THE CASE FOR COMPUTER-BASED SPATIAL MODELS IN GEOLOGY

A progress report on Project 22A

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Introduction

Spatial modelling is a new name for an old concept. It refers to what has long been a core activity of a geological survey, namely, piecing together a picture of the geometrical configuration and disposition of sequences of strata or other rocks, their constituent materials, characteristics, and properties, and relating that picture to ideas of their history and origin. The novelty lies in basing the model on computer methods rather than the conventional reports, maps and cross-sections. This is not to suggest that the interpretation and ideas can come from anywhere other than the geologist. But it is suggested that a computer model could give a better medium for the geologist to express his ideas; build up his interpretations; organise, analyse, summarize and share his observational data; explore the consequences of his hypotheses; reconcile information from diverse sources with his expectations and background knowledge; display the results of his work and transmit them to the users.

The observational data available to a geologist typically refers to only a small part of the rocks of interest. Data from various sources are likely to have imprecise operational definitions, different sampling patterns and resolution. They are thus not directly comparable. Even where the sampling scheme is carefully designed, data analysis is likely to be of help largely with linear relationships. The relationship of gravity data and geochemical stream sediment analyses, for example, would generally be too complex for computer data analysis to be useful. A large interpretational element is therefore inevitable in arriving at an overall picture. The interpretation, that is, the spatial model, should be internally consistent, and consistent with reasonable geological expectations and with all available data within the limits of its reliability. The aim of computer implementation of the spatial model, hitherto attempted only in limited fields, such as gravity or magnetism, is to improve on existing methods and overcome some of their limitations. This implies that the model must be quite comprehensive. In designing the model, all aspects must be considered together because of their complex interactions. The stage has now been reached where a general line of approach can be proposed. It is hoped that in due course this can be extended and refined by experiment and discussion into a design for a production system.

In what follows, an attempt is made to show why it is important to create a comprehensive, computer-based model, why it has not been done before but why it now merits urgent attention, what effects modelling might have on geological survey, what can be expected of the model and how it can meet the requirements, and which directions are appropriate for further research.

Benefits

It may be useful to consider first the potential benefits of a computer model as opposed to conventional methods. A map can give only a limited view of a geological sequence of three-dimensional surfaces and their evolution through time. In the computer model, internal representation is in three or four dimensions throughout, and the functions of data recording, storage and retrieval, analysis and interpretation, and display are handled separately, rather than being combined in one document in a static, two-dimensional format. The results of separation are greater flexibility, higher precision leading to more accurate results, and a more comprehensive record of the geology. A production system should also offer quicker and easier access to the information, lower costs, and savings of manpower. The benefits from the computer model for each separate function will now be considered in more detail.

In data recording, it is clear that a wide range of quantified information can be supplied to the model. The model, unlike the map, has no defined scale, although it has the comparable attribute of resolution, that is, the distance apart of two points that can be separately distinguished. It is suggested later that models should be stored for each area at a number of predetermined levels of resolution. Nevertheless, there is no limit to the fineness of detail, as this can vary from one point to another. The geologist’s broad overall impressions can also be quantified. For example the nature of internal variations in a rock body, such as lateral and vertical changes and gradients, minor discontinuities and repetitive sequences, could be recorded as a summary description rather than showing an artificial homogeneity within formations. Surfaces which can be correlated only locally can be included in a spatial model as well as widespread "mappable units". Recent work on seismic stratigraphy illustrates their significance (e.g. Mitchum et al., 1977). The spatial relationships between items in one or more models can be defined. For example, the relationships that a fossil locality is on a Carboniferous outcrop which is on the downthrown side of a fault, or that a formation boundary converges upstream with a river, can be indicated explicitly, as can the locational accuracy of observations.

Information from the geologist’s interpretation or background knowledge can be recorded, and distinguished from observations. The geologist might, for example, have expectations about the general form or shape of a surface from his knowledge of its mode of deposition or structural deformation. On similar grounds, he might expect similarity or accentuation of form between two surfaces, such as the thickness of a sequence of sand bars being greatest at high points on the underlying surface. The expected form of structural surfaces, such as faults or axial planes, may also be known, and the geometrical consequences of their interaction with stratigraphic horizons may be predictable. If the geologist is able to specify an evolving pattern of changes of form through time then this too could be included in the model.

