A REVIEW OF CONCEPTUAL RAINFALL-RUNOFF MODELS USING DIGITALLY DISTRIBUTED DATA

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Abstract

Distributed rainfall-runoff models are usually associated with physically-based models which require the use of complex numerical solutions schemes and are difficult to apply. There exist, however, a few data distributed models which are based on simple concepts and can be envisaged for operational applications. These are reviewed in this document. Suggestions are then made for the use in flood modelling supported by digital of river network, elevations and soil-types maps available at IH.

Contents

Executive summary

A review of simple conceptual rainfall-runoff models using digitally distributed data is presented. The objective is to assess the potential offered by the digital maps being produced at IH for the prediction of floods. Distributed models, like SHE or IHDM, are deliberately omitted, as their application is very demanding in terms of data and calibration time and are not yet ready for operational use. Only a limited number of models were found in the literature survey, reflecting perhaps the fact that this is a new area of research and that there is scope for new developments. Six models are described, four using a map of the river network, and two using a Digital Terrain model.

MODELS USING THE RIVER NETWORK

The models based on the river network are quite similar, in their principle, to the time-area diagram: they synthesise catchment response from the knowledge of isochrones throughout the catchment and the knowledge of the spatial distribution of effective rainfall.

The Geomorphological Instantaneous Unit Hydrograph (GIUH) is a well known model relying on this idea. Like the time-area diagram, it assumes that the effective rainfall is uniformly distributed over the catchment, so that the amount of flow reaching the outlet after a given time, say t, is directly proportional to the area of the catchment corresponding to a travel time of t. However, the resemblance with the time-area diagram stops at this point. The GIUH is a probabilistic model in which the movement of water is represented by the passage through a chain of probabilistic states, rather than by the transport in the river. This model quite complicated and subject to many criticisms in the literature.

The three other models are simpler than the GIUH and have a better physical basis. They are also more closely related to the principle of the time area diagram. Like the latter, they assume that the time of travel in the catchment is a direct function of the distance to the outlet and use the river network to specify the distribution of points situated at a certain distance in the catchment. The distribution of travel times can then be deduced by specifying the velocities along the flow paths, or alternatively, the probability distribution of the time spent along them. Catchment response can then be constructed from the map of isochrones.

If rainfall is assumed to be spatially uniform, catchment response can be represented by a unit hydrograph. Two of the models adopt this assumption. One proposed by Calver et al (1972) is very simple. The second one, proposed by Mesa and Mifflin (1986), is rather complicated and difficult to apply, showing that simple concepts can result in complicated models in terms of their mathematical formulation. One of the differences between the two models comes from the way they represent the transport of water in the network. In the first model, a simple translation is used and the velocity is assumed to be spatially uniform. In the second, the transport is modelled

as a convection-diffusion process which is kept as simple as possible but nevertheless results in complicated equations. The third model, proposed by Surkan (1969) is both simple and flexible. It assumes that the velocity of water is constant everywhere in the network, but unlike the two other models, it allows a spatially variable input.

Models based on the river network share a common feature which is the modelling, in a deliberately simple way, of the transport of water in the river network. The representation of subsurface flow is however either neglected or simplistic. This shortcoming is remedied in the two topography-based models which focus on the generation of flows fromt hillslopes.

MODELS USING A DIGITAL TERRAIN MODEL

TOPMODEL, proposed by Beven and Kirkby (1979) and teh Saturation Zones Model, proposed by O'Loughlin (1981), are both based on two assumptions which allow them to represent, very simply, the distribution of wetness in the catchment and also the baseflow. The first hypothesis concerns the representation of subsurface flow for a given profile. It is assumed that at a given point, subsurface flow is a function of the local slope and wetness of the profile. The steeper the slope or the wetter the profile, the larger the flow. The second hypothesis is that subsurface flow is uniformly distributed in the catchment. It varies in time as a function of catchment wetness but does not vary in space. So, if two points drain equal areas, they will have the same subsurface flow. If, they happen to have different slopes, the wetness will be different too, to accommodate the requirement for equal flows. This shows the role played by the topography in predicting the spatial distribution of wetness in the catchment. Both models relate the baseflow of the catchment to its overall wetness and can then predict the wetness at a given point from the knowledge of the overall wetness, the local slope and the area drained.

There are however differences between the two models. One difference concerns the representation of subsurface flow in unsaturated zones. The second difference is in their application. The Saturation Zones Model is an event-based model which allows the contributing area to vary between events, but not within an event, whereas TOPMODEL works on a continuous basis, allowing the contributing area to vary continuously.

Although the transmissivity of the soil is accommodated in both models, it is assumed to be spatially uniform. The influence of soil-type is therefore not considered in a distributed way and this is true for all the models of the review. This can be explained by the fact that digital maps of soil-types are rare. It might also be due to the difficulty of incorporating this type of information in a model.

