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Interpolation of external magnetic fields over large sparse arrays using Spherical Elementary Current Systems

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Introduction

Estimation of external magnetic field disturbances to a point remote from an observatory can be made, for example, by using the latitudinal-weighted arithmetic mean of the field measured at two other observatories, one to the north and one to the south of the point of interest. An alternative technique for interpolating external magnetic field disturbances across large spatial areas can be achieved with the Spherical Elementary Current System (SECS) method using data from a number of ground-based magnetic observatories. The SECS method represents the complex electrical current in the ionosphere as a simple set of equivalent currents placed at a specific altitude. The magnetic field recorded at observatories can be used to invert for the electrical currents and subsequently used to interpolate or extrapolate the magnetic field across a large area.

Here we apply the SECS method to high geomagnetic latitudes using a series of observatory networks to test how well the external field can be interpolated over large distances. We found that co-estimating for induced subsurface current systems in addition to ionospheric current systems can result in strong improvements to the interpolated magnetic field, particularly in the radial (or Z) component of the field. We demonstrate that relatively few observatories are required to produce an estimate which is better than the null hypothesis (i.e. assuming no change in the field) or interpolation using latitudinal weighting of data from other observatories.

Spherical Elementary Current Systems

The basic concept of SECS is to construct the equivalent current system using a linear superposition of divergence-free elementary current systems, all of which can be placed freely within the current plane. Amm and Viljanen (1999) introduced a method for continuation of the magnetic field disturbance from the ground to the ionosphere. Pulkkinen et al. (2003) extended the complex image method developed by Pirjola and Viljanen (1998) to show that the field could be calculated from superposition of the magnetic effect of two horizontal current layers composed of divergence-free elementary current systems. Using a spherical coordinate system (r, θ, φ), the magnetic fields (**B**) induced by a system of internal (*i*) and external (*e*) current sheets (**I**) can be defined for the subsurface and the ionosphere, as a function of two linear equations derived in Amm and Viljanen (1999) and Pulkkinen et al. (2003), which relate current and magnetic fields in a spherical reference frame (*T*):

$$\mathbf{B}(r,\vartheta,\varphi) = \sum_{j=1}^{M} I_j^i \underline{\underline{T}}_{df}^i (R_G,\theta_j,\phi_j,r,\vartheta,\varphi) + \sum_{k=1}^{S} I_k^e \underline{\underline{T}}_{df}^e (R_I,\theta_k,\phi_k,r,\vartheta,\varphi)$$

$$\mathbf{B}(r,\vartheta) = I_1^i \underline{\underline{T}}_{df}^i (R_G,0,0,r,\vartheta) + I_1^e \underline{\underline{T}}_{df}^e (R_I,0,0,r,\vartheta)$$

$$= \frac{\mu_0 I_1^i}{4\pi r}\left(\frac{R_G}{r}\left[\frac{1}{\sqrt{1-\frac{2r\cos(\vartheta)}{R_I}+\left(\frac{R_G}{r}\right)^2}}-1\right]\mathbf{e}_r - \frac{1}{\sin\vartheta}\left[\frac{r-R_G\cos\vartheta}{\sqrt{r^2-2rR_G+R_g^2}}-1\right]\mathbf{e}_\vartheta\right)$$

$$+\frac{\mu_0 I_1^e}{4\pi r}\left(\left[\frac{1}{\sqrt{1-\frac{2r\cos(\vartheta)}{R_I}+\left(\frac{r}{R_I}\right)^2}}-1\right]\mathbf{e}_r - \frac{1}{\sin\vartheta}\left[\frac{\frac{r}{R_I}-\cos\vartheta}{\sqrt{1-\frac{2r\cos\vartheta}{R_I}+\left(\frac{r}{R_I}\right)^2}}+\cos\vartheta\right]\mathbf{e}_\vartheta\right)$$

where $\underline{\underline{T}}_{df}^i$ and $\underline{\underline{T}}_{df}^e$ are the geometric parts of the internal and external magnetic fields produced by each elementary current system located at (R_G, θ_j, φ_j) and (R_I, θ_k, φ_k). The subscripts *j* and *k* denote the number of current systems of each part. The derivation is constructed for a point between the current systems in the ground and in the ionosphere, $R_G < r < R_I$.

We assume the magnetic field B has been measured at a set of points and construct a linear system of equations relating the measured field to the geometric parts and scaling factors of both the internal and external elementary current systems. Scaling factors which best fit observations of ground magnetic field disturbances are determined by solving the inverse problem for **I**, using Singular Value Decomposition, and then computing the field over the entire area of interest or at a specific point.