Storage and retrieval within a spatial model offer new possibilities. Alternative explanations or multiple hypotheses could be stored and the selection and justification of a preferred explanation could be explicit. The consequences of new data or new hypotheses could be readily explored and the model revised to include new or amended information. As the information is in digital form, it could be stored for an indefinite time and transmitted rapidly to remote devices without loss of precision. The items of information to any level of detail could be identified separately, thus allowing precise cross-referencing within the model and externally from other models, text descriptions or data files.

The observations, interpretation and background knowledge recorded in the spatial model can each be identified and used in analysis and display. The analysis can give rise to additional information which may assist the interpretation. For example, geometrical features, such as lines of curvature or small anomalies on a regional slope, which may be of scientific and economic interest, can be computed from a digital model and displayed appropriately, although conventional contouring methods would not reveal them. The geometrical characteristics of one or more surfaces, including such aspects of shape, and probable sizes and numbers of closures, could be summarized numerically from a model. The similarities of form of successive strata, and the relationship of biostratigraphic, lithostratigraphic and seismic reflector horizons with one another and with faults, axial planes and other structural surfaces could be defined and quantified. The shape descriptors could form the basis of quantitative background knowledge of regional and vertical variations of shape, and of the influence of mode of deposition and structural deformation. This in turn could be used explicitly in the interpretation, with each step explained and justified in a linked comments file. At each stage, uncertainty could be estimated and its sources defined.

Derived information, such as areas, volumes, and slope or curvature distributions could be calculated directly from a digital model for use in resource estimation, prospectivity analysis, sediment budget calculations, reconciliation of interpreted deformation patterns with the theoretical stress field, etc. Some important sources of data, such as gravity or regional geochemical measurements, may not define the geometry of the spatial model uniquely. However, the model could be used to predict such values, and adjusted to reconcile its predictions with the observed data, within the limits of statistical significance. There is thus scope for integrating many diverse sources of information within a model. Derived or implicit relationships (such as: because A is younger than B, and B is younger than C, then A is younger than C) can be computed and any inconsistencies brought to the attention of the geologist for correction.

The computer model can offer a wide variety of forms of output, and remote access to the information through the telephone network. Printed output might be required for such results as: prognosis of depths to defined horizons at a proposed well; area or volume calculations; shape descriptors; frequency and orientation distributions of closures of various sizes, etc. Graphic display can be flexible, and such forms as line and symbol maps, posted value maps, contours, isopachs, perspective views, cross-sections, fence diagrams, stereograms, correlograms, form-surface and lines of curvature maps are envisaged as possible outputs. The content, scale and type of computer-drawn displays could be selected by the user, and it is practicable to generate series of diagrams to examine different aspects of the geology. They could be based on the most recent data and interpretation.

Computer graphics techniques, including raster display with texture and colour, stereo, ciné and perspective views, can assist in visualizing three-dimensional form, changes through time, levels of uncertainty, and variability within formations. Where there is sufficient compatibility, selected information from the model could be made available to users in digital form, for manipulating or displaying with other digital spatial data, such as mine plans, proposed motorway routes, catchment areas, land use maps, etc.

Overall, in the face of growing specialisation and fragmentation of geology, the computer-based spatial model should help the geologist to bring together information from several disciplines; to explore hypotheses and analyse his information more fully; and to display results appropriately for the user needs of the future. It could help to restore the balance between the large amounts of data potentially available to the geologist, and the limited ability of the geologist to analyse and interpret his data. The model could give greater coherence to computer developments in geology in such areas as digital cartography, text handling, data banks and graphics by relating them to one core activity.

Timeliness

Computer applications in geology are well established in data management, display, analysis and geophysical modelling, but the spatial model envisaged here would be a new departure. It does not lack precedents, however, and meteorological, econometric and ecological models are examples of computer models which have a central role within their discipline. There is no similar documented development in the main stream of geology, perhaps because of the complexity of the spatial patterns, the success of long-established methods, the conservatism of workers in this field, and the need for a basis in data banks and automated cartography. In the last ten years, however, extensive geological data banks and cartographic systems have been developed. Work outside geology in computer graphics, computer aided design, digital terrain analysis and finite element methods has led to greater understanding of methods of handling complicated surfaces on the computer.