SUGGESTIONS FOR FUTURE WORK

The Institute of Hydrology is producing digital maps of catchment geomorphology for the whole of Great Britain. The types of maps being made available are: a map of soil-types (HOST), a Digital Terrain Model and a map of river networks. These contain all the information required for the application of the models reviewed. The Digital Terrain Model and the river network can be used together to model the generation and the transport of flow. The map of soil-types (HOST) could also be used if, for instance, it could provide values of transmissivity or soil depth or the depth of the water table. A framework is proposed, in which various models could be explored and developed. For example, Surkan's assumption of a constant velocity in the network could be relaxed and a velocity varying with the area or the wetness of the catchment could be introduced. Event data as well as continuous hydrographs could be modelled, the latter being useful for the modelling of subsurface flow.

Introduction

The Flood Studies Report (1975) provides a method of deriving the distribution of annual maximum flows by means of a rainfall-runoff model, for any site in the United Kingdom. This method has since been widely used and its performance and limitations are now well known. For instance, Boorman et al (1990) in an assessment of the method found that it tends to overestimate the flow corresponding to a given return period.

Sixteen years later, it is worth updating the method. First, because new data are now available, either as extended hydrological records, or in the form of digital maps which provide spatial information about catchment physiography. Second, because a new generation of computers allow, if necessary, more complex models to be applied. Finally, because new expertise has been gained which might be used in the method.

In order to come up with a method of flood frequency estimation, a number of steps must be undertaken:

1. A rainfall-runoff model must be chosen and calibrated at gauged sites
2. A method of flood frequency derivation must be selected and applied to the gauged sites
3. A way of applying the method at ungauged sites must be devised

The present review is intended to assist in the choice of a model and does not consider steps 2 and 3. It does not consider all existing types of models, but is restricted to those which use distributed data to describe catchments, and at the same time are simple enough to be used in operational applications. The reasons for selecting this type of model, rather than other types are given below.

There is a large number of rainfall-runoff models available which might be classified by increasing order of complexity, as follows:

1. Lumped models
2. Semi-distributed models
3. Conceptual distributed models
4. Physically-based distributed models

Physically-based models, like SHE or IHDM, are thought to be too demanding in terms of data and computing requirement and are not yet envisaged for operational use. Lumped models, on the other hand, are too simple, in that they cannot fully exploit the potential offered by the spatial representation of catchment characteristics. In addition, it is quite likely that a lumped model would not improve significantly on the unit hydrograph used in the Flood Studies Report. Semi- distributed models, like those used in urban hydrology are not fundamentally different from lumped models. However, they are not very easy to apply, because they require the subdivision of the catchment into sub-catchments whose choice is not always easy to determine.

Conceptual distributed models appear as a good compromise between physically-based and lumped models. They are fairly simple to apply and at the same time incorporate spatial information about the catchment. This last feature makes them interesting, for two reasons: first, it allows a spatial description of hydrological properties, like for instance soil moisture. Second, it provides a better way of describing catchment properties and as such should improve the transfer to ungauged sites.

Reviews of conceptual data distributed models can be found in the literature, for instance by Beven et al (1988), Gupta and Mesa (1988), Bras and Rodriguez-Iturbe (1989) and Bras (1990), each one being different, in the level of details given, in its objectives or in the range of models described.

The purpose of this review is to describe and classify both the concepts involved in the modelling and the type of maps used. The classification turns out to be quite easy, as the models seem to be tailored to suit the data available. Two groups of models can be distinguished: models of the first group aim at routing the effective rainfall through the river network, thus giving emphasis to the timing of floods. This type of model only requires a map of the rivers and is usually applied to large catchments in which hillslope effects are relatively unimportant by comparison with the river effects. Models of the second group use the topography to represent the spatial variation of wetness in the catchment, thus emphasizing the production of flow volumes. This type of model only needs a Digital Terrain Model (DTM) and is usually applied to small catchments in which hillslope effects are dominant. It should be noted that only a small number of models were found through the survey, and that to the author's knowledge, none uses soil-types.

The plan of the review follows the above classification: the first part describes and discusses the models using the river network, the second part describes and discusses those using the topography. A third part is then added which gives suggestions for future work. In the description of the models, only the part concerning the use of distributed data is considered. This means that other aspects, like the modelling of interception and evaporation, or the calibration method are not included.

1 Models using the river network

The models based on the river network use the concept of travel times and as such can be considered as deriving from a common ancestor: the time-area diagram, which is briefly described in section 1.1.

These models need to map travel times for the whole catchment and to achieve this, they need to map all possible paths to the outlet and then specify the time spent travelling along them. The classification adopted here distinguishes two groups of models by the way the paths are defined.