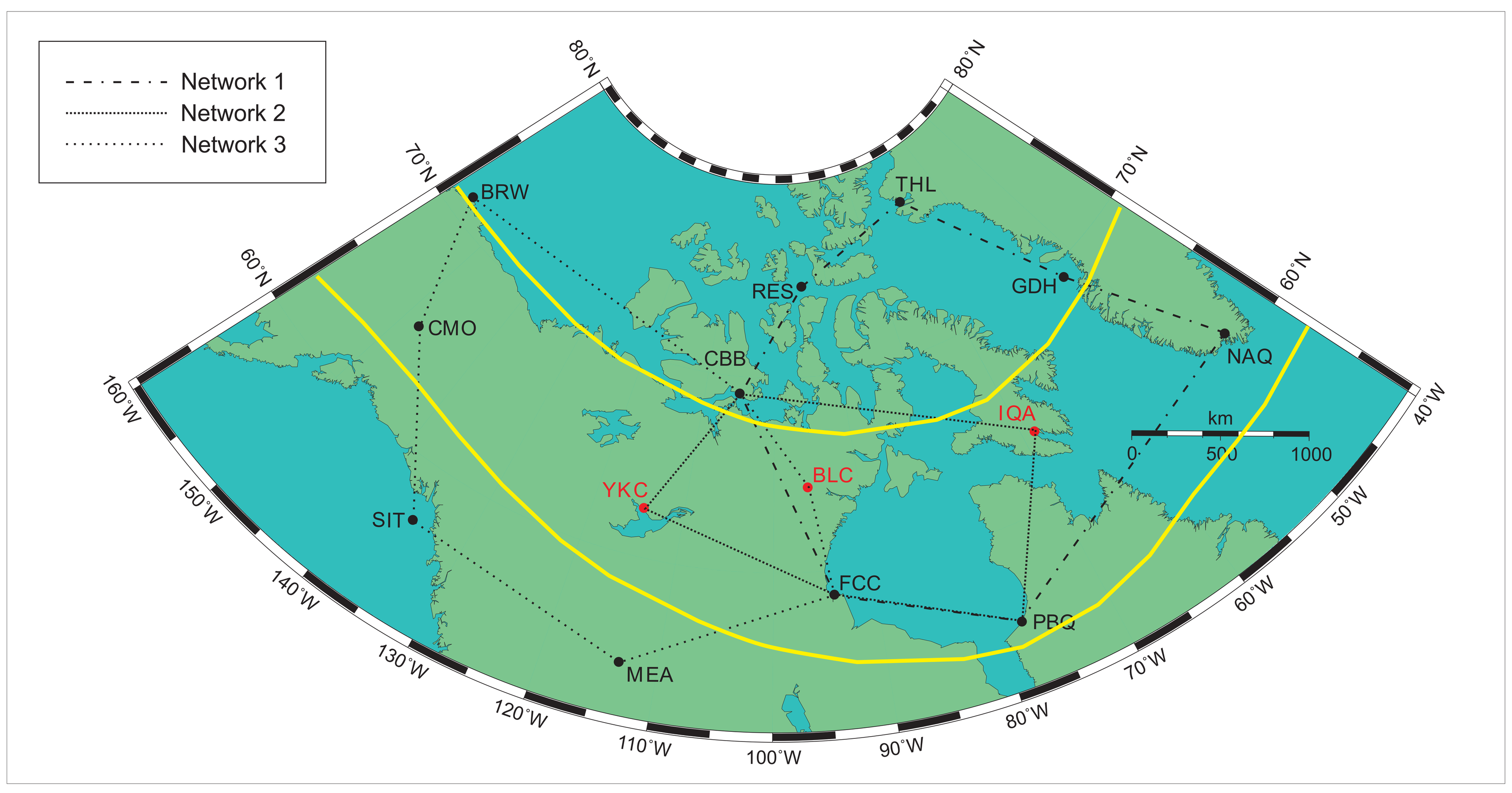

Figure 1: Example of three networks of ground-based observatories (black joined by dotted lines) used to estimate the ionospheric disturbance at a remote observatory (red). Many observatories are in the auroral zone at high geomagnetic latitudes (approximately denoted by the yellow lines).

An example: solving for SECS in N. America

Figure 1 shows three networks of observatories in North America. For example, Network 1 consists of eight observatories surrounding Iqaluit (IQA). The minute means values (with the daily mean constant subtracted off) from each observatory were used to solve for the current systems over an area of two million square km and then to estimate the magnetic field disturbance each minute at IQA. The difference between the estimate and the observed measurements are compared using the root-mean-square misfit (in units of nanoTesla) defined as:

$$RMS_{misfit} = \sqrt{\frac{1}{N}\sum_{i}^{N}(Obs_i - Estimate_i)^2}$$

The SECS estimates were compared with an estimate from the latitudinally weighted average of two close observatories. Power describes the misfit of the null hypothesis (i.e. no change of the field). Figure 2 shows an example of the SECS and latitudinal weighted estimate of the field over one day for the three networks. It also shows the daily misfit over a three month period (May-July 2005).