It is anticipated that automation of the map production line will be cost-effective within a few years, and selective digitization of more geological maps is therefore likely. There are also large amounts of existing computer data, particularly from seismic surveys and borehole logging, which are at present largely untapped. Many customers for geological maps, in industry, government and the universities, already make extensive use of computer data banks and display, and are increasingly likely in the future to require geological information in digital form. Most of the required hardware, communication networks, and software in such areas as database management, graphics, cartography, and contouring, are already available and provide the basis of a system in which the spatial model could be embedded. Development work on spatial models now is thus justified by the availability of information, the means of handling it and the requirement for the product.

Technical developments in other fields, such as video, telecommunications and text processing suggest that the office of the future, including the geologist’s office, will have facilities for generating, communicating, receiving, processing, and storing voice, text, data, software, facsimile and graphics all in digital form. Many existing forms of map and memoir may be uneconomic in that environment. Their replacement by forms more appropriate to the technology will depend on the implementation of spatial models from which "electronic maps" can be generated.

The main benefits of the spatial model are long-term. A successful production system might set a pattern that would persist for the foreseeable future, but years of research, discussion, experimentation, training and experience will be required before a full-scale system could be implemented, not least because it must rely on the skills of geologists at present unused to computer techniques. The project to which this report refers is intended to meet the urgent need to explore and develop the concepts and strategy in what remains a neglected area of computer applications despite its central importance in geology.

Implications

A high level of consistency has been achieved in interpretation and representation of regional geology in maps and other publications. No major technological change in handling knowledge of the disposition and properties of rocks and of their origin and evolution could be seriously contemplated unless it can both maintain that consistency and overcome important limitations in conventional techniques. The potential benefits of a computer model are considerable and far-reaching to the extent that they will be fully achieved only as areas are resurveyed. It is reasonable to assume, therefore, that computer models will coexist with conventional methods for the foreseeable future. Nevertheless, the geological map may progressively lose its role in helping to maintain uniform standards and definitions. The same standards must, therefore, be maintained within the spatial model.

The proposed model is intended to represent geologists' interpretations, which should be consistent with their expectations and with all sources of relevant information. The geologist should be able to work with a greater diversity of information and more detail and precision than before. Interpretations could be more wide-ranging because of the ease of manipulating and displaying geometrical information on the computer. Because of the high value, within the model, of quantitative data collected on a defined sampling scheme, field data collection is likely to become more rigorous, and directed to measuring critical attributes pinpointed by the model.

In the long run, these developments would make heavy demands on the interpretational skills of the geologist and the design skills of the cartographer. Computer support is obviously essential, and mathematical aspects of geology important. The model is related to most other aspects of computer applications in geology and would help to establish links and maintain compatibility between them and with outside data files, such as topographic databases.

User view

The main requirements for a spatial model are to provide efficiently the benefits previously listed. Some implied characteristics, such as economy, accessibility, ease of maintenance and portability, apply to most computer systems. Others, related particularly to the user's view, are more specific and need more detailed consideration here.

Users with widely varying levels of geological knowledge would require access to information from the model. Three levels of access are suggested. The simplest level would be a collection of maps and associated material prepared from the model on paper or facsimile. Other than selection of items suited to his interest profile, the user would have no direct control of the displays. A second level of access would be to provide digital display files which the user could amend without affecting the model. The user could program the style of presentation, in terms of output device and hence precision, scale, colour, ornament, symbols, registration and overlaying. The ability to overlay, assuming careful separation of information between display files, would allow some control of content, but not of interpretation.

A third level of access would be appropriate for the geologist involved in mapping or reinterpretation. It would provide the spatial model for a defined area and stratigraphic interval, together with associated data files and explanatory text, and programs for constructing, amending and displaying the model. One of the products of the interpretation could be the display files mentioned previously.

A master archival collection of authorised versions of models, display files and maps could be maintained centrally. This would give a single source from which all users could draw, and would provide a means of controlling quality, including adherence to standards, matching of adjacent models at their boundaries, and the like. It would also help to monitor areal coverage and detect gaps and inconsistencies in the range of models.

