output. For our example environment of under sea ice, we use the ASPeCt sea ice characterization protocol [19] and probability distributions of ice thickness and concentration within a rigorous process to quantify risk given a range of sea ice conditions and with ships of differing ice capabilities. Complementary expert knowledge is included within the conditional probability tables of the BBN. In [20] we showed how a BBN model can be combined with Monte-Carlo simulation to generate risk 'envelopes' for the AUV operation. The role of the BBN here was to update cumulative risk distributions for a given operational environment. This cumulative distribution would then be integrated in a Monte-Carlo framework to randomly generate Kaplan Meier survival plots of the AUV survivability with distance. This approach ignored the criticality of specific faults. A fault that was once considered of high criticality could later be deemed of low criticality and vice-versa. The approach presented in this paper is a significant improvement on previous work because instead of using the BBN for updating the cumulative risk profile for a given environment and operational constraints we show how the BBN can be used for updating the likelihood of loss for a failure for a given environment and operational conditions. This required fine-tuning of the conditional probability tables.

In the third stage, the extended Kaplan Meier estimator is used for updating the risk profile in light of the revised probability of loss given failure.

2. Autonomous Underwater Vehicle Risk Modelling and Analysis for Extreme Environment Missions

Our Autonomous Underwater Vehicle risk model is based on the vehicle's intrinsic failure history and expert judgments on the impacts of failure in the target operating environment. As subjective probability is a belief assessment on the likelihood of a hypothesis being true, this will differ between individuals when the uncertainty is epistemic, that is, due to imperfect knowledge. There remains controversy among statisticians over the validity of subjective probability, between the frequentists and the adherents of Bayes' theorem [13]. However, O'Hagan and colleagues argue that "this controversy does not arise" for the practical elicitation of subjective probability [13]. Hence, in this work a formal process of eliciting expert judgment was followed [21].

2.1 Nominal Risk Models for Open Waters, Coastal Waters, Sea ice and Ice shelf

Several formal expert judgment elicitation methods have been developed over the years [14]. For this work, we draw upon the formal expert judgment elicitation that was conducted in order to build the risk model for Autosub 3 deployment underneath the Pine Island Glacier. The static risk model was based on expert judgment on the criticality of each failure in the failure history [21, 22]. The subsequent analysis sought to identify biases arising from various causes [21, 23]. When making probability assessments, people tend to follow a number of mental shortcuts, denoted as heuristics, these may be based on how quickly the occurrence of an identical event comes to mind, or the impact of the event or how one anchors his or her assessment to a known event. Representativeness, availability and anchoring are the most common type of heuristics. Research has shown that people can introduce biases when following heuristics [23, 24]. Reference cases for risk of loss in different environments were obtained from an earlier study in which ten independent experts were asked to consider the simple question, "What is the probability of loss of the Autosub3 AUV in the given environment X given the fault/incident Y?" [14]. X comprised four example environments: open water, coastal, under sea ice, under ice shelf and Y comprised the set of 63 faults/incidents recorded on 29 missions to April 2007; 10 missions had no faults or incidents. The experts, from the USA, Canada and Australia, had a wide range of backgrounds, encompassing academic research (two graduate students and two full professors, both with polar experience, working on AUVs), research laboratories (three experts, two with polar experience), military research and development (two experts), and industry (one expert, with polar experience).

The full results are contained in a detailed 198-page report, available online [15], which contains nearly 2000 individual judgments, together with the expert's own assessment of their confidence in making each judgement. The authors augment the experts' reasons for their judgments with a commentary on points of agreement and disagreement, and conclude that where there were bimodal distributions, the experts seem to have fallen into two camps - optimists and pessimists. While noting these differences of opinion, the overall aggregated outcome was formed using a linear opinion pool for each fault or incident[21]. The final

results were visualized as relative frequency distributions for the assigned probabilities for each environment, Figure 1.

<Figure 1 goes here>

In reaching their judgments on the risk when operating in these four environments the experts were provided with brief descriptions of the characteristics of each environment that affect risk of loss. However, each expert also drew on their own knowledge of the operating environments, and in the supporting comments to their judgements gave reasons for reaching their probability of loss estimate for each fault. Rather than mathematically aggregating these judgements into a single probability of loss in each environment, which would over-simplify the assessment, and give a false sense of confidence, Figure 1 shows the probability frequency distribution for each environment from the judgements of the experts.

For sea ice, the experts were asked to keep in mind an area of first year ice (0.3–2.0m thick), 50% ice concentration, with ice keels to 15m, sporadic icebergs and a ship capable of breaking 2m ice at 2kt. These parameters form the particular reference case – the prior information for the Bayesian approach – for the motivating example in this study. To help extend the risk modelling to other ice conditions, the experts' judgments on risk in open water and under ice shelf are important. In the Bayesian sense they provide new information as they bound the risks under the two extremes of sea ice conditions. Where there is a high fraction of open water between the sea ice, the risk should tend to that of open water. That will also be the case with thin films of ice, including frazil, shuga and grease ice [19]. The risk here is virtually independent of ship ice-breaking capability, these forms of ice posing no difficulty to even a low-capability vessel. In contrast, where multi-year ice is present, and the open water fraction is small, the risk will tend towards that of operating under an ice shelf, with the risk markedly dependent on the ice breaking capability of the support ship. These effects are captured in the Bayesian Belief Network topology proposed in section 3.

2.2 Survival Modelling and analysis

The expert judgment on the criticality of failures in specific environments enables the identification of critical design and operational failures. This model alone does not allow the quantification of the probability of survival with travelled distance. Statistical survival modelling is a well known approach for representing systems' survival with operating time or distance. This method is based on the assumption that survival data can be classed as censored, that is, the system survived at least a known time or distance, or not censored, this is, the system is known to have failed at a given time or distance. This binary approach to modelling survival data does not suit the use of expert subjective judgment. In [11] an extended Kaplan Meier statistical survival estimator survival was proposed that combined the expert judgment on the likelihood of failure

1   leading to loss and the distance at which the failure has emerged to build a survival profile for vehicle loss in a

2   given environment. This extended Kaplan-Meier estimator is:

3
$$\hat{S}(r) = \prod_{r_i < r} 1 - \left(\frac{1}{n_i} \right) P(e_i) \qquad (1)$$

4   Where n_i is the number of events at risk at range r_i , and $P(e_i)$ the probability of fault leading to loss. Thus

5   if $P(e_i)$ is zero we have a censored case; that is, no loss is observed during the interval of interest. If $P(e_i)$

6   equals one, loss is observed during the interval of interest. For these extremes, the approach reduces to the

7   original version of the Kaplan-Meier method [25].

8

3. The Bayesian Belief Network topology

A Bayesian Belief Network is a graphical representation of a set of random variables (the nodes) together with directed interconnecting links (arcs). The arrow forming an arc sets the direction of a causal relationship between parent and child [26].

For the BBN discussed here, each node has a set of discrete states, either numeric or as ordered descriptions. At each node, a conditional probability table (CPT) captures the relationships between the states of the parent nodes and those of each child node. These conditional probabilities are assigned by an expert or a panel of experts, based on their knowledge of the interactions between the factors described by the parent nodes and how they affect the child nodes.

A BBN model is typically composed of target, intermediate and observable nodes. Target nodes are nodes that represent variables for which we will compute a probability distribution. Observable nodes represent variables that are measurable or directly observable; for example, ice thickness, ice concentration and vessel characteristics. Intermediate nodes are mainly defined to help manage the size of the conditional probability tables, adding transparency by representing hidden variables or highlighting hidden interactions between variables. The following sections show how causal relations between observable, intermediate and target nodes representing variables for this problem domain were established for a probabilistic model to be defined and used to predict AUV risk of loss under sea ice.

3.1 Network Design

In designing a BBN the essential causal influences need to be included. However, there are advantages to achieving a compact network, which results in tractable CPTs. Before detailing a network design for the motivating example of under sea ice AUV operations, the generality of this approach is demonstrated by two examples from other scenarios, Figure 2.

<center>< Figure 2 goes here ></center>

The first example, Figure 2a, is from a scenario where the AUV is tasked with long-range unattended exploration of a mid-ocean ridge using sonar and sampling systems [27]. The revised probability of loss is affected by two factors: recovery effectiveness and AUV Susceptibility, represented as intermediate nodes. Recovery effectiveness acknowledges that the distance between the AUV and the base may be some hundreds of kilometres and that the availability of a suitable ship for an unscheduled recovery may be uncertain. The observable nodes being an index of ship availability, which may be an ordered set, and a numeric AUV-Base separation distance. The AUV Susceptibility when on the surface and in mid-water would be captured in an appropriate risk reference case, which would also capture the risk for a defined reference seabed morphology. In the BBN, the risk due to the reference seabed morphology would be modified by the expected seabed roughness and slope probability density functions at the particular exploration site. It may well be the case that there would be insufficient information to generate the slope and roughness within the

1 working area, especially at a sufficiently high resolution appropriate to assessing risk to the AUV. In such a

2 case, the uncertainty would be expressed through a wider slope-roughness probability distribution.

3 The second example, Figure 2b, captures key risk elements when working in coastal waters. Recovery

4 effectiveness is affected by the capability of the ship or craft and the metocean conditions, which would

5 include the currents, wind, visibility and waves at the time of recovery. The AUV Susceptibility from an

6 appropriate reference case is modified by the particular operating environment, here characterized by the

7 prevailing metocean conditions, the seabed slopes and roughness, an index of underwater obstacles or

8 hazards such as kelp and an index of surface traffic. In both these examples, the risk reference case as

9 determined by the expert judgment process needs to be sufficiently close to the scenario of interest to be

10 meaningful. That is, an open water risk reference case would be inappropriate.

11 A prototype network that captures the essence of AUV operation in sea ice is shown in Figure 3, and is used

12 here as the basis for a detailed worked example. The observable nodes are: Vessel Characteristics, Ice

13 Concentration, Ice Thickness and Risk Reference Case. Other observable nodes could be added in the future

14 such as Floe Size or Separation of Vessel and AUV. The chosen topology, with each child node having only two

15 parents, results in CPTs of the lowest possible dimension.

