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NATURAL ENVIRONMENT RESEARCH COUNCIL

INSTITUTE OF HYDROLOGY

SUBSURFACE SECTION

REPORT NO.66

PLATEAU TIMES FOR DILUTION GAUGING BY THE CONSTANT  
RATE INJECTION METHOD.

K GILMAN  
AUGUST 1976

Institute of Hydrology  
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## Plateau times for dilution gauging by the constant rate injection method

The constant rate injection method has been used in calibrating the Plynlimon structures, following the procedure outlined by Greenland (1975). Greenland recommended the use of the formula

$$t_p = t_a + 2t_d$$

to estimate the time taken to reach steady (plateau) conditions, where  $t_a$  is the time of arrival of a fluorescein slug at the sampling point, and  $t_d$  is the time from arrival to disappearance. The injection of the fluorescein tracer for each gauging is inconvenient and indeed unnecessary, so it seems likely that in the field operations this part of the procedure has been discarded in favour of a more subjective means of estimating the plateau time. In view of the demands on the time of field staff, it could be expected that this subjectively estimated time would tend to be an underestimate rather than an overestimate, and that there would be a real danger of failure to achieve plateau conditions. This is particularly so as the systematic error due to this cause is poorly understood.

This report describes experiments with salt injections and a recording conductivity meter, which show that in many cases the time taken to reach plateau has been underestimated, resulting in a serious overestimate of the stream discharge.

## The relation between residence time and plateau time

The residence time of a fluid element in a gauging reach is the time between entry at the injection point and exit at the sampling point. Obviously residence times vary a great deal, depending on the path taken by the element through the reach: for instance a deep pool (e.g. the sediment traps) can add many minutes to the residence time, and while some of the water will cross the pool quickly, much will be caught up in the large scale circulation of the pool and will reach the outlet much later. The residence times can be measured by performing a sudden injection of tracer at the injection point, and recording the concentration of the tracer at the sampling point. As all the marked elements passed the injection point at the same moment, the area of the concentration-time curve (neglecting background) to the right of a given ordinate  $\theta$ , expressed as a fraction of the total area, gives the fraction of the discharge with

residence times greater than  $\theta_1$  (Figure 1). The concentration-time curve, when normalised to unit area, defines a *residence time distribution*,  $E(\theta)$ . The point at which this function starts to rise from zero is the arrival time  $t_a$ , while the centroid of the area defines the mean residence time (or time of travel)  $\bar{\theta}$ .

The rise of downstream concentration to a plateau level during a constant rate injection is governed by the form of  $E(\theta)$ . In fact the downstream concentration  $C_2$  is given by

$$C_2(t) = C_o + (C_p - C_o) \int_0^t E(\theta) d\theta$$

where  $C_o$  is background concentration and  $C_p$  is the plateau concentration at the sampling site. For a more detailed explanation of the rise to plateau the reader is referred to the paper on changing discharge by Gilman (1975). The error in the discharge estimate due to failure to reach plateau is

$$\delta(t) = \frac{1}{\int_0^t E(\theta) d\theta} \quad (\text{see Gilman 1975})$$

So if  $t$  is large the error  $\delta(t)$ , which is a fractional error (e.g.  $\delta(t) = 0.10 \bar{=} 10\%$  of discharge), tends to zero, while if  $t$  is small the integral is less than 1 and  $\delta(t)$  is positive, i.e., the discharge is overestimated. Approximately, if the integral is 0.99 the systematic error will be 1% and so on. The form of  $E(\theta)$  is usually such that the integral never quite reaches unity, so that we have to be content with a value of say 0.995 and define the time taken to reach this value as  $t_p$ , the plateau time.

#### Time of travel studies in the minor structures

Table 1 summarises the experimental results obtained from five of the minor structures at Plynlimon. All of the experiments were performed at low flows, when plateau time problems can be expected to be most serious. The residence time distributions were obtained by injecting sufficient salt to reach a peak concentration of about 50 mg/litre at the sampling point. Injection points were at likely sites for a gauging injection and were sketched carefully to ensure that they would be located accurately on future occasions. The sampling points were at the nearest convenient spot to the foot of the flume.

The Hafren flume proved to be difficult; residence time distributions were not constant with time. It was thought that this was due to the large, slowly circulating pool immediately upstream of the ramp. However the general principles of this report probably apply to the Hafren, and even to the Severn trapezoidal flumes.

Figure 1 shows a typical residence time distribution. The tail of the curve was produced to the right (beyond the range of the field measurements) by an exponential approximation, and the total area under the conductivity-time curve was determined using Simpson's rule. This area was then used to *normalise* the curve, i.e., adjust the values of the ordinates so that the area of the curve in the figure is unity. Using Simpson's rule, the area to the right of any given ordinate, for example the value  $\theta$ , can be calculated. When this area is plotted on log probability paper against a time axis a straight line is obtained, as was to be expected from the similarity of the curve to a lognormal probability distribution function. For the time axis, a ratio of the time elapsed since injection to the mean residence time  $\bar{\theta}$  was chosen. Using the straight line, it is possible to determine the time at which 99% or 99.5% (for example) of the area of the curve has passed, leaving 1% or 0.5% respectively in the 'tail'. Figure 2 shows a typical log probability plot. The extrapolation is lent some support by the fact that, in the two cases when the experiment continued until the 1% point was passed, the straight line continued to be a good approximation (Figure 3).