The purpose of the ready-prepared maps and display files would be to give the user immediate access to an expert interpretation, as he has now with a geological map, without unnecessary involvement in detail and technicalities. The form of display, however, is likely to change from present practice to take advantage of the flexibility of the model and the possibility of frequent revision. Many kinds of map could be generated from a computer model, all of which might convey useful information. Cheap, ephemeral displays would be appropriate for many such maps, using, for example, a colour raster screen as in television sets or equivalent higher resolution devices. Raster displays are likely to be widely used in the office of the future. One possibility for distribution of a geological "electronic atlas" would be videotape or videodisc, with a mechanism for selecting individual maps or pages, together with text, data, and synchronised sound. A microprocessor and random-access memory could give additional facilities for modifying and overlaying maps or repetitive display of a cycle of maps. Although lacking the high resolution of the conventional map, a raster display has some compensating advantages, such as movement, adjustable scale, and the availability of a wide variety of colours and textures. Movement in particular is a largely unexplored tool in presenting geological information to show relative depths, uncertainty, changes in time, and relative spatial configurations of different variables.

The selection of standard displays to illustrate a particular spatial model requires considerable geological knowledge of the area and is likely to be the responsibility of the geologists who prepared or revised the model for that area. Their interaction with the computer is much deeper than that of the casual user, and one or their main requirements is likely to be ease of use. In the long run, the techniques of "expert systems" (see, for example, Duda et al., 1978) may be relevant. In the short run, appropriate use of menus, formatted screens, keywords and pointers on a screen or tablet may be helpful. It is essential, however, that the system should be designed, first, to be user-friendly, and, second, to encourage a constructive interchange of information between the geologist and the computer. These points are therefore considered further.

Each computer user has his own preconceptions, accurate or otherwise, of what takes place within the processor. A user-friendly system meets these preconceptions and behaves as the user expects it to. The geologist is likely to visualize the spatial model as an automated map or atlas. He might begin a computer session by consulting a table of contents or an index. He might select an area; a topic, such as drift geology, solid geology, contours on lithostratigraphic units; and scale. The information that would normally appear on a map border might be available on request, such as a list of formations present; the meanings of symbols; a diagrammatic stratigraphic succession; the authors' names and dates of preparation, revision and approval; and representative cross-sections. Cross-references might be required to detailed explanatory text (the "electronic memoir"), relevant data files, and diagrams indexing adjacent sheets, information on larger and smaller scale maps, and maps on other topics in the same area. The ability to select part of the area for more detailed examination at a larger scale would also be relevant.

The spatial model lends itself well to a hierarchical structure, reflecting the geologist's need for an overall, then increasingly detailed, view of a region. He might follow a regional overview by more detailed parallel investigations of, say, structure, stratigraphy and palaeogeography. Just as geological maps are available at several fixed scales and with predetermined sheet boundaries, so the spatial model could be available at several levels of detail and resolution, subdivided at each level into segments for defined areas, topics and stratigraphic intervals. The model segment would probably be a subdivision of the existing map sheet, of a size convenient for graphic display. Each segment could carry a summary of its contents and pointers to sources of detailed information at lower levels in the hierarchy. This design would give the basis for a well-structured dialogue between geologist and computer, with the geologist refining and narrowing the discourse by selecting relevant information, the computer searching for wider implications and introducing more detail.

The model can only be constructed piece by piece, with separate groups of geologists working on individual segments. Each segment must conform with those for other areas and other resolutions. The model should therefore be designed to assist in maintaining the correct relationships between components and ensuring that the necessary interactions can be achieved. The present preliminary investigation can guide further work by indicating topics where existing methods are inadequate to support the model and where further experimentation is needed to decide between alternative methods. To avoid extensive back-tracking, or becoming locked in to inappropriate solutions, these points must be resolved and a comprehensive design tested before full-scale implementation.