16

17 <Figure 3 goes here >

18

19 Ice Concentration and Ice Thickness are combined twice, into two separate intermediate nodes, one

20 describing the Vessel Environment Constraints, the other AUV Environment Constraints. Both CPTs at these

21 nodes output a set of five ordered states: Very low, Low, Moderate, High and Very high. These are combined

22 with the Vessel Characteristics and Risk Reference Case observable nodes to evaluate Vessel Effectiveness

23 and AUV Susceptibility. The Vessel Effectiveness CPT outputs an ordered set with five states: 0 nautical miles

24 per hour (kts), 1.5kts, 3kts, 5kts, 10kts. The AUV Susceptibility node outputs sixteen probability class states,

25 which are combined with the five Vessel Effectiveness states in the final P(loss) CPT to give sixteen probability

26 class states as the output. The number of states chosen for each node provides an adequate representation

27 of the data.

28 **3.2 Observable Node Data**

29 **3.2.1 Vessel Characteristics**

30 There are many individual parameters that are available to describe the characteristics of ships operating in

31 Polar Regions, including among others tonnage, propulsion power, and the presence or absence of induced

32 roll tanks. It would be possible to use such multiple characteristics as the input to the Vessel Effectiveness

33 node, but the dimensions of the CPT would be large. Instead, to simplify the problem, the vessel

34 characteristics are summarized within a simple ordered set. Unfortunately, there is no universal standard set

of descriptions, associated classes and class designators for ice capable ships, hence there is no mandated single set of ordered descriptions. For its simplicity and accessibility, the system chosen for this study is the Russian Maritime Register of Shipping Rules (2003) for independent navigation in arctic seas [28]. The ordered set of descriptions for ice classes LU4 (least capable) to LU9 (most capable) forms the state set for this node. Accepting that equivalents may not be exact between these rules and those of the American Bureau of Shipping or Lloyds Register among others, examples of polar vessels and classes from which it is conceivable to deploy an AUV are:

LU4 – RV L. M. Gould (USA)

LU5 – RRS James Clark Ross (UK)

LU7 – RV Nathaniel B. Palmer (USA)

LU8 – RV Polarstern (Germany)

LU9 – NS Yamal (Russian Federation)

When using the BBN, and the vessel is known, the vessel class may be instantiated, that is, the class probability is set to 100%. If there were uncertainty over which class of vessel may be used two or more vessel classes may be included to capture the increased range of risk that this would imply.

3.2.2 Ice Concentration and Ice Thickness

The ordered set for ice concentration covers 0% to 100%, representing, to the nearest 10%, the fraction of the ocean surface covered by ice of any type. Spot observations using the ASPeCt code [19] would give a single ice concentration value, which would be instantiated. However, when it is required to estimate risk over a mission, or over a period of time, where several ice environments would be traversed by the AUV, a representative ice concentration frequency distribution would be derived over a set of observations of primary and subsidiary ice types.

Ice thickness is an ordered set that has values of 0, 0.1, 0.5, 1, 2 and 5 m in the prototype. This captures a subset of the ice classes from the ASPeCt code covering early development, first– and multi–year ice. Thickness values can be instantiated, or a frequency distribution used.

3.2.3 Risk Reference Case

The risk reference case for a nominal probability of loss, P(loss) is that determined by the group of experts though eliciting their judgments [14] represented as a relative frequency distribution. To keep the CPTs tractable, the risk reference case distribution is grouped into sixteen classes, where f=P(loss|failure):

$0 < f < 0.0001$	$0.0001 \leq f < 0.0003$	$0.0003 \leq f < 0.001$	$0.001 \leq f < 0.003$
$0.003 \leq f < 0.01$	$0.01 \leq f < 0.03$	$0.03 \leq f < 0.1$	$0.1 \leq f < 0.2$
$0.2 \leq f < 0.3$	$0.3 \leq f < 0.4$	$0.4 \leq f < 0.5$	$0.5 \leq f < 0.6$
$0.6 \leq f < 0.7$	$0.7 \leq f < 0.8$	$0.8 \leq f < 0.9$	$f \geq 0.9$

This set of values provides sufficient range and resolution to capture risk of loss in all of the environments from open water to ice shelf.

3.3 Populating the Conditional Probability Tables

Given the high-dimensional conditional distribution required to populate some CPTs, we followed the common practice of making strong assumptions about the form of the conditional probability distribution [28]. The key assumptions made to design the CPTs were as follows; all of the CPTs are provided as tables in the Supplementary Material.

3.3.1 Environment constraints on the AUV

The CPT for this intermediate node was based on the sensitivity analysis of the BBN model output. The CPT had to comply with two conditions to ensure plausibility of the results. The aim here was to ensure that the two scenarios of singularity, the most benign operating scenario, open water and the most critical scenario, ice shelf, were points of singularity in the BBN model. These conditions are presented below.

Let ν be the random variable that represents the environment constraints on the AUV, the following assumptions were implemented in the CPT:

$$P(\nu = very\ low \mid \vartheta_O, \zeta_O) = 1 \ \wedge \ P(\nu = very\ high \mid \vartheta_I, \zeta_I) = 1 \quad (2)$$

where the suffix 'O' stands for open water environment and 'I' for ice shelf environment. ϑ represents the ice thickness set and ζ the ice concentration set.

The reference case corresponds to a first-year sea ice scenario (0.3–2.0m thick), with ice keels to 15m, and sporadic icebergs, at a concentration of 50% [14, 15]. To capture the reference case, the following condition was set on this node.

$$P(\nu = low \mid \vartheta = 2m, \zeta = 50\%) = 0.5 \ \wedge \ P(\nu = mod\ erate \mid \vartheta = 2m, \zeta = 50\%) = 0.5 \quad (3)$$

3.3.2 Environment constraints on the Vessel

The reference support vessel was deemed able to break ice up to 2m thick. Let υ be the random variable that represents the environment constraints on the vessel, the following assumptions were implemented in the CPT:

$$P(\upsilon = mod\ erate \mid \vartheta_r, \zeta_r) = 1 \ \wedge \ P(\upsilon = very\ low \mid \vartheta_O, \zeta_O) = 1 \ \wedge$$
$$P(\upsilon = very\ high \mid \vartheta_I, \zeta_I) = 1 \quad (4)$$

where the suffix 'r' stands for reference case, 'O' stands for open water environment and suffix 'I' for ice shelf environment. ϑ represents the ice thickness set and ζ the ice concentration set.

3.3.3 AUV Susceptibility

This CPT captures the effect of different environment conditions on the reference AUV probability of loss distribution. If conditions considered in the reference case are observed the AUV Susceptibility distribution should be identical to the probability of loss distribution elicited for the reference sea ice case. This is:

$$P(\phi = P_{range} \mid \nu = \mod erate_r, P_{ref} = P_{range}) = 1 \quad (5)$$

Where ϕ stands for AUV Susceptibility, P_{range} is the probability range and P_{ref} is the reference probability of loss.

Ice concentration and ice thickness are the two factors considered. The CPT captures scenarios such as (a) damage to navigation and relocation appendages such as GPS antennas and acoustic beacon transducers that can occur with modest ice thickness, and whose frequency of occurrence increases with ice concentration, (b) damage to the AUV hull affecting its buoyancy or water-tight integrity, which will be minimal at ice thicknesses of 0.1m and less, but becomes substantial at 0.5m and above, with ice concentration again affecting the probability of occurrence, (c) if the AUV surfaces under ice, its ability to receive a navigation fix would be compromised, and the acoustic propagation conditions could be poor, leading to difficulties in locating the vehicle, affected by both thickness and concentration.

Other key assumptions were:

1) As the environmental conditions approach those of an ice shelf the AUV Susceptibility distribution should be become identical to the P(loss) distribution elicited for ice shelf conditions [15], where the judgments elicited for ice shelf are known.

2) Similarly, if open water conditions were observed the AUV Susceptibility should approach a P(loss|failure) distribution as elicited for open water [15].

3.3.4 Vessel Effectiveness

Vessel class LU7 most closely resembles the type of vessel set out in the reference case. Environment constraints have a negative effect on vessel effectiveness, whilst there is a positive association between vessel class and vessel effectiveness. This node quantifies the progress that the ship can make under given environment conditions. This is measured in terms of speed. The node has five states: 0 kts, 0.5 kts, 3 kts, 5 kts and 10kts. Where kts stands for nautical mile per hour. The underlying assumption was that if the environment constraint was deemed moderate the updated risk estimates reflected the reference case. Vessel effectiveness was reduced sharply as the ice thickness became comparable with the standard for each class, weighted by the sea ice concentration. Table entries ensured that at low ice concentrations effectiveness was high, and less susceptible to either ice thickness (as the vessel would be able to find open water easily) or vessel class (as icebreaking capability would not be called upon).

3.3.5 Revised Probability of Loss

To define the CPT for this node, the main question addressed was: What is the effect of the vessel effectiveness on the AUV Susceptibility? Given the probability judgments on AUV Susceptibility are in a particular range; will the vessel effectiveness move the probability judgments lower or higher? An important assumption was that, if the vessel effectiveness was deemed as 3 kts the output P(loss) distribution should be identical to the reference case. Additionally, if the vessel effectiveness was deemed lower than 3 kts, a

1   fraction of the probability judgments were moved to a range of greater risk. Conversely, when vessel

2   effectiveness was deemed higher than 3 kts a fraction of the probability judgments moved towards a range of

3   lower risk.

4

4.   Updating Mission Survival Estimates

The new methodology proposed in this paper uses the BBN model to update a single judgment, for the probability of loss given fault. In order to update a single judgment the BBN must read each judgment as a frequency distribution, over the sixteen classes defined earlier. This, therefore, becomes a discretization problem.

The result of the elicitation exercise [15] shows that the expert judgments can be as small as 0.0001 or as high and precise as 0.98. This wide range of judgments does not accommodate the easy use of Fuzzy set theory. A more suitable solution would be to use a continuous variable to model individual expert judgment. In addition to the actual probability judgment, the user must also specify a variance. Most BBN tools allow the user to specify continuous variables in the form of a Gaussian distribution [29].

The Beta distribution offers an alternative and more suitable approach for modeling expert judgments [13]. The mean (μ_j) and variance (σ_j^2) for the Beta distribution can be obtained using the formulation presented in (6) and (7).

$$\mu_j = \frac{\alpha}{\alpha + \beta} \qquad (6)$$

$$\sigma_j^2 = \frac{\alpha\beta}{(\alpha+\beta)^2 \cdot (\alpha+\beta+1)} \qquad (7)$$

Equations (6) and (7) are central to the risk updating method proposed in this paper. Given that the risk judgment for a particular failure is known, the hyper-parameters of the Beta distribution (α and β) can be estimated using (6) and (7). The expressions for obtaining alpha and beta parameters are presented in (8) and (9).

$$\alpha = \frac{\mu_j^2}{\sigma_j^2} - \frac{\mu_j^3}{\sigma_j^2} - \mu_j \qquad (8)$$

$$\beta = \frac{\alpha}{\mu_j} - \alpha \qquad (9)$$

A discrete distribution over the sixteen probability judgment classes can then be created using the Beta distribution. These are steps 1 and 2 of the risk updating process; automatically executed using Matlab. Then, a series of sequential steps must be performed in order to complete the risk updating process. In brief:

1.   Encode probability judgment in a Beta distribution. For each fault, a pooled expert probability judgment and its variance are used to calculate the hyper-parameters of the Beta distribution, using (8) and (9).