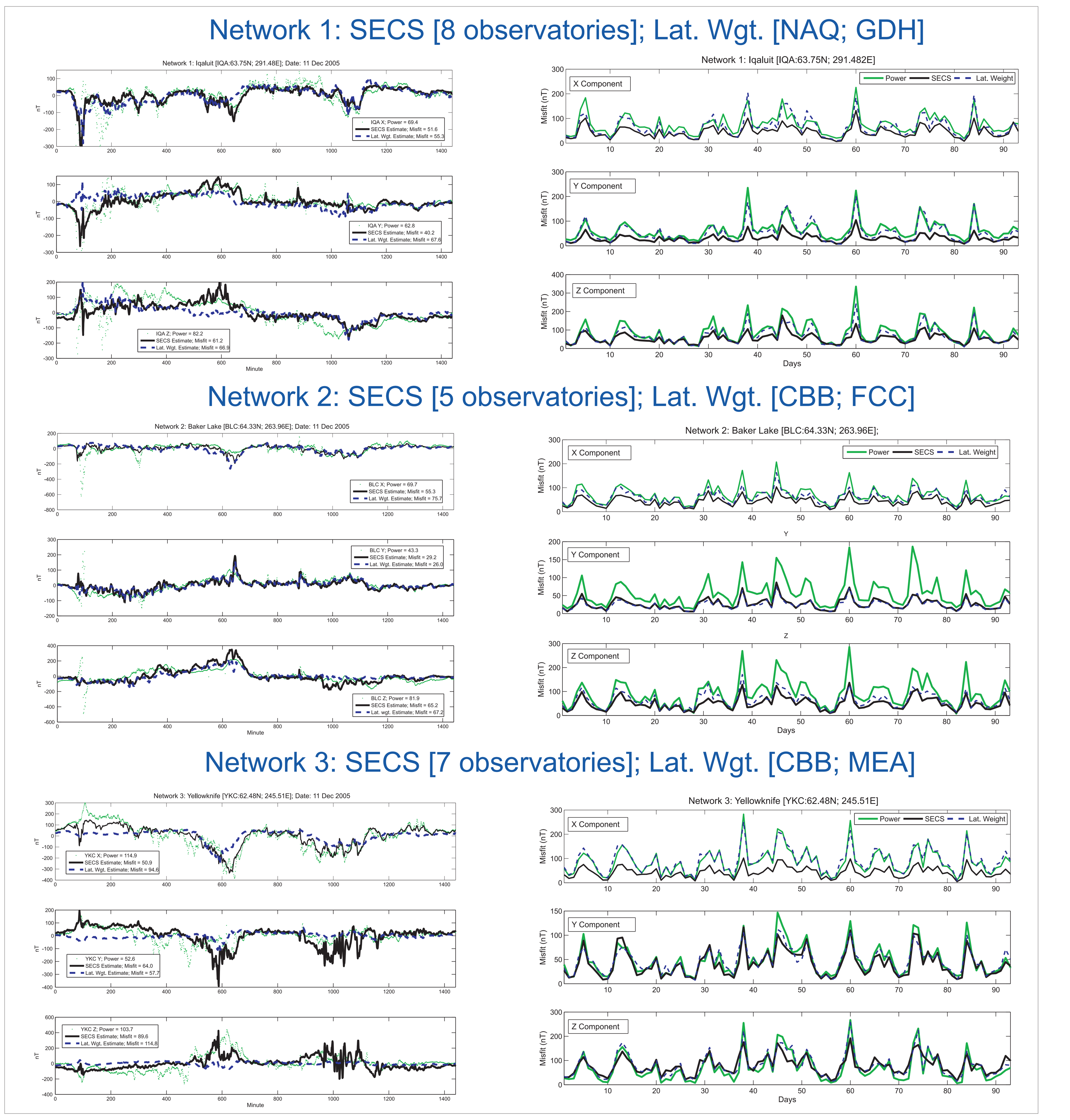

Figure 2: (Left Panels) Comparison of the measured external field disturbance (green points) and the estimate from the SECS (black solid) and Latitudinal Weighted (blue dashed) methods for a single day (11th December 2005).
(Right Panels) Comparison of the daily power of the measured external disturbances (green) with the misfit of the estimate from the SECS (black) and Latitudinal Weighted (blue) method over a 3 month period (May-July 2005). Note the ~7-9 day solar harmonic peaks. The SECS method tends to fit better on these days.

Discussion

It is interesting to note that using the SECS to estimate the external field disturbance does better, on average, than the simpler latitudinal weighting of data from two observatories, though it does employ more observatories to achieve this result. In particular, the SECS method produces a better fit to the data on more disturbed days. Previously, the SECS method has only been applied to relatively small spatial regions with large numbers of observatories. This work indicates that the method can be applied (with suitable caveats e.g number of observatories, size of area covered) to larger regions sparsely covered with observation points. McLay and Beggan (2010) will discuss the work in more detail.

Acknowledgments

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