Methods

It may be desirable to hide the internal complexities of computer representation from the user, but the methods and the design should be clearly visible, so that at each stage the geologist can control the model, and understand the implications of his instructions to the computer. Only an outline of the proposed methods can be attempted here, but additional reports dealing with individual aspects of the model are in preparation. Several requirements of a spatial model are unusual in current geological applications, and at first sight the problems of implementation seem quite formidable. However, when the model is considered in its entirety, it appears that many of the apparent difficulties reflect incompatible views of parts of the same problem, and can be solved by a unified approach. Once a satisfactory overall design has been achieved, it need not all be implemented simultaneously, provided that there can be good interchange of information between the geologist and the computer. This must rely on effective graphic presentation.

The importance of the interpretative element and the consequent need for a model rather than a database has already been stressed. Generally, the entities of interest are surfaces or solid bodies which have to be reconstructed from rather haphazardly distributed data at points or on lines. Interpolation of lines and surfaces in a spatial model must be piecewise, that is, the surface is regarded as composed of a set of segments each of which is separate but related to adjacent segments. This is necessary because of the complexity of the geological surfaces. It is also desirable because many formative processes were local in their effect and the model should be capable of reflecting their variation. The piecewise approach makes it easier to introduce discontinuities, such as faults, which would be represented as segment boundaries. Changes to the model, resulting from, say, a new well or seismic line, can be localised, thus keeping the computing load within bounds. Changes of this kind can be monitored to ensure that the cumulative results of many small changes lead when necessary to changes at other levels of the model. As the segment size can be varied, the full detail of the data can be reproduced in the model.

At digitizing time, lines can be segmented into nodes and links, comparable to the structure used by the Ordnance Survey (Thompson, 1978) with nodes at all line intersections and elsewhere as required for ease of editing (Loudon, et al., 1980). Surfaces known along lines, such as seismic lines, can be divided into segments (patches) bounded by lines. Surfaces known at points can be segmented by triangulation (McCullagh and Ross, 1980; Nelson, 1978). The vertices of the patches can be regarded as nodes. An appropriate data structure can link the nodes and the segments (Gold et al., 1977). The need to link patches between and within surfaces and to subroutines and parameters for interpolation calls for network structures of greater complexity than is usual in geology.

Low-order power-series polynomials seem most appropriate for interpolating within segments of lines or surfaces, for reasons explained in the literature on finite element methods and computer graphics (for example, Strang and Fix, 1973; Rogers and Adams, 1976). A convenient procedure might be to determine elevation, slope and curvature at nodal points, at a resolution appropriate to the size of the segments to be interpolated. Data from two nodes could define a cubic polynomial for a line segment, from three nodes a quintic polynomial for a triangular patch, and from four nodes a bicubic polynomial for a quadrilateral patch. Points, lines and surfaces would thus match across patch boundaries with second order continuity at nodes. Discontinuities of elevation or slope (faults or hingelines) can be introduced as required by the geologist, by specifying nodes in which one or more of the values of elevation, slope and curvature differ by a stated amount depending on the side from which the node is viewed.

The elevation, slope and curvature of the surface in the vicinity of a node can be calculated at more than one resolution to allow for composite surfaces which combine features of different sizes. These nodal data can provide orientations of lines of curvature, useful in analysing the form of the surface and the pattern of deposition or structural deformation. They can also be summarized statistically to describe shape characteristics or transformed geometrically to reconstruct a surface which fits the elevation data but also conforms to some expected shape characteristics. Analysis of the nodal data can indicate the proportion of variability of surface elevation due to irregularities of different size and orientation. Background knowledge may suggest estimates for variability over distances less than the data spacing. The resulting graph of variability versus distance, akin to the variogram of geostatistics (Journel and Huijbregts, 1979), is a basis for defining a probability envelope around the interpolated surface, indicating the limits within which the real surface is likely to lie, and the extent to which the real surface is likely to be more variable than the interpolated one.

There are spatial relationships between entities, both observed and reconstructed, such as: coinciding with; near to; above; inside; containing; bounding; overlapping; parallel to; converging with; crossed or cut by; displaced by; faulted against; unconformable on; oblique to; asymptotic to; continuous with; grading into; interfingering with; adjacent to; touching; branching; accentuating; together with their opposites and approximations. Spatial relationships may be significant in analysis, interpretation, retrieval and display and should therefore be held in the model. They apply to probability envelopes as well as to interpolated surfaces, on the grounds, for example, that there is zero probability of separate and distinct rock bodies simultaneously occupying the same space. Although no comprehensive account can be given here, it appears that many spatial relationships can be defined in terms of equalities or inequalities, continuity, restricted geometrical transformations, dependencies and set memberships, all of which can be handled in computer analysis. It is desirable, not only to record spatial relationships in a model, but also to link in the inferred consequences.