2.   Discretization of the Beta distribution. A discretization algorithm is used to create a probability distribution over all sixteen classes of the 'Nominal PLoss' Node.

3.   Bayesian inference for updating P(loss). The discretized probability distribution is used to instantiate the 'Nominal PLoss' node. The 'Revised P(loss)' outputs the updated PLoss judgment.

4.   Fit a Beta distribution to the 'Revised P(loss)'. The maximum likelihood algorithm is used to fit a Beta probability function to the 'Revised P(loss)' distribution.

5.   Mean and variances for the 'Revised P(loss)' are calculated using equations (1) and (2).

6.   Once steps 1 to 5 have been carried out for all failures. The probability of survival with distance is calculated using the extended Kaplan Meier method.

5.   Using the network to reason with risk

The inference algorithms embedded within most commercially available BBN tools can support four types of reasoning: predictions, diagnostics, combined and intercausal [16]. Here, we are interested in using the BBN for computing predictions as to how factors described in the previous section influence and revise the probability of loss given a fault. The software tool's inference algorithm propagates the observable evidence through the network updating the belief in the states of its child nodes; a relative frequency distribution is calculated for each of the latter nodes, conditioned on all of the hard and soft evidence.

This section provides two examples that demonstrate the use of the BBN model topology presented in Figure 3 to estimate risks in under sea ice AUV missions. The model was implemented in Hugin 6.5 [29, 31](there is no connection with the Hugin AUV).

5.1 Historic: Greenland – Autosub2 on the James Clark Ross

This example is taken from an Autosub2 cruise to North East Greenland in 2004 [32]. The vessel capability state is instantiated to LU5, representing the capability of the James Clark Ross. We choose, as an example, 23–24 August 2004 when the 12-hour, 80km distance run Autosub2 mission 366 took place in the vicinity of 80˚ 3'N 14˚ 23' W.

Hourly ice observations were made by members of the science party trained in using the ASPeCt code. Overall ice concentration varied from 0–60%. At the start of the vehicle's mission the primary ice type was fast ice 1.5m thick, with the secondary type being first-year ice floes also 1.5m thick at the edge of the fast ice field. The vehicle then traversed beneath a region of scattered multi-year floes, 2m thick, then beneath a region of higher-concentration of first year floes before reaching open water for its recovery. This information is captured in the node probability tables of the observable nodes Ice Concentration and Ice Thickness.

Figure 4 shows the BBN topology with node state tables corresponding to a set of states after the reasoning embedded within the CPTs. In this example, the output probability distribution is skewed towards lower probability of loss compared with the reference case. While the vessel is of lower capability (LU5) than the reference (LU7), and 60% of the ice present was 1m or greater in thickness, at the upper end or exceeding the vessel's breaking ability, for the mission as a whole, ice concentration was low with 87% at a concentration of 30% or below.

The probability of loss given fault is considered to be that assessed for failure 387_1 of Autosub2. In this failure, Autosub2 failed to home in to the acoustic command sent by the ship, the vehicle headed off in an

uncontrolled direction. This fault was caused by combination of uncalibrated receiver array and a network failure. The probability of loss given this fault, for the optimistic group is 0.0189 [15].

The CPT assessed the Vessel Effectiveness as predominantly able to progress at 10 kts. The high open water fraction reduced the direct risk to the AUV over the reference case, with the median risk class reducing by one class from 0.03–0.1 to 0.01 to 0.03. When combined with Vessel Effectiveness, this reduced risk was carried through to the output probability distribution.

<Figure 4 goes here >

5.2 Predictive: Amundsen Sea – Autosub3 on the Nathaniel B. Palmer

This example draws upon historic sea ice thickness and concentration measurements from Autosub2 in the Amundsen Sea, Antarctica [33] during its mission 323 to predict the risk for the vehicle's successor – Autosub3 – on missions in 2009 in the same general area, but operating from the RV Nathaniel B. Palmer. Figure 5 shows the BBN topology with node state tables corresponding to a set of states given in each observable node after the reasoning embedded within the CPTs.

The support vessel is more capable in this second example (LU7 rather than LU5) and equal to the reference case. However, the ice concentration is higher at 100%. The ice thickness distribution had a strong peak at 2m with a small contribution from multiyear ice over 2m thick. In combination, the Ice Concentration and the Ice Thickness lead to more severe Environmental Constraints on the AUV compared with the Greenland case, with 68% rated 'high' compared with 0%. Vessel effectiveness was predominantly below 0.5kts in these conditions. In the resulting revised assessment of P(loss) the mode, at ~34%, was in the risk class of]0.01, 0.03] compared with the mode at ~ 38% for the same class in the reference case. The 95% quantile has jumped two risk classes, increasing from]0.03, 0.1] to]0.2, 0.3]. As a risk profile, under these conditions the BBN outcome is more akin to that of the experts' judgment for operations under an ice shelf, Figure 1.

<Figure 5 goes here >

6.   Sensitivity analysis

There are many approaches available for modelling uncertainty. But no method can claim to be 100% infallible. Each method needs to be tailored to a specific problem. Nevertheless, when developing knowledge-based systems, it is imperative to ensure that the model is complete and coherent. Validation takes a different form from what is typically considered for deterministic systems. Knowledge-based systems encode expert knowledge, thus validation implies assessing whether the model outputs are consistent with the expert judgements. Here the sensitivity analysis consisted of running several simulations to check whether model predictions comply with a set of axioms that formalise the expected plausibility.

This section presents a summary of the sensitivity analysis conducted for the model presented in section 3. We use three axioms to guide the sensitivity analysis. These axioms capture rules that the BBN model must meet and were defined to demonstrate the validity of the conditional probability tables of the intermediate nodes.

a. Axiom 1. An icebreaker can break ice up to its maximum specification at a speed of 3kts. If the ice thickness, is greater than the vessel's specified ice thickness then the vessel can navigate only by ramming the ice, during which the vessel speed is 0.5kts. In mathematical notation: $T(x_i) > T(x_v) => V(x_i) = 0.5kts$.

This axiom can be verified by setting the ice concentration to 10/10 and varying the ice thickness from 0m to 5m.

Figure 6 shows the results of the sensitivity studies conducted on the node vessel effectiveness. We considered two vessel classes: LU7 and LU9, results for which are presented on the left and right plots respectively.

<Figure 6 goes here>

For open water conditions, results show that both vessels can travel at full speed of 10kts. For an ice thickness of 0.1m both vessels can make similar progress, the same for 0.5m ice thickness.

A vessel travels at a speed of 3kts when it is breaking ice thickness that meets its capability. Results show that while for the LU7 vessel the mode of the distribution is at ice thickness of 1m for the LU9 type vessel the mode of the distribution is at 2m. The ramming speed is different for both vessels. The output shows that an LU7 vessel starts ramming ice at an ice thickness of 2m while an LU9 vessel is likely to start ramming at an ice thickness of 5m. An LU7 class vessel runs to a stop when the ice thickness reaches 5m.

b. Axiom 2. The AUV Susceptibility is a monotonically increasing function with ice thickness. If ice concentration $I(xi) > I(xj)$ and $T(xi) = T(xj)$, then $P(xi) > P(xj)$.

1　The AUV Susceptibility variable captures the influence of the environment on the AUV probability of loss

2　given fault estimate. To verify this axiom, the nominal probability of loss was instantiated with state] 0.03,

3　0.1] and in a second simulation with state]0.1, 0.2]. Two sets of simulations were conducted. One simulation

4　was conducted with ice concentration set to 50% and one simulation was conducted with ice concentration

5　set to 100%

6　Figure 7 shows the results of running the BBN model for all combinations of ice thickness. The contour plots

7　were generated using the contour function of Matlab R2011b. Figure 7a shows the distribution of the AUV

8　Susceptibility node when the nominal Probability of loss is instantiated to] 0.03, 0.1] and ice concentration is

9　50%. Figure 7b shows the distribution of the AUV Susceptibility node when the nominal Probability of loss is

10　instantiated to]0.03, 0.1] and ice concentration is 100%. Figure 7c shows the AUV Susceptibility distribution

11　when the nominal Probability of loss is instantiated to]0.1, 0.2] and ice concentration is instantiated to 50%.

12　Figure 7d presents the distribution of AUV Susceptibility for the same nominal Probability of loss but ice

13　concentration of 100%.

14　The results show that for ice concentration of 50%, for ice thickness between 0.5m-2m, nominal sea ice

15　environment, the AUV Susceptibility stays in the same range as nominal P(loss). Results here show that axiom

16　2, AUV Susceptibility is monotonically increasing with the ice concentration.

17　<Figure 7 goes here>

18

19　*c. Axiom 3. The AUV Susceptibility is a monotonically increasing function with ice concentration. If ice*

20　*concentration I(xi) > I(xj) and thickness T(xi) = T(xj), then P(xi) > P(xj).*

21

22　To verify this axiom, the BBN model was instantiated with a range for the nominal probability of loss and an

23　ice thickness value. The ice concentration was then increased from 0% to 100%. We present the result of

24　running these simulations for nominal probability of loss ranged between]0.03, 0.1] and]0.1, 0.2]. Figure 8

25　shows the contour graphs of the BBN output for the AUV Susceptibility for four ice thickness conditions: 0.5m

26　(8a and 8b), 1m (8c and 8d), 2m (8e and 8f) and 5 m (8g and 8h). The contour graphs show that the AUV

27　Susceptibility remains constant only when nominal sea ice conditions are met. For all other instance the AUV

28　Susceptibility is monotonically increasing with ice concentration.

29

30　<Figure 8 goes here>

31　The BBN output for AUV Susceptibility for nominal Probability of loss in the range of]0.1, 0.2], ice

32　concentration of 100%, ice thickness of 5m, gives 37% confidence that the AUV Susceptibility is in the range

33　of]0.3, 0.4], 20% confidence that the AUV Susceptibility is in the range of]0.4, 0.5] and 4% confidence that

the AUV Susceptibility is in]0.6, 0.7]. Ice shelf has an ice thickness greater than 5m. The simulation results show that the BBN model captures the effect of extreme operation conditions.

