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Spectral analysis of regional main field and secular variation in CHAOS-4 using spherical Slepian functions

CIARÁN D. BEGGAN [ciar@bgs.ac.uk]

British Geological Survey, West Mains Road, Edinburgh, United Kingdom

1. Introduction

Magnetic models such as CHAOS-4 represent the global field using Spherical Harmonic (SH) functions weighted by a set of numbers known as Gauss coefficients. This representation allows values of the field to be calculated at any location and altitude above the core-mantle boundary, but has limitations when attempting to isolate the contribution to the field from specific areas or regions.

Spherical Slepian functions provide an alternative mathematical basis to represent the field [Ref. 1]. They have the advantage of allowing an area of interest to be optimally described in a spatio-spectral sense. In addition, spherical Slepian functions can also be used to separate and decompose the Gauss coefficients from a SH magnetic field model into the components that represent the contribution to the model from individual regions of the globe [Ref. 2].

We investigate the spectral and spatial changes of the main magnetic field of CHAOS-4q [Ref. 3] at the Earth's surface between spherical harmonic degrees 12-35 in eight different regions across the globe: the Americas; Africa; Australia; Eurasia; Antarctica; the Pacific Ocean; the Atlantic Ocean and the Indian Ocean between 1997 and 2011.

2. Slepian Functions

Rather than spherical harmonics, spherical Slepian functions can be employed to produce a locally and also globally orthogonal basis in which to optimally represent the available data in a region at a given degree. Slepian functions can be tailored to be either band- or space-limited, allowing a trade-off between spectral and spatial concentration in the region and leakage beyond. Only N Slepian coefficients are required to be solved for to optimally concentrate the energy of the Slepian functions into the region of interest, that is: $N = (L+1)^2 R/4\pi$, where N is the Shannon Number, L is the degree and R is the size of the region as a fraction of the full sphere.

We optimally separate the spherical harmonic coefficients of the CHAOS-4q model into regions in order to investigate the spectral content of each. However, leakage (or coupling) between regions, means that the separation is not perfect. Coupling is related to the size of the region of interest, its shape and the degree resolution of the model. Coupling between degrees and orders arises from the partial selection of information thus breaking the orthonormality. It cannot be completely avoided.

3. Regional Separation of the field model

CHAOS-4q, the latest version of the CHAOS model series [Ref. 3], is derived from satellite and ground-based observatory data. It has a time-varying core field up to degree and order 20 and a fixed crustal field up to degree and order 100. As we are interested in the higher degree core field, we truncate the model at degree 45 and use Slepian functions to separate the model's Gauss coefficients into individual regions spectrally and spatially every two years, following the methodology of Beggan *et al.* (2013).

Figure 1 shows the regional separation for 2011.0, while Figure 2 shows the change between 2011.0 and 2001.0. Note the spatial leakage from each region.