The spatial model as described above provides a means of estimating the location in space of a defined geological entity. The end product is either information for further computation or for display. There is, however, another class of requirements which calls for determining the geological entities present at a specified point, or within an area or volume, defined by spatial coordinates. The conventional map is well suited to retrieval either by entity or location. On the one hand, the use of legends, colours and symbols leads the eye from an index of geological entities to their positions on the map. On the other hand, the grid of geographic coordinates, or the topographic base, provides a convenient means of locating an area, and inspecting the geological entities present there. For searching the computer model in this way, it would be possible to create an inverted file on a fine coordinate grid with pointers to the geological entities at each point. But even with data compression, this unwieldy mechanism would probably not be well suited to the geologist's needs. Instead, it is suggested that the model should be stored hierarchically on a coarse geodetic grid, possibly with position on a medium grid stored as an attribute for selective retrieval. Retrieval at a more detailed level would not be by computer but by inspection of a display of the relevant entities, thus ensuring that the retrieved information is seen in context.

Analysis is likely to involve comparison of patterns rather than individual points, and this requires further development of techniques for statistical description of form and spatial pattern. The spatial model concerns interpretations of material as well as geometry and the interaction of the two aspects in stratigraphic classification has yet to be placed on an explicit, objective basis for computer analysis. Methods of computer interpolation to reconstruct lines or surfaces between data points must also be reconsidered to take into account the full range of available, relevant geological information.

There are extensive software requirements for a spatial model. At the early exploratory stage, it may be best to meet these as far as possible by the use of existing programs for contouring, graphic display and data management, even at the cost of frequent reformatting to move between systems. In the longer run, it seems preferable to restrict the spatial model in the strict sense to data, estimates and opinions related directly to the interpretation of the geometry and materials of the chosen region, and software which is specifically and solely required for modelling purposes. Programs with wider applicability, for data management, display and analysis are probably better regarded as being part of the computer system in which the model is embedded, and thus available for other purposes. Predictive models in specialist areas, such as gravity or stream sediment geochemistry, could usefully be based on information from the spatial model, thus checking that the model is consistent with these important sources of information. They need not, however, be an integral part of the model.

There is also a need to link the model to data banks which are maintained independently of the model, such as a topographic data base. A referencing system will therefore be required within the model to identify links or sources of information. Extensive text files will be needed for explaining decisions and giving additional information, but again these may best be regarded as external files accessed through the referencing system. In the medium term, it seems likely that computers will be increasingly used, for economic reasons, in the production of conventional geological maps. Before such systems are implemented, it will be desirable to ensure that easy exchange of digitized data is possible between the map production line and the spatial model, with a view, in the longer term, to integration of the two activities.

Future work

The basic activities of geological surveying, field and subsurface mapping have so far been affected only marginally by computer methods. The present assessment suggests that there are major long-term advantages in linking these activities to comprehensive spatial models implemented on the computer. The model would fit well with future methods of exchanging information. It can offer greater flexibility, precision, comprehensiveness and efficiency than conventional methods, and overcome the limitations of static two-dimensional displays. It can provide a focal point for computer applications in geology.

A unified design is the key to obtaining the full benefits of the model. However, gaps are apparent in existing computer methods in the following areas:

1) shape description;
2) interpolation utilising background knowledge;
3) use of spatial relationships;
4) structuring spatial data;
5) quantitative stratigraphy.

Work is in progress to consider these aspects and to document suggested procedures more fully.

Experimentation with real geological examples will be essential to determine the relative contribution of various aspects of geological information to the accuracy of the model, and for this small-scale implementations on the computer are required. The subsequent development of a robust production system still lies some years ahead. As such a system would affect the pattern of geological investigation, it should be developed through close collaboration with a broad range of geologists.

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