7. Updating Survival Estimates - Amundsen Sea – Autosub3 on the Nathaniel B. Palmer

To show the application of the method proposed in this paper we consider the judgments obtained for the optimistic group of the judgments provided for sea ice [11]. The observable nodes 'Vessel', 'Ice concentration' and 'Ice thickness' were set with the states depicted in Figure 5.

Table I, in the Appendix, shows the results of the updated risk judgments for each of the 63 failures used to create the Autosub3 risk model [11]. A variance must be specified for all failures. For judgments where the variance was 0 (where experts all agreed on a single value), we decided to assign a variance of 10-7; this occurred for 10 out of 63 failures.

The updated risk model allows us to quantify the criticality of each fault when the AUV is deployed in a new operating environment. This information is important since it allows decision makers to address the question of whether or not it is cost effective to completely remove a fault.

Quantitatively, from Figure 9, the probability of survival drops quickly in the first 32km. This suggests a mitigation strategy, however, a full discussion of mitigation measures are beyond the scope of this paper; examples have been discussed elsewhere [15, 34], including ensuring a period of monitoring the AUV for a set distance in waters where recovery can be effected should there be a problem at the start of a mission. Such a strategy proved effective during the Autosub3 campaign under Pine Island Glacier, Antarctica in 2009.

8.   Conclusion

The importance of this work is that it provides a structured approach to AUV risk management in hazardous environments or where loss could lead to damage to, or contamination of, the environment. Such an approach will be essential for the use of expensive or high profile AUVs for science in the Polar Regions, and is likely to prove necessary for AUVs that may be used by the offshore industry in the Arctic. The process draws upon an extensive and time-consuming exercise in eliciting expert judgment and has sought to maximize use of the pooled opinions by applying Bayesian reasoning to extend the applicability of the judgments beyond the original settings. Its use for AUV missions under sea ice is particularly appropriate; in that the BBN copes with the indeterminate or probabilistic elements and with uncertainty – for example, at the mission planning stage we may, or we may not, know what vessel will provide support.

A more detailed analysis would involve assessing each failure separately and quantifying the effect of the vessel effectiveness, ice concentration and ice thickness on each particular failure. Such a study would help us to build a more precise probability distribution. However, for the purpose of this paper, the aim is to expose BBNs as a means to capture arguments of this nature as well other arguments relevant to the AUV risk prediction in Polar missions.

The approach can be extended to other factors affecting AUVs under sea ice by adding to the observable nodes, for example, including observations on ice keels and icebergs, seabed topography including ice and strudel scour if operating in shallow water, distance from support vessel and availability of rescue or support tools such as an ROV, helicopter or acoustic navigation. Other AUV operating environments with complexity and uncertainty posing risk can be similarly modeled.

It is possible, that for some operating areas there is a combination of variation of two or more environments, for example, a combination of coastal, Pc, and sea ice environments, Pi. In this case the combined probability of loss can be calculated using the following expression: $Pt = 1 – (1\text{-}Pc)*(1\text{-}Pi)$.

Fundamental to this work is the source data on reliability and faults and incidents with the vehicle. Accurate and complete recording of this information is essential to assess and control risk on AUV missions.
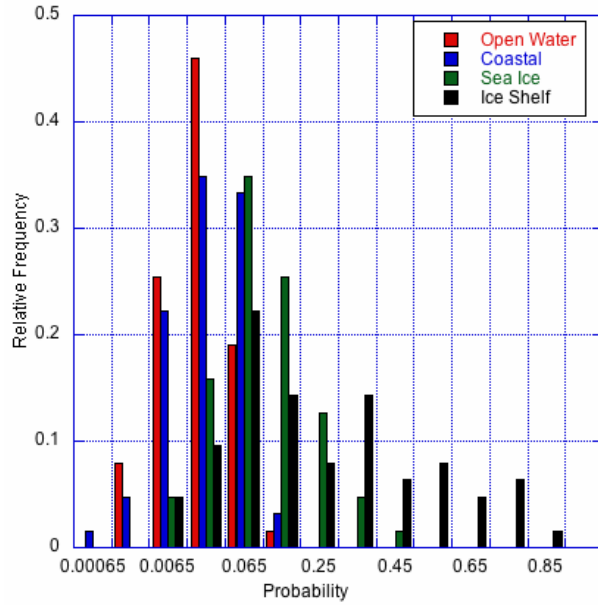
1 **FIGURES**

2

3 Figure 1. Frequency distribution of linear opinion pooled probabilities assigned by the experts for the set of 63 faults or
4 incidents in each of the four example environments. The data have been grouped into the probability classes used in the
5 BBN as described in section 3.

6
7
8
9
10
11
12
13
14
15
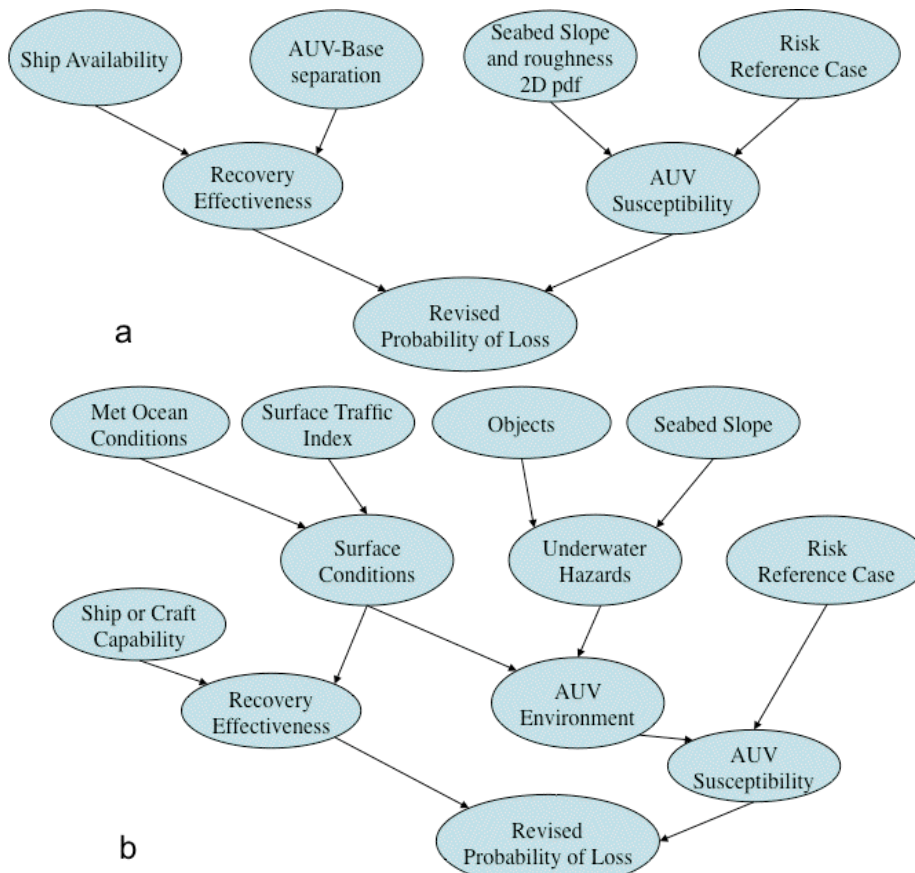
16
17
18
19
20
21
22
23
24
25
26
27
28
29 Figure 2. Topology for simple Bayesian Belief Networks that enable reasoning over the risk to an AUV. A) Scenario where
30 the AUV is used for long-range unattended exploration of mid-ocean ridges, B) for operations in coastal waters.