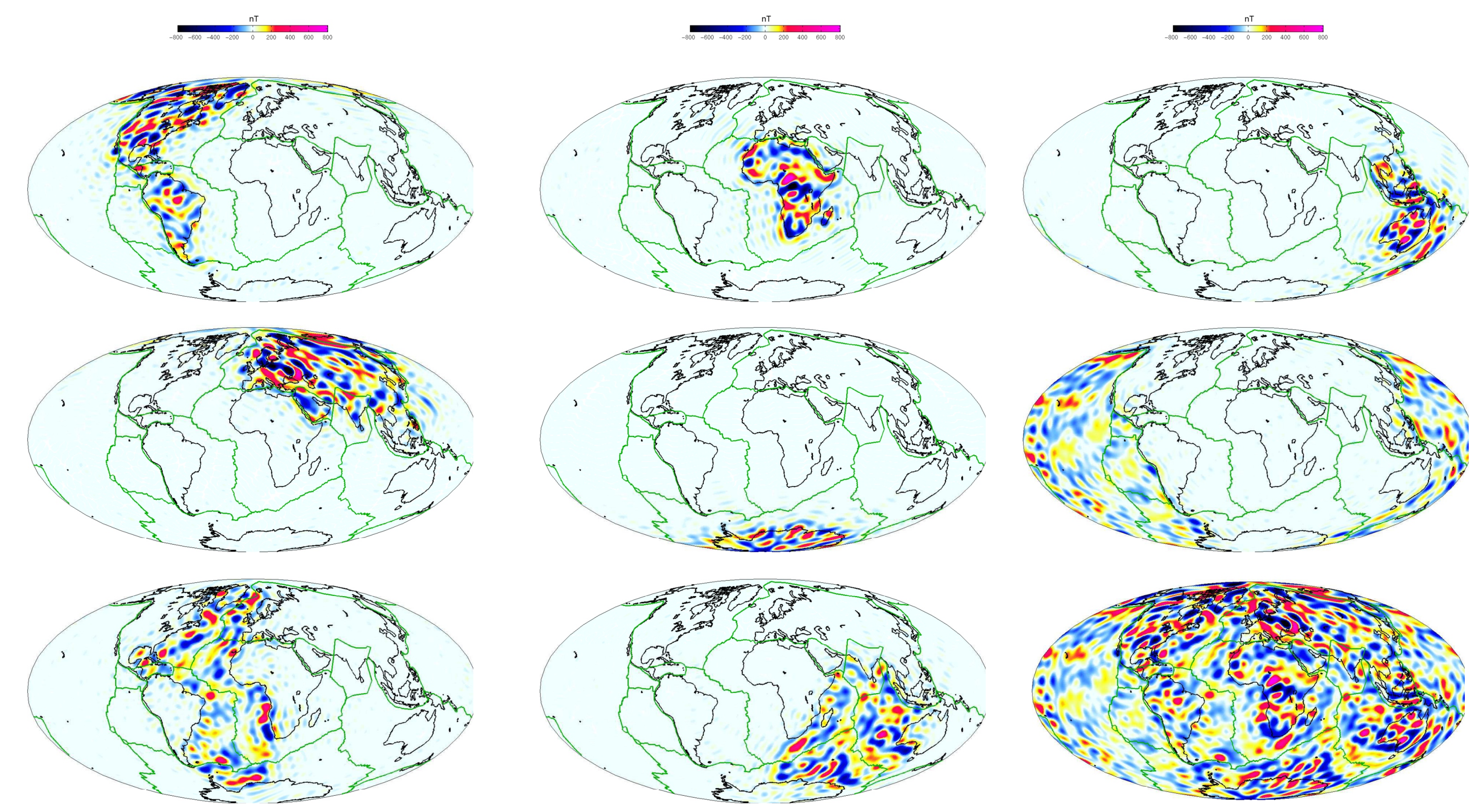

Figure 1: Separation of CHAOS-4q model for 2011.0 over degree and order L = 12 – 45 into eight regions. From upper left: the Americas; Africa; Australia; Eurasia; Antarctica; the Pacific Ocean; the Atlantic Ocean and the Indian Ocean. Bottom right: CHAOS4-q model for 2011.0. Scale: +/- 800nT.