In the first group, described in section 1.2, the paths are represented by a chain of probabilistic states. In the second group, described in section 1.3, the paths are represented by the trajectories of the runoff. In some cases, the description of the model refers to concepts which are used to characterise river networks. A summary of the terminology and concepts involved is given in the Appendix.

[Section 1.2 which describes a probabilistic model is longer than all other sections, and this is uniquely due to the complexity of the model. It does not necessarily reflect the importance given to it.]

1.1 THE TIME AREA DIAGRAM

The time-area diagram was proposed by Ross (1921) and popularised by Clark (1945). It assumes that catchment response to effective rainfall is linear and time invariant, and thus can be represented by a unit hydrograph. The unit hydrograph can be synthesised from the knowledge of isochrones in the catchment, as illustrated in Figure 1. If, for any point in the catchment, the travel time to the outlet is known, it is possible to construct isochrones, as shown in Figure 1(a). The isochrones shown on this Figure define class limits of travel times with an interval of one hour. This map of isochrones can be used to derive the area of the catchment corresponding to each class, as illustrated in Figure 1(b). This diagram, if normalised (by dividing the areas by the total area and by the class intervals), can be interpreted as the unit hydrograph of the catchment.

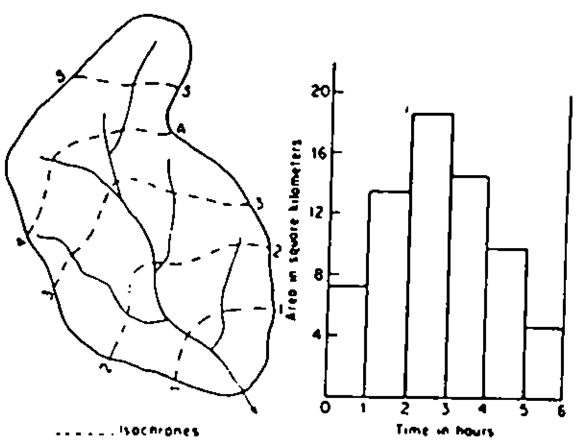

Figure 1 Time area diagram (from Linsley et al., 1982)

Although the time area is very simple in its principle, in practice, it is not easy to apply, because the method does not specify how the isochrones may be derived. The models described below attempt to overcome this deficiency by specifying the spatial distribution of travel times to the outlet. This can only be achieved by listing all the possible paths (or at least a sample of them) and the time spent along each one. The specification of travel times from distributed data is therefore one direction in which the development of the method has been taken. Another development, which has been taken by one of the models in section 1.3, relaxes the assumption of time invariance associated with the unit hydrograph. In this model, a spatially variable rainfall input is allowed and this results in a variable response at the outlet.

Like the time area diagram, the model described in the next section uses the concept of unit hydrograph which it derives from the spatial distribution of travel times. The resemblance, however, stops at this point, because this model is entirely probabilistic, both in the way it describes the paths and in the way travel times are represented. The paths are conceived as the passage through a Markov chain whose states represent different types of channel. The probability distribution of waiting times is specified for each state, so that travel times, expressed in terms of a probability distribution, can be estimated for each possible path.

1.2 A MODEL WITH PROBABILISTIC FLOW PATHS: THE GEOMORPHOLOGIC INSTANTANEOUS UNIT HYDROGRAPH (GIUH)

The GIUH is the only model in this review which is entirely probabilistic. Both the path and travel time along this path are defined in terms of probabilities. Part 1 of this section gives a detailed description of the model as it was presented originally. The model was later restated in a simpler and more general form and this alternative representation is given in part 2. Part 3 briefly describes a few variants and extensions which have since been introduced. Part 4 then gives an account of the comments and criticisms found in the literature about the GIUH.

(1) The original model

The GIUH was first introduced by Rodriguez-Iturbe and Valdes (1979). At the basis of the model are the following hypotheses:

1. Catchment response to rainfall is linear and time invariant, although it can vary from storm to storm.
2. The effective rainfall input is known.
3. The whole basin contributes to runoff.
4. Hydrological properties of the river network are adequately represented by the Strahler ordering scheme (see Appendix).
5. The time spent in a given stream is exponentially distributed. The parameter of the distribution depends on the order of the stream and is a function of its length.
6. The time spent in the hillslope phase is negligible by comparison with the time spent in the river.

Rodriguez-Iturbe and Valdes point out that, the time integral of the IUH which represents the proportion of input water to have left the basin by a given time, also represents the probability that a drop of water chosen at random will take less than a given time to reach the outlet. So, like in the time-area diagram, the instantaneous unit hydrograph can be derived as the probability density function of the time spent in the catchment.