31
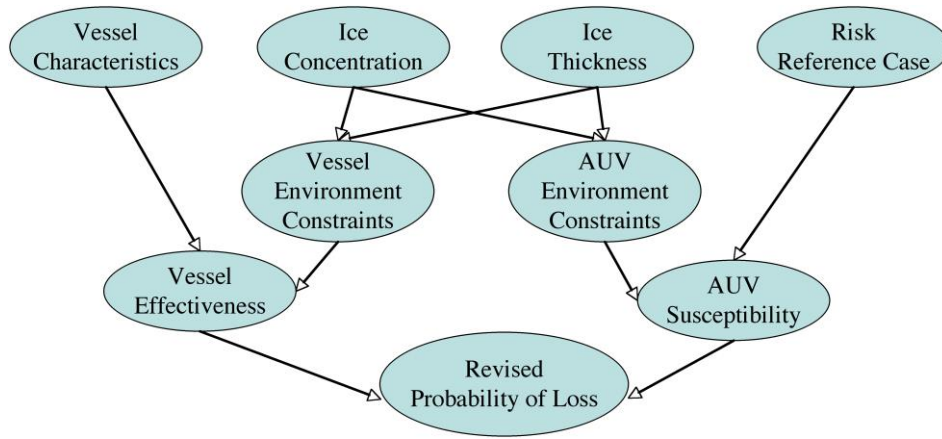
1
2 Figure 3. Topology for a simple Bayesian Belief Network that enables reasoning over the risk to an AUV in sea ice
3 environments when operating from a support vessel.
4
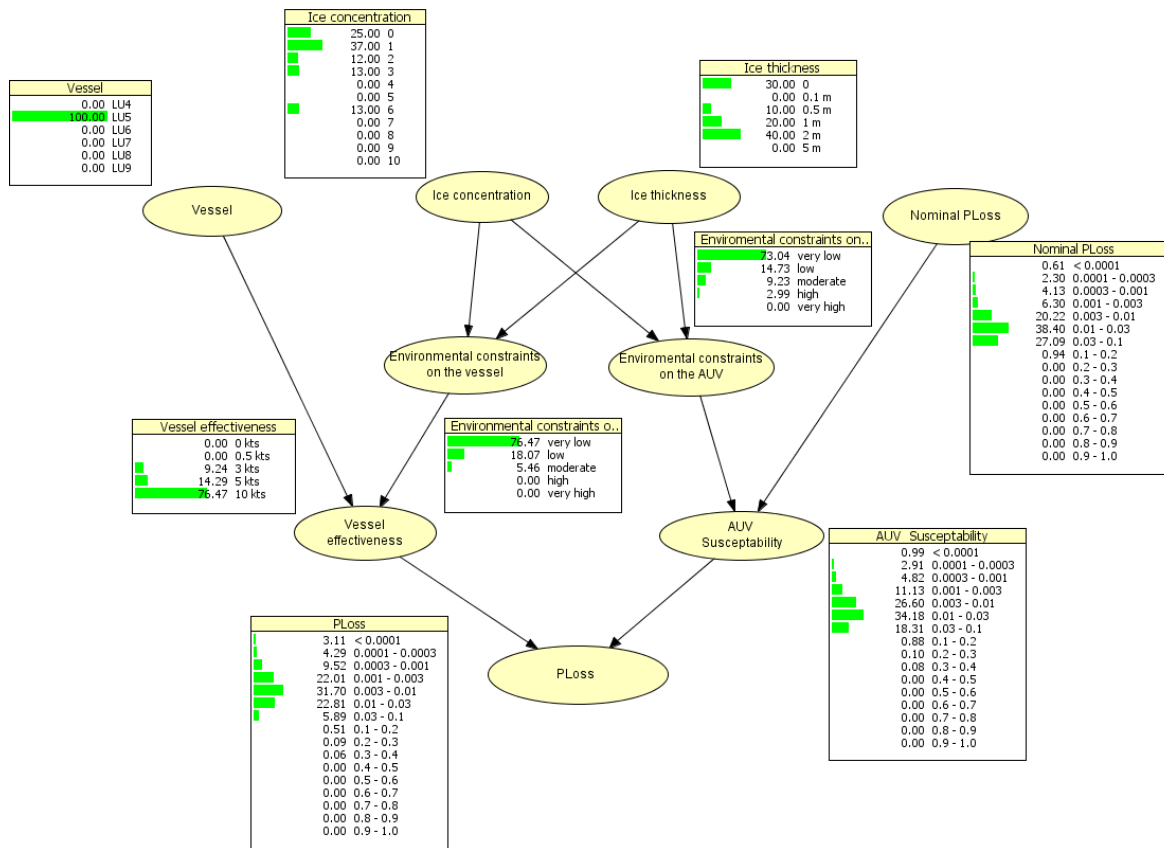
5
6 Figure 4. Bayesian Belief Network for evaluating risk to an AUV in sea ice environments showing the input states to the
7 four root nodes for Autosub2 operation from the RRS *James Clark Ross* off NE Greenland on 23–24 August 2004. The
8 states of the child nodes are shown. Fault 387_1.
9
10
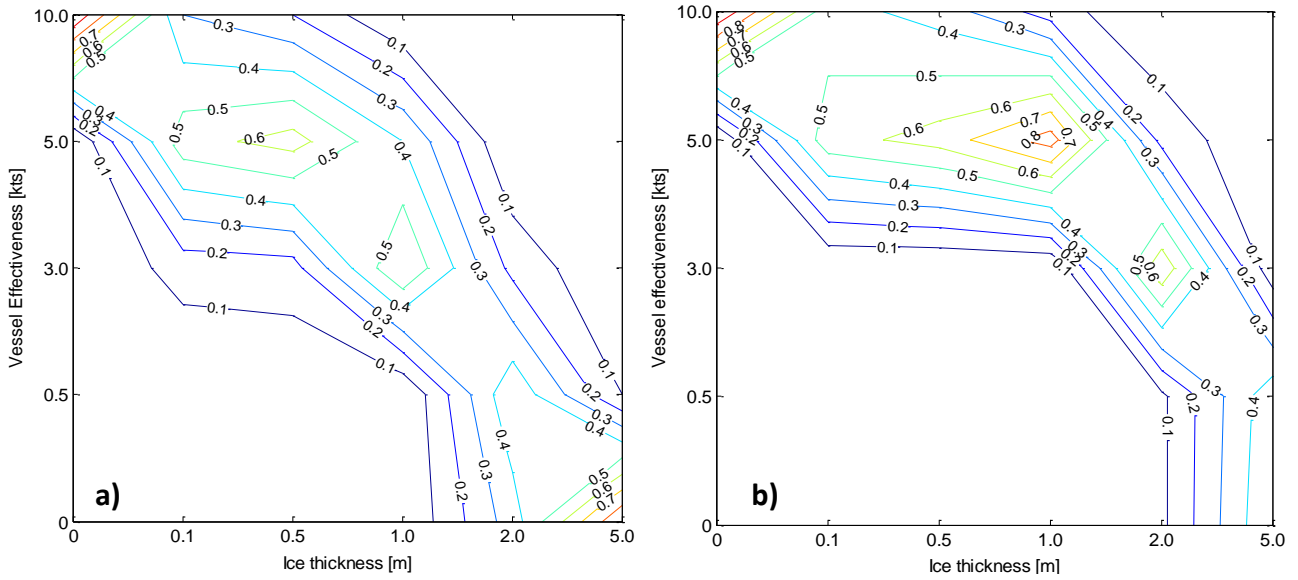
Figure 6. Variation of vessel speed with ice thickness for ice concentration of ten tenths. On the left is the distribution for vessel class LU7, on the right is the distribution for vessel class LU9.
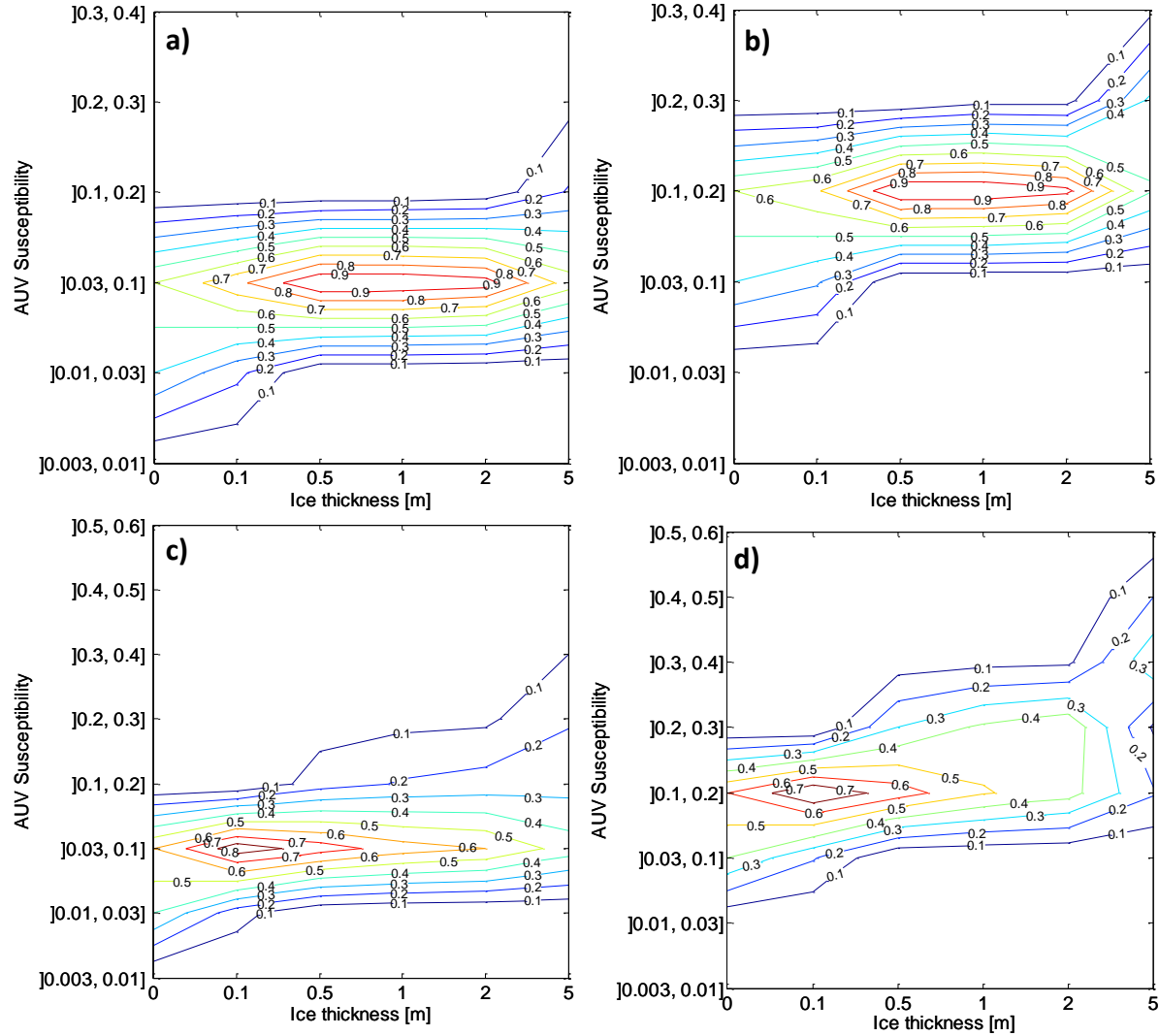
Figure 7. Variation of AUV Susceptibility with ice thickness. a) Nominal P(loss) instantiated at]0.03, 0.1], ice concentration set to 50%. b) Nominal P(loss) instantiated at]0.03, 0.1], ice concentration instantiated to 100%. c) Nominal P(loss) instantiated at]0.1, 0.2], ice concentration set to 50%. d) Nominal P(loss) instantiated at]0.1, 0.2], ice concentration set to 100%.

Figure 8.   Variation of AUV Susceptibility with ice concentration. a) Ice thickness of 0.1m, P(loss) instantiated at]0.03,0.1]; b) Ice thickness of 0.5m, P(loss) instantiated at]0.1,0.2]; c) Ice thickness of 0.5m, P(loss) instantiated at]0.03,0.1]; d) Ice thickness of 0.5m, P(loss) instantiated at]0.1, 0.2]; e) Ice thickness of 1m, P(loss) instantiated at]0.03,0.1]; f) Ice thickness of 1m, P(loss) instantiated at]0.1,0.2]; g) Ice thickness of 2m, P(loss) instantiated at]0.03,0.1]; a) Ice thickness of 2m, P(loss) instantiated at]0.1, 0.2].