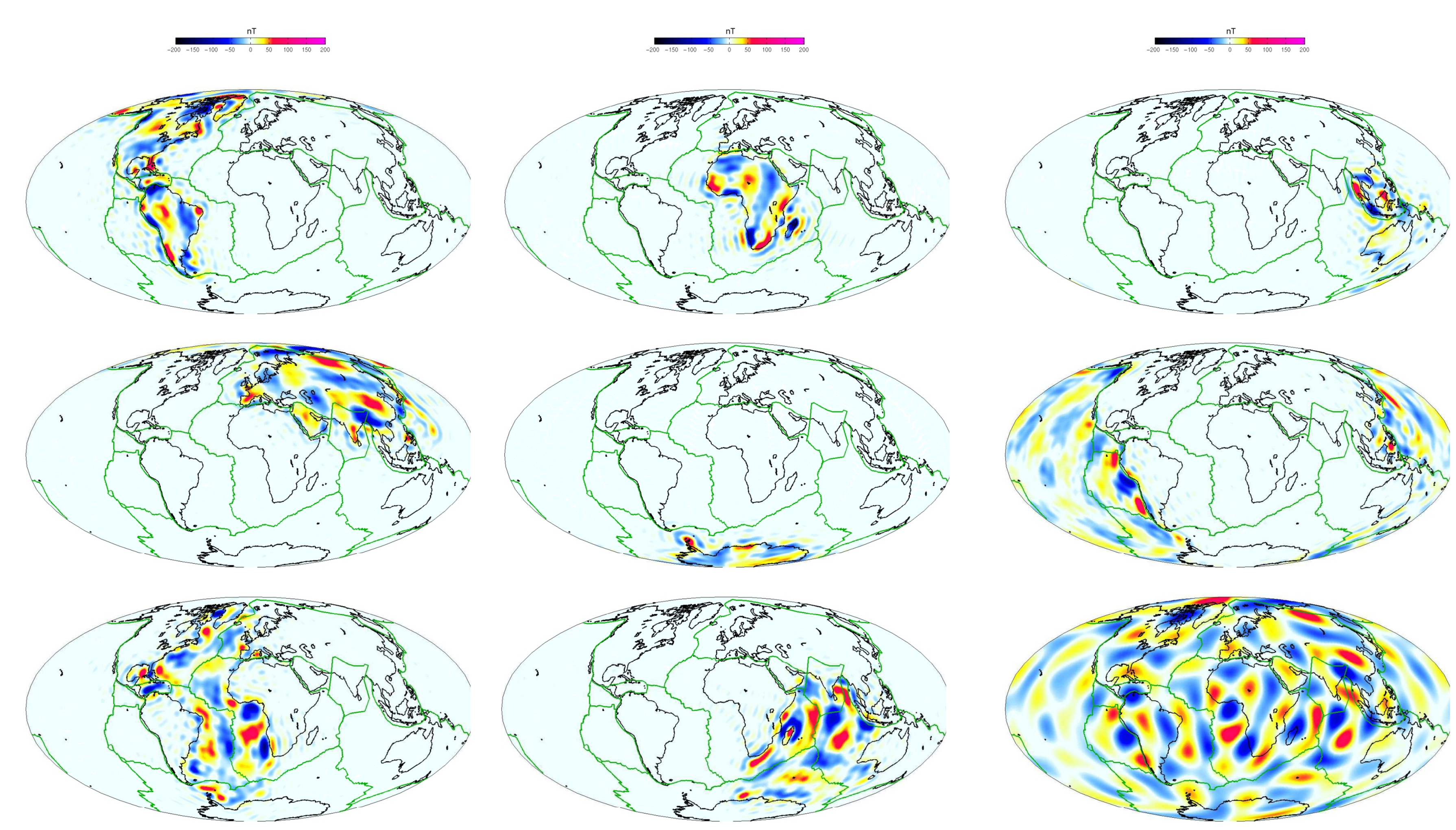

Figure 2: Difference of CHAOS-4q model between 2001.0 and 2011.0 over degree and order L = 12 – 45 into eight regions. From upper left: the Americas; Africa; Australia; Eurasia; Antarctica; the Pacific Ocean; the Atlantic Ocean and the Indian Ocean. Bottom right: CHAOS4-q model differences between 2001.0 2011.0. Scale: +/- 200nT.

4. Spectral Analysis of the regions

The power spectra for each separate region computed for L = 12 – 35 every 2 years from 1997–2009 are shown in Figure 3.

Figure 3 (a) shows the area-weighted power spectra obtained directly from the decomposition of CHAOS-4q every two years. Figure 3 (b) shows the same data, normalised relative to the 2005 curve in each region, to emphasise the variation of the spectra over time.

There is clearly an issue with the Slepian decomposition because there is strong variation in the crustal field degrees (L = 20–35), which do not change within the CHAOS-4q model. The variation must be attributed to the imperfect decomposition due to leakage between regions, even when using the optimal number of basis functions. Indeed, spatial leakage is evident in Figure 1.