Using the 0.5% point as the plateau time (see previous section) Table 2 summarises the results for the 16 residence time distributions of Table 2, the plateau times being expressed in seconds and as a multiple of the arrival time  $t_a$ . Both columns show a large amount of scatter, but a plot of the second column of values A, where

$$t_p = At_a$$

on probability paper suggests that an upper limit of 14.5 times the arrival time will be adequate in 97.5% of cases (29.6 A being treated as a result from a distinct distribution). This contrasts strongly with the Water Research Association manual (WRA 1970) which gives a value of 3.5 times the arrival time as a 'safe overestimate' of  $t_p$ .

## Comparison with gauging practice

Using the value  $A = 14.5$ , the average value of plateau time for the results of Table 1 is 53 minutes: for the one 'rogue' result 29.6A  $t_p$  is 31 minutes. Table 3 shows the sampling periods used in gaugings of low flows in the minor structures: these are all the available gaugings of flows under 50 l/s for the five structures considered. Five of the gaugings, those asterisked, had injection points on the flume ramp, and should be compared with 29.6A. Out of a total of 11 gaugings, only two have sampling periods exceeding the plateau times mentioned above (53 minutes and 31 minutes), and in only two cases was any estimate of plateau time recorded.

From the 16 log probability graphs (of which Figure 2 is a representative) it appears that a 50% underestimate of plateau time can result in a positive systematic error of from 5 to 20%. It is therefore very important to ensure that the plateau condition has been reached.

## Conclusions

Failure to achieve plateau conditions is a possible cause of systematic error in constant rate injection dilution gauging. It is recommended that a return be made to the practice of determining plateau times by dye injection, either using Greenland's formula

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Where gaugings are performed entirely in the flumes, the plateau time is far longer than would be expected intuitively - it is recommended that a minimum of 30 minutes be used.

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4.5		33	26.0	89	270
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31.5 B		33	12.6	160	492
31.5 D		66	12.6	472	1755
29.6 A*		13	4.1	57	161
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3.8		33	5.0	279	901
4.8 A	Iago	43	4.5	223	811
4.8 B	"	43	4.5	275	800
4.8	Gwy	53	22.0	144	338
4.8	Cyff	64	9.6	388	1006
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Date	Structure	Plateau time (0.5% point)	
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31.5 B		1671	10.4
31.5 D		5492	11.6
29.6 A		1871	33.1
29.6 B		4971	19.8
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$$t_p = t_a + 2t_d$$

or the simpler formula developed in this report

$$t_p = 14.5 t_a$$

The transport processes in the river are so complex that a subjective estimate of plateau time is inadequate, particularly under the constraint that gaugings should be made as quickly as possible in order to free staff for other commitments (or other gaugings).

Where gaugings are performed entirely in the flumes, the plateau time is far longer than would be expected intuitively - it is recommended that a minimum of 30 minutes be used.

## References

- GILMAN K 1975 Application of a residence time model to dilution gauging with particular reference to the problem of changing discharge, Bull. intern Ass Sci Hydrol. 20(4), 523-537
- GREENLAND P C 1975 Dilution gauging - an example using Cyff flume calibration No. 6 31.1.75, Inst. Hydrol. Subsurface Section Report 65
- WATER RESEARCH ASSOCIATION 1970 River flow measurement by dilution gauging, Water Res. Ass. Tech. Paper TP74

Table 1 Salt injection experiments at the minor structures 1976

Date	Structure	Length of gauging reach	Discharge (from latest flume calib.)	Arrival time	Mean time of travel
13.4	Tanllwyth	33m	13.5 l/s	124 sec	461
21.4 A	"	33	13.5	143	448
21.4 B		33	13.5	153	433
4.5		33	26.0	89	270
31.5 A		33	12.6	170	518
31.5 B		33	12.6	160	492
31.5 D		66	12.6	472	1755
29.6 A*		13	4.1	57	161
29.6 B		33	4.1	251	1281
3.8		33	5.0	279	901
4.8 A	Iago	43	4.5	223	811
4.8 B	"	43	4.5	275	800
4.8	Gwy	53	22.0	144	338
4.8	Cyff	64	9.6	388	1006
5.8	Hore	34	18.0	115	348
5.8	Tanllwyth	33	4.1	308	974

\*injection point on flume ramp.

Table 2 Plateau times

Date	Structure	Plateau time (0.5% point)	
		Seconds	x arrival time
13.4	Tanllwyth	1552	12.5
21.4 A	"	1645	11.5
21.4 B		1493	9.7
4.5		850	9.6
31.5 A		1989	11.7
31.5 B		1671	10.4
31.5 D		5492	11.6
29.6 A		1871	33.1
29.6 B		4971	19.8
3.8		3551	12.7
4.8 A	Iago	3252	14.6
4.8 B	"	3110	11.3
4.8	Gwy	872	6.0
4.8	Cyff	2687	6.9
5.8	Hore	884	7.7
5.8	Tanllwyth	3369	10.9

Table 3 Gaugings in the minor structures

Gauging	Discharge (measured by dilution gauging)	Estimated plateau time	Sampling period
Hore 53	26 l/s		30-42 min
Hore 79	39 l/s		31-39 min
Iago 13	27 l/s	10 min	30-42 min
Iago 61	20 l/s		30-38 min
Iago 82	11 l/s		10-18 min
Iago 94*	8 l/s		5-16 min
Gwy 54	40 l/s		20-32 min
Tanllwyth 78*	8 l/s		15-23 min
Tanllwyth 86*	32 l/s	10 min	10-21 min
Tanllwyth 104*	11 l/s		30-41 min
Tanllwyth 105*	8 l/s		30-41 min

\* denotes a gauging with an injection point on the flume ramp