1
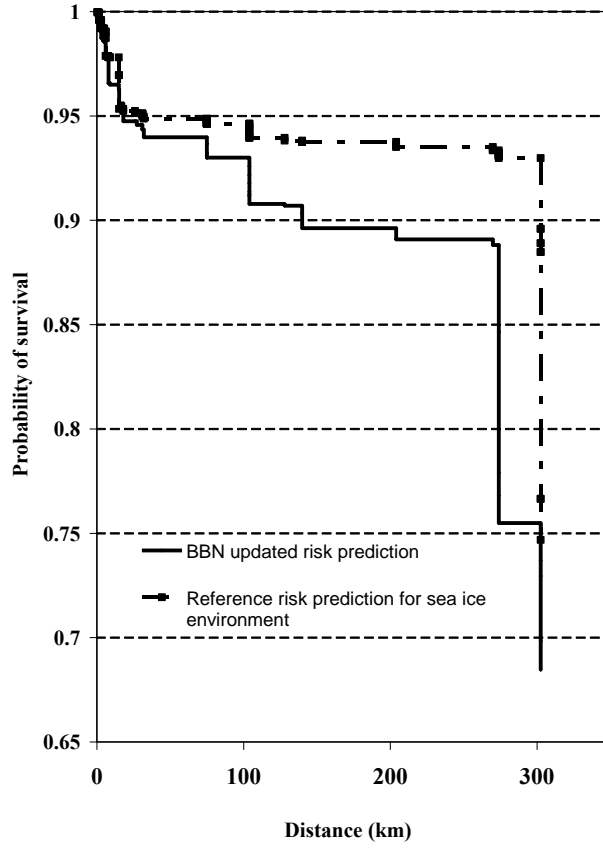
Figure 9. Probability of survival with range.

2
3
4
5

1 **Supplementary Material**

2 Table 1 Environment constraints on AUV CPT.

Ice thickness	0 m										
Ice concentration	0	10	20	30	40	50	60	70	80	90	100
Very low	1	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91
Low	0	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
Medium	0	0	0	0	0	0	0	0	0	0	0
High	0	0	0	0	0	0	0	0	0	0	0
Very High	0	0	0	0	0	0	0	0	0	0	0

3 Table 1 Environment constraints on AUV CPT. Cont.

Ice thickness	0.1 m										
Ice concentration	0	10	20	30	40	50	60	70	80	90	100
Very low	1	0.9	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.45
Low	0	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55
Medium	0	0	0	0	0	0	0	0	0	0	0
High	0	0	0	0	0	0	0	0	0	0	0
Very High	0	0	0	0	0	0	0	0	0	0	0

4 Table 1 Environment constraints on AUV CPT. Cont.

Ice thickness	0.5 m										
Ice concentration	0	10	20	30	40	50	60	70	80	90	100
Very low	1	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.45	0.4	0.35
Low	0	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65
Medium	0	0	0	0	0	0	0	0	0	0	0
High	0	0	0	0	0	0	0	0	0	0	0
Very High	0	0	0	0	0	0	0	0	0	0	0

5

6 Table 1 Environment constraints on AUV CPT. Cont.

Ice thickness	1 m										
Ice concentration	0	10	20	30	40	50	60	70	80	90	100
Very low	1	0.7	0.65	0.6	0	0	0	0	0	0	0
Low	0	0.3	0.35	0.4	0.7	0.65	0.6	0.55	0.5	0.1	0
Medium	0	0	0	0	0.3	0.35	0.4	0.45	0.5	0.9	1
High	0	0	0	0	0	0	0	0	0	0	0
Very High	0	0	0	0	0	0	0	0	0	0	0

7 Table 1 Environment constraints on AUV CPT. Cont.

Ice thickness	2m										
Ice concentration	0	10	20	30	40	50	60	70	80	90	100
Very low	1	0.6	0.55	0.5	0	0	0	0	0	0	0
Low	0	0.4	0.45	0.5	0.6	0.5	0.3	0.2	0	0	0
Medium	0	0	0	0	0.4	0.5	0.5	0.45	0	0	0
High	0	0	0	0	0	0	0.2	0.35	0.75	0.65	0.55
Very High	0	0	0	0	0	0	0	0	0.25	0.35	0.45

8 Table 1 Environment constraints on AUV CPT. Cont.

Ice thickness	5m										
Ice concentration	0	10	20	30	40	50	60	70	80	90	100
Very low	1	0.5	0.45	0.4	0	0	0	0	0	0	0
Low	0	0.5	0.55	0.6	0	0	0	0	0	0	0
Medium	0	0	0	0	0	0	0	0	0	0	0
High	0	0	0	0	0.6	0.5	0.35	0.2	0.05	0.05	0
Very High	0	0	0	0	0.4	0.5	0.65	0.8	0.95	0.95	1

9
10

1 Table 2. Environment Constrains on the vessel CPT.

Ice thickness	0m										
Ice concentration	0	10	20	30	40	50	60	70	80	90	100
Very low	1	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91
Low	0	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
Medium	0	0	0	0	0	0	0	0	0	0	0
High	0	0	0	0	0	0	0	0	0	0	0
Very high	0	0	0	0	0	0	0	0	0	0	0

2 Table 2. Environment Constrains on the vessel CPT. Cont.

Ice thickness	0.1m										
Ice concentration	0	10	20	30	40	50	60	70	80	90	100
Very low	1	0.9	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.45
Low	0	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55
Medium	0	0	0	0	0	0	0	0	0	0	0
High	0	0	0	0	0	0	0	0	0	0	0
Very high	0	0	0	0	0	0	0	0	0	0	0

3 Table 2. Environment Constrains on the vessel CPT. Cont.

Ice thickness	0.5m										
Ice concentration	0	10	20	30	40	50	60	70	80	90	100
Very low	1	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.45	0.4	0.35
Low	0	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65
Medium	0	0	0	0	0	0	0	0	0	0	0
High	0	0	0	0	0	0	0	0	0	0	0
Very high	0	0	0	0	0	0	0	0	0	0	0

4 Table 2. Environment Constrains on the vessel CPT. Cont.

Ice thickness	1m										
Ice concentration	0	10	20	30	40	50	60	70	80	90	100
Very low	1	0.7	0.65	0.6	0	0	0	0	0	0	0
Low	0	0.3	0.35	0.4	0.7	0.65	0.6	0.55	0.5	0.1	0
Medium	0	0	0	0	0.3	0.35	0.4	0.45	0.5	0.9	1
High	0	0	0	0	0	0	0	0	0	0	0
Very high	0	0	0	0	0	0	0	0	0	0	0

5 Table 2. Environment Constrains on the vessel CPT. Cont.

Ice thickness	2m										
Ice concentration	0	10	20	30	40	50	60	70	80	90	100
Very low	1	0.6	0.55	0.5	0	0	0	0	0	0	0
Low	0	0.4	0.45	0.5	0.6	0.5	0.3	0.2	0	0	0
Medium	0	0	0	0	0.4	0.5	0.5	0.45	0	0	0
High	0	0	0	0	0	0	0.2	0.35	0.75	0.65	0.55
Very high	0	0	0	0	0	0	0	0	0.25	0.35	0.45

6 Table 2. Environment Constrains on the vessel CPT. Cont.

Ice thickness	5m										
Ice concentration	0	10	20	30	40	50	60	70	80	90	100
Very low	1	0.5	0.45	0.4	0	0	0	0	0	0	0
Low	0	0.5	0.55	0.6	0	0	0	0	0	0	0
Medium	0	0	0	0	0	0	0	0	0	0	0
High	0	0	0	0	0.6	0.5	0.35	0.2	0.05	0.05	0
Very high	0	0	0	0	0.4	0.5	0.65	0.8	0.95	0.95	1

7

1 Table 3. Vessel Effectiveness CPT.

Ice breaker class	C4=LU4				
Environment constraints	C3=very low	C3=low	C3=moderate	C3=high	C3=very high
0 kts	0	0	0	0.8	1
0.5 kts	0	0.143577	0.4	0.2	0
3 kts	0	0.251889	0.6	0	0
5 kts	0	0.604534	0	0	0
10 kts	1	0	0	0	0

2 Table 3. Vessel Effectiveness CPT. Cont.

Ice breaker class	C4=LU5				
Environment constraints	C3=very low	C3=low	C3=moderate	C3=high	C3=very high
0 kts	0	0	0	0.3	1
0.5 kts	0	0	0	0.6	0
3 kts	0	0.3	0.7	0.1	0
5 kts	0	0.7	0.3	0	0
10 kts	1	0	0	0	0

3 Table 3. Vessel Effectiveness CPT. Cont.

Ice breaker class	C4=LU6				
Environment constraints	C3=very low	C3=low	C3=moderate	C3=high	C3=very high
0 kts	0	0	0	0.3	0.95
0.5 kts	0	0	0	0.7	0.05
3 kts	0	0.7	0.8	0	0
5 kts	0	0.3	0.2	0	0
10 kts	1	0	0	0	0

4 Table 3. Vessel Effectiveness CPT. Cont.

Ice breaker class	C4=LU7				
Environment constraints	C3=very low	C3=low	C3=moderate	C3=high	C3=very high
0 kts	0	0	0	0.25	0.9
0.5 kts	0	0	0	0.55	0.1
3 kts	0	0.2	0.6	0.2	0
5 kts	0	0.8	0.4	0	0
10 kts	1	0	0	0	0

5 Table 3. Vessel Effectiveness CPT. Cont.

Ice breaker class	C4=LU8				
Environment constraints	C3=very low	C3=low	C3=moderate	C3=high	C3=very high
0 kts	0	0	0	0	0.8
0.5 kts	0	0	0	0	0.15
3 kts	0	0	0	0.8	0.05
5 kts	0	0.97	0.99	0.2	0
10 kts	1	0.03	0.01	0	0

6 Table 3. Vessel Effectiveness CPT. Cont.

Ice breaker class	C4=LU9				
Environment constraints	C3=very low	C3=low	C3=moderate	C3=high	C3=very high
0 kts	0	0	0	0	0.8
0.5 kts	0	0	0	0	0.2
3 kts	0	0	0	0.75	0
5 kts	0	0.8	0.85	0.25	0
10 kts	1	0.8	0.7	0	0

7

1   Table 4. AUV Susceptibility

NPLoss	"<=0.0001"				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	1	0.9	0.8	0.4	0.3
0.0001- 0.0003	0	0.1	0.2	0.6	0.7
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

2

3   Table 4. AUV Susceptibility. Cont.

NPLoss	0.0001- 0.0003				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0.1	0.7	0	0	0
0.0001- 0.0003	0.9	0.3	1	0.4	0.3
0.0003-0.001	0	0	0	0.6	0.7
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

4

5   Table 4. AUV Susceptibility. Cont.

NPLoss	0.0003-0.001				
EnvCA	Very low	Low	Moder	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0.3	0.2	0	0	0
0.0003-0.001	0.7	0.8	1	0	0
0.001-0.003	0	0	0	0.1	0
0.003-0.01	0	0	0	0.9	0.95
0.01-0.03	0	0	0	0	0.05
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

6
7

1 Table 4. AUV Susceptibility. Cont.

NPLoss	0.001-0.003				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0.35	0.2	0	0	0
0.001-0.003	0.65	0.8	1	0	0
0.003-0.01	0	0	0	0.5	0.4
0.01-0.03	0	0	0	0.5	0.6
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

2
3 Table 4. AUV Susceptibility. Cont.