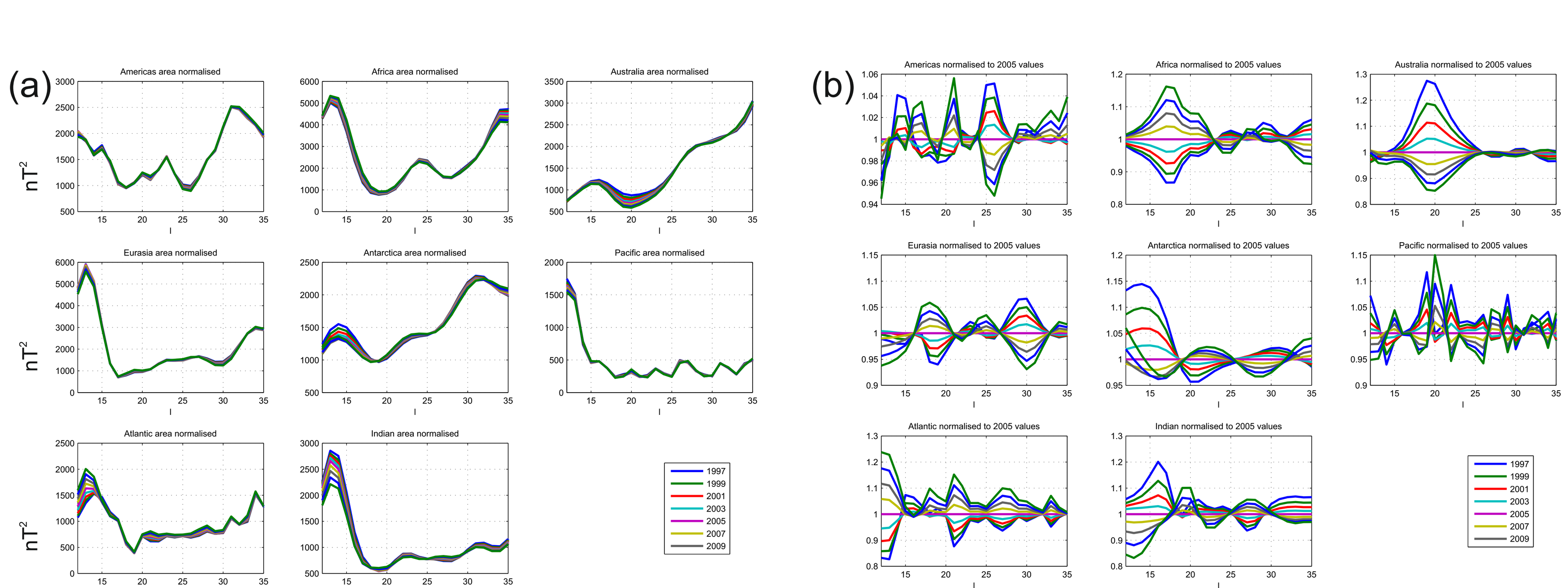

Figure 3: Spectra of CHAOS-4q model over degree and order L = 12 – 45 into eight regions. From upper left: the Americas; Africa; Australia; Eurasia; Antarctica; the Pacific Ocean; the Atlantic Ocean and the Indian Ocean. (a): Area weighted spectra; (b): Normalised to 2005.0. Note the change of the crustal field due to leakage between regions and imperfect separation.

5. Conclusions / Future Work

We are interested in determining the change of the higher degree *core* field (L = 12 – 20) that is typically masked by the long-wavelength crustal field, particularly over the oceans where the crustal field is assumed to be relatively weak. However, the Slepian decomposition technique, as applied in this study, shows obvious and significant leakage between regions. This suggests the resulting spectra in Figure 3 are not correct. Examination of the coupling matrices (not shown) indicates that the smaller regions suffer most leakage.

There are several possible paths to improve these results. Rather than using the optimal number of Slepian functions, all the functions can be used to gain a better representation of the regional signal.

In addition, a comparison between a forward model of crustal magnetisation over the oceans [Ref. 4] and the spectra from the Slepian decomposition of the ocean regions will be made to investigate the differences.

This poster as a pdf:
http://nora.nerc.ac.uk/504430/

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