NPLoss	0.003-0.01				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0.4	0.3	0	0	0
0.003-0.01	0.6	0.7	1	0.99	0.98
0.01-0.03	0	0	0	0.01	0.02
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

4
5 Table 4. AUV Susceptibility. Cont.

NPLoss	0.01-0.03				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0.4	0.3	0	0.055	0
0.01-0.03	0.6	0.7	1	0.9	0.558824
0.03-0.1	0	0	0	0.02	0.441176
0.1-0.2	0	0	0	0.025	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

6
7

1   Table 4. AUV Susceptibility. Cont.

NPLoss	0.03-0.1				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0.4	0.2	0	0	0
0.03-0.1	0.6	0.8	1	0.5	0.45
0.1-0.2	0	0	0	0.3	0.25
0.2-0.3	0	0	0	0.1	0.2
0.3-0.4	0	0	0	0.1	0.1
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

2

3   Table 4. AUV Susceptibility. Cont.

NPLoss	0.1-0.2				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0.4	0.3	0	0	0
0.1-0.2	0.6	0.7	1	0.3	0.2
0.2-0.3	0	0	0	0.6	0.03
0.3-0.4	0	0	0	0.1	0.405
0.4-0.5	0	0	0	0	0.221
0.5-0.6	0	0	0	0	0.0365
0.6-0.7	0	0	0	0	0.0475
0.7-0.8	0	0	0	0	0.04
0.8-0.9	0	0	0	0	0.02
0.9-1.0	0	0	0	0	0

4   Table 4. AUV Susceptibility. Cont.

NPLoss	0.2-0.3				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0.7	0.8	0	0	0
0.03-0.1	0.3	0.2	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	1	0.14	0.0145
0.3-0.4	0	0	0	0.06	0
0.4-0.5	0	0	0	0.8	0
0.5-0.6	0	0	0	0	0.13
0.6-0.7	0	0	0	0	0.29
0.7-0.8	0	0	0	0	0.4655
0.8-0.9	0	0	0	0	0.1
0.9-1.0	0	0	0	0	0

5

1   Table 4. AUV Susceptibility. Cont.

NPLoss	0.3-0.4				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0.2	0	0	0
0.1-0.2	0.4	0.8	0	0	0
0.2-0.3	0.6	0	0	0	0
0.3-0.4	0	0	1	0	0
0.4-0.5	0	0	0	0.222222	0.4
0.5-0.6	0	0	0	0.611111	0.4
0.6-0.7	0	0	0	0.166667	0.2
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

2
3   Table 4. AUV Susceptibility. Cont.

NPLoss	0.4-0.5				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0.4	0	0	0	0
0.1-0.2	0.6	0.8	0	0	0
0.2-0.3	0	0.2	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	1	0	0
0.5-0.6	0	0	0	0.7	0.4
0.6-0.7	0	0	0	0.3	0.6
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

4
5   Table 4. AUV Susceptibility. Cont.

NPLoss	0.5-0.6				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0.65	0.8	0	0	0
0.3-0.4	0.3	0.2	0	0	0
0.4-0.5	0.05	0	0	0	0
0.5-0.6	0	0	1	0	0
0.6-0.7	0	0	0	0.7	0
0.7-0.8	0	0	0	0.3	0.8
0.8-0.9	0	0	0	0	0.2
0.9-1.0	0	0	0	0	0

1   Table 4. AUV Susceptibility. Cont.

NPLoss	0.6-0.7				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0.05	0	0	0	0
0.3-0.4	0.75	0.8	0	0	0
0.4-0.5	0.2	0.2	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	1	0.1	0
0.7-0.8	0	0	0	0.6	0
0.8-0.9	0	0	0	0.3	0.8
0.9-1.0	0	0	0	0	0.2

2
3   Table 4. AUV Susceptibility. Cont.

NPLoss	0.7-0.8				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0.75	0.8	0	0	0
0.5-0.6	0.2	0.2	0	0	0
0.6-0.7	0.05	0	0	0	0
0.7-0.8	0	0	1	0.9	0
0.8-0.9	0	0	0	0.1	0.1
0.9-1.0	0	0	0	0	0.9

4
5   Table 4. AUV Susceptibility. Cont.

NPLoss	0.8-0.9				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0.75	0.8	0	0	0
0.6-0.7	0.2	0.2	0	0	0
0.7-0.8	0.05	0	0	0	0
0.8-0.9	0	0	1	0.8	0
0.9-1.0	0	0	0	0.2	1

1

2 Table 4. AUV Susceptibility. Cont.

NPLoss	0.9-1.0				
EnvCA	Very low	Low	Moderate	High	Very High
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0.4	0.35	0.222222	0	0
0.9-1.0	0.6	0.65	0.777778	1	1

3

4

1   Table 5. Probability of Loss CPT

Asus	"=< 0.0001"				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0.2	0.3	1	1	1
0.0001- 0.0003	0.8	0.7	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

2
3   Table 5. Probability of Loss CPT. Cont.

Asus	0.0001-0.0003				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0.7	0.8
0.0001- 0.0003	0.6	0.7	1	0.3	0.2
0.0003-0.001	0.4	0.3	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

4
5   Table 5. Probability of Loss CPT. Cont.

Asus	0.0003-0.001				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0.7	0.8
0.0003-0.001	0.6	0.7	1	0.3	0.2
0.001-0.003	0.4	0.3	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

6

1   Table 5. Probability of Loss CPT. Cont.

Asus	0.001 - 0.003				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0.7	0.8
0.001-0.003	0.6	0.7	1	0.3	0.2
0.003-0.01	0.4	0.3	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

2
3   Table 5. Probability of Loss CPT. Cont.

Asus	0.003 - 0.01				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0.7	0.8
0.003-0.01	0.6	0.7	1	0.3	0.2
0.01-0.03	0.4	0.3	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

4
5   Table 5. Probability of Loss CPT. Cont.

Asus	0.01 - 0.03				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0.7	0.8
0.01-0.03	0.6	0.7	1	0.3	0.2
0.03-0.1	0.4	0.3	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

1   Table 5. Probability of Loss CPT. Cont.

Asus	0.03 - 0.1				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0.7	0.8
0.03-0.1	0.6	0.7	1	0.3	0.2
0.1-0.2	0.4	0.3	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

2
3   Table 5. Probability of Loss CPT. Cont.

Asus	0.1 - 0.2				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0.55	0.6
0.1-0.2	0.6	0.7	1	0.45	0.4
0.2-0.3	0.4	0.3	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

4
5   Table 5. Probability of Loss CPT. Cont.

Asus	0.2 - 0.3				
Veff	0	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0.55	0.6
0.2-0.3	0.6	0.7	1	0.45	0.4
0.3-0.4	0.4	0.3	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

1 Table 5. Probability of Loss CPT. Cont.

Asus	0.3 - 0.4				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0.55	0.6
0.3-0.4	0.6	0.7	1	0.45	0.4
0.4-0.5	0.4	0.3	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

2
3 Table 5. Probability of Loss CPT. Cont.

Asus	0.4 - 0.5				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0.55	0.6
0.4-0.5	0.6	0.7	1	0.45	0.4
0.5-0.6	0.4	0.3	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

4
5 Table 5. Probability of Loss CPT. Cont.

Asus	0.5 - 0.6				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0.55	0.6
0.5-0.6	0.6	0.7	1	0.45	0.4
0.6-0.7	0.4	0.3	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

1   Table 5. Probability of Loss CPT. Cont.

Asus	0.6 - 0.7				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0.55	0.6
0.6-0.7	0.6	0.7	1	0.45	0.4
0.7-0.8	0.4	0.3	0	0	0
0.8-0.9	0	0	0	0	0
0.9-1.0	0	0	0	0	0

2
3   Table 5. Probability of Loss CPT. Cont.

Asus	0.7 - 0.8				
Veff	0 kts	0.5 kts	3	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001-	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0.55	0.6
0.7-0.8	0.6	0.7	1	0.45	0.4
0.8-0.9	0.4	0.3	0	0	0
0.9-1.0	0	0	0	0	0

4
5   Table 5. Probability of Loss CPT. Cont.

Asus	0.8 - 0.9				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0.7	0.8
0.8-0.9	0.8	0.9	1	0.3	0.2
0.9-1.0	0.2	0.1	0	0	0

1

2   Table 5. Probability of Loss CPT. Cont.

Asus	0.9 - 1.0				
Veff	0 kts	0.5 kts	3 kts	5 kts	10 kts
"<=0.0001"	0	0	0	0	0
0.0001- 0.0003	0	0	0	0	0
0.0003-0.001	0	0	0	0	0
0.001-0.003	0	0	0	0	0
0.003-0.01	0	0	0	0	0
0.01-0.03	0	0	0	0	0
0.03-0.1	0	0	0	0	0
0.1-0.2	0	0	0	0	0
0.2-0.3	0	0	0	0	0
0.3-0.4	0	0	0	0	0
0.4-0.5	0	0	0	0	0
0.5-0.6	0	0	0	0	0
0.6-0.7	0	0	0	0	0
0.7-0.8	0	0	0	0	0
0.8-0.9	0	0	0	0.3	0.5
0.9-1.0	1	1	1	0.7	0.5

3

4

TABLE I

Failure	Distance	Reference risk case	Updated Risk case	Failure	Distance	Reference risk case	Updated Risk case
	km	μ_r,σ_r	μ_u,σ_u		km	μ_r,σ_r	μ_u,σ_u
384_1	1.5	0.0173, 0.00039	0.0639, 0.00569	**404_3**	75	0.055, 0.00236	0.147, 0.0183
384_2	1.5	0.0173, 0.00039	0.0639, 0.00569	**404_4**	75	0.00038, 0.0	0.00038, 0.0
385_1_	15.2	0.194, 0.141	0.280, 0.292	**404_5**	75	0.0156, 0.00035	0.059, 0.00504
386_1	26	0.001, 0.00001	0.00163, 0.0	**404_6**	75	0.00069, 0.0	0.0007, 0.0
387_1	27.2	0.0189, 0.00038	0.0693, 0.0062	**404_7**	75	0.00227,	0.00258, 0.0001
388_1	0.5	0.00231, 0.00002	0.00259, 0.0001	**405_1**	2.5	0.00064, 0.0	0.00065, 0.0
388_2	0.5	0.00082, 0.0	0.00105, 0.0	**405_2**	2.5	0.00064, 0.0	0.00065, 0.0
389_1	3	0.03, 0.00040	0.105, 0.0096	**406_1**	104	0.0386, 0.00175	0.112, 0.0144
389_2	3	0.162, 0.050	0.219, 0.0562	**406_2**	104	0.0383, 0.00191	0.109, 0.0143
389_3	3	0.00662, 0.00002	0.0224, 0.00043	**406_3**	104	0.034, 0.0022	0.0958, 0.0129
391_1	31	0.0182, 0.00044	0.0666, 0.00617	**406_4**	104	0.0343, 0.00192	0.0988, 0.0130
391_2	31	0.0037, 0.00002	0.0162, 0.00032	**406_5**	104	0.01, 0.00001	0.0302, 0.00063
391_3	31	0.00033, 0.0	0.00033, 0.0	**406_6**	104	0.0032, 0.00002	0.0156, 0.00033
392_1	32	0.07, 0.0124	0.129, 0.0237	**406_7**	104	0.026, 0.00212	0.08, 0.0107
393_1	5	0.0119, 0.00004	0.04, 0.00156	**407_1**	204	0.01, 0.00001	0.0772, 0.00291
394_1	3	0.14333, 0.0330	0.201, 0.0438	**407_2**	204	0.00036, 0.0	0.00036, 0.0
395_1	8	0.41, 0.109	0.501, 0.109	**408_1**	302.5	0.00036, 0.0	0.00036, 0.0
396_1	4	0.001, 0.00001	0.0139, 0.0	**408_2**	302.5	0.026, 0.0005	0.0907, 0.00886
397_1	4	0.0144, 0.00049	0.0561, 0.00524	**408_3**	302.5	0.01, 0.00001	0.0302, 0.00063
398_1	8	0.00064, 0.0	0.00065, 0.0	**408_4**	302.5	0.0157, 0.00055	0.0854, 0.0107
401_1	7.5	0.0112, 0.00009	0.0452, 0.00253	**408_5**	302.5	0.00486,	0.0198, 0.00048
401_2	7.5	0.00377, 0.00002	0.0163, 0.00031	**409_1**	1.5	0.01, 0.00001	0.0302, 0.00063
402_1	274	0.355, 0.162	0.469, 0.43	**410_1**	9	0.01, 0.00001	0.0302, 0.00063
402_2	274	0.667, 0.239	0.717, 0.339	**411_1**	128	0.0037, 0.00002	0.0162, 0.00032
402_3	274	0.04, 0.00203	0.112, 0.0148	**412_1**	270	0.0037, 0.00002	0.0162, 0.00032
402_4	274	0.0424, 0.00203	0.119, 0.0155	**412_2**	270	0.005, 0.00002	0.0185, 0.00033
402_5	274	0.00064, 0.0	0.00065, 0.0	**415_1**	6	0.182, 0.0494	0.237, 0.0571
403_1	140	0.0144, 0.00049	0.0561, 0.00524	**415_2**	6	0.0307, 0.00219	0.0886, 0.0119
403_2	140	0.005, 0.00002	0.0185, 0.00033	**415_3**	6	0.0143, 0.00002	0.0356, 0.00103
403_3	140	0.04, 0.00203	0.113, 0.0148	**416_1**	18	0.267, 0.142	0.347, 0.308
404_1	75	0.036, 0.00200	0.102, 0.0136	**418_1**	15	0.0257, 0.00029	0.0954, 0.00837
404_2	75	0.0028, 0.00002	0.00285, 0.0001				

Result of the risk update for all 63 judgments, optimistic expert group, sea ice environment. The failure code is structured as follows: 'mission number'_'failure number'. The mean and variance are given for the reference case and for the updated risk case.

1

2 References

3

4 [1] Banks CJ, Brandon MA and GarthWaite PH. Measurement of sea ice draft using upward-looking ADCP

5 on an autonomous underwater vehicle. Annals of Glaciology 2006; 44: 211–6.

6 [2] Bellingham JG and Rajan K. Robotics in remote and hostile environments. Science 2007; 318: 5.

7 [3] Wadhams P, Wilkinson JP and McPhail SD. A new view of the underside of Arctic sea ice. Geophysical

8 Research Letters 2006; 33: L04501.

9 [4] Nicholls KW, Abrahamsen EP, Buck JIH, Dodd PA, Goldblatt C, Griffiths G, Heywood KJ, Hughes NE,

10 Kaletzky A, Lane-Serff GF, McPhail SD, Millard NW, Oliver KIC, Perrett JR, Price MR, Pudsey CJ, Saw K,

11 Stansfield K, Stott MJ, Wadhams P, Webb AT and Wilkinson JP. Measurements beneath an Antarctic ice shelf

12 using an autonomous underwater vehicle. Geophysical Research Letters 2006; 33: L08612.

13 [5] Roman C. University of Rhode Island, Narragansett, RI, private communication, October 2008.

14 [6] Strut J. Report of the inquiry into the loss of Autosub2 under the Fimbulisen. National Oceanography

15 Centre, Southampton 2006.

16 [7] Department of Defense USoA. US MIL-HDBK-217F: Military Handbook, Reliability prediction of

17 electronic systems. Virginia. ed, 1991.

18 [8] Management SI. Offshore Reliability Data Handbook (4th ed.) Trondheim, Norway: OREDA – Offshore

19 Reliability Data Handbook (4th ed.) 2002.

20 [9] SINTEF. (1984). Available: htpp://www.sintef.no/static/tl/projects/oreda/results.htm

21 [10] Cushing MJ, Mortin DE, Stadterman TJ and Malhotra A. Comparison of electronics-reliability

22 assessment approaches. IEEE Transactions on Reliability 1993; 42: 542-6.

23 [11] Brito MP, Griffiths G and Challenor P. Risk Analysis for Autonomous Underwater Vehicle Operations in

24 Extreme Environments. Risk Analysis 2010; 30: 1771-88.

25 [12] Jenkins A, Dutrieux P, Jacobs SS, McPhail SD, Perrett JR, Webb AT and White D. Observations beneath

26 Pine Island Glacier in West Antarctica and implications for its retreat. Nature Geoscience 2010; 3: 468-72.

27 [13] O'Hagan A, Buck CE, Daneshkhah A, Eiser JR, Garthwaite PH, Jenkinson DJ, Oakley JE and Rakow T.

28 Uncertain judgments: Eliciting experts' probabilities. Chichester, UK: Wiley;2006.

29 [14] Griffiths G and Trembanis A. Eliciting expert judgment for the probability of AUV loss in contrasting

30 operational environments. New Hampshire, US, 2007.

31 [15] Brito M, Griffiths G and Trembanis A. Eliciting expert judgment on the probability of loss of an AUV

32 operating in four environments. National Oceanography Centre, Southampton 2008.

33 [16] Korb KB and Nicholson AE. Bayesian artificial intelligence. Boca Raton, FL: Chapman and Hall,; 2004.

34 [17] Hänninen M and Kujala P. Influences of variables on ship collision probability in a Bayesian belief

35 network model. Reliability Engineering & System Safety 2012; 102: 27-40.

[18]   Matellini DB, Wall AD, Jenkinson ID, Wang J and Pritchard R. Modelling dwelling fire development and occupancy escape using Bayesian network. Reliability Engineering & System Safety 2013; 114: 75-91.

[19]   ASPeCt. (2015, 01.06.2015). Antarctica Sea Ice Processes & Climate. Available: http://aspect.antarctica.gov.au/

[20]   Tsoy L. Draft of new regulations for navigation on the seaways of the Northern Sea Route, requirements to ships and interrelation between ice classes. in In Proc. ARCOP Workshop 7, Oslo, ed, 2005.

[21]   Otway H and von Winterfeldt D. Expert judgment in risk analysis and management: process, context, and pitfalls. Risk Analysis 1992; 12: 83-93.

[22]   Keeney RL and Winterfeldt Dv. Eliciting Probabilities from Experts in Complex Technical Problems. IEEE Transactions on Engineering Management 1991; 38: 191-201.

[23]   Kynn M. The 'heuristics and biases' bias in expert elicitation. Journal of the Royal Statistical Society: Series A (Statistics in Society) 2008; 171: 239-64.

[24]   Kahneman D and Tversky A. Subjective probability: A judgment of representativeness. Cognitive Psychology 1972; 3: 430-54.

[25]   Kaplan EL and Meier P. Nonparametric estimation from incomplete observations. J. Am. Stat. Assoc. 1958; 53: 457-81.

[26]   Pearl J. Probabilistic reasoning in intelligent systems. San Mateo, CA: Morgan Kaufmann;1988.

[27]   Griffiths G and Brito MP. Predicting Risk in Missions under Sea Ice with Autonomous Underwater Vehicles. . Woods Hole, MA, 2008.

[28]   Littlewood B and Wright D. The use of multilegged arguments to increase confidence in safety claims for software-based systems: A study based on a BBN analysis of an idealized example. Ieee Transactions on Software Engineering 2007; 33: 347-65.

[29]   A/S H. (01.06.2015). Hugin Expert Available: http://www.hugin.com

[30]   O'Hagan A, Buck CE, Daneshkhah A, Eiser JR, Garthwaite PH, Jenkinson DJ, Oakley JE and Rakow T. Uncertain judgements: Eliciting experts' probabilities. Chichester, UK: Wiley;2006.

[31]   Madsen AL, Jensen F, KjJÆrulff UB and Lang M. The Hugin Tool for Probabilistic Graphical Models. International Journal on Artificial Intelligence Tools 2005; 14: 507-43.

[32]   Wadhams P, Ackley A, Collins KJ, Cooper P, Edwards T, Evans J, Griffiths G, Hughes N, Kaletzky A, McPhail SD, Mercer D, Cofaigh CÓ, Rutherford K, Webb AT, Wilkinson JP, Williams G and Willis D. JR106 Cruise Report: Autosub under arctic sea ice: James Clark Ross cruise in Cruise Report, ed: National Oceanography Centre, 2004.

[33]   Banks CJ, Brandon MA and Garthwaite PH. Measurement of sea-ice draft using upward-looking ADCP on an autonomous underwater vehicle. . Annals of Glaciology 2006; 44: 211-6.

1   [34]   Griffiths G and Trembanis A, "Towards a risk management process for autonomous underwater

2   vehicles. ," in In Masterclass in AUV technology for polar science, G. Griffiths and K. J. Collins, Eds., London:

3   Society for Underwater Technology 2007, pp. 103-18.

4