The migration of water is modelled as the passage through a chain of hypothetical states which are defined by the Strahler ordering scheme. If a particle is in a channel of order i, it is said to be in state i ($i=1,2,\ldots,\Omega$), where Ω is the order of the catchment (see Appendix). A map of the catchment is used to define drainage divides and it is assumed that, if the particle is in the part of the hillslope phase which drains to a channel of order i, then the particle is in state i. In this way, the state of the particle is defined at any location in the basin.

A particle starting its journey in a given state travels through the catchment, making transitions from states of lower order to those of higher order, until it reaches the outlet which has state $N=\Omega+1$. It is important to note that, although the successive states take increasing values, the increment being 1 or greater. For example, if the initial state is 2, the state which follows can be 3, as well as 4, 5, ... or $\Omega+1$.

In order to determine the probability density function of waiting time in the catchment, i.e. the GIUH, the following parameters have to be defined:

1. The initial probabilities which express the probability of starting the migration in a given state.
2. The transition probabilities, p_{ij}, which express the probability of moving from state i to state j.
3. The probability distribution of the time spent in a given state.

Initial probabilities

A drop of water chosen at random from a spatially uniform storm is equally likely to fall anywhere on the catchment. So, it starts its journey in state i with probability

$$\theta_i(0)\ A_i/A, \qquad i=2,3\ldots,\Omega \tag{1}$$

where A_i represents the area draining to streams of order i and A is the area of the basin.

Transition probabilities

After spending some time in state i, the drop makes a transition to a different state, say j, with probability p_{ij}, where

p_{ij}=(number of streams of order i draining into streams of order j) / (total number of streams of order i).

The values of p_{ij} are easy to calculate if a map of the river network is available.

Probability distribution of travel times

The probability distribution of the waiting time in state i is assumed to be exponential with distribution function

$$F_i(t) = 1 - \exp(-\alpha_i . t), \qquad i = 2,3,...,\Omega \tag{2}$$

The value of α_i is calculated as

$$\alpha_1 = v/L_i, \tag{3}$$

where L_i is the average length of streams of order i. This relationship between α_i and L_i is the consequence of two assumptions:

1. The time spent on the hillslope is supposed to be small compared to the time spent in the river. The time spent in state i is then taken to be the time spent in a stream of order i.
2. The velocity is spatially uniform in the river. So, the average time spent in a stream of order i is $m_i = L_i/v$. Hence, because the distribution is exponential, $\alpha_i = 1/m_i = v/L_i$.

Derivation of the GIUH

Let $N = \Omega + 1$

- **P** be the (N,N) matrix of transition probabilities p_{ij}
- **A** be a diagonal (N,N) matrix of the parameters α_i
- **I** be the (N,N) identity matrix
- $\Theta(t)$ be the (N,1) matrix of probabilities $\theta_i(t)$=P(a drop of rain selected at random is in state i at time **t**)
- $\Phi(t)$ be a matrix giving the probabilities of going from one state to another after a time t.

If the holding times are independent of the destination and are exponentially distributed, then the process is Markovian and $\Theta(t)$ is given by:

$$\Theta(t) = \Theta(0).\Phi(t) \tag{4}$$

where $\Phi(t) = \exp[A(P-I)t] = \exp[Xt]$ and where $\exp(Xt)$ is defined as $I + Xt + (X^2t^2/2!) + ...$

The authors give a method for solving this equation and give its solution for a third order basin. The expression of this solution is quite complicated and is not reported here.

The GIUH, h(t), is obtained by differentiating the last term of $\Theta(t)$ with respect to t:

$$h(t) = d\theta_N(t)/dt = \sum_{i=1}^{N} \theta_i(0).d_{iN}(t)/dt \tag{5}$$

where ϕ_{iN} is an element of the matrix Φ and $\theta_i(0)$ is an element of $\Theta(t)$.

In this way, an expression for the unit hydrograph is given as a function of the transition probabilities, the proportions of the various states and the travel time parameters (p_{ij}, $\theta_i(0)$, α_i, $i=1,2\ldots N$). These parameters are determined by the structure of the river network and the velocity of water in the channels which is unknown and is obtained through calibration.

The procedure described above is quite lengthy and difficult to apply. In order to simplify the method, Rodriguez-Iturbe and Valdes have taken the method a step further, in which the time-to-peak and flow peak of the GIUH were related, by statistical analysis to the geomorphologic parameters of the river network.

In this further step, Rodriguez-Iturbe and Valdes derived the GIUH for a set of four natural and three synthetic networks characterised by various basin orders, Horton ratios channel velocities and link lengths. The values of velocity, time to peak and flow peak were obtained from a rainfall-runoff model and then regressed against these parameters. The relationships established are:

$$q_p = \frac{1.31\ R_L^{0.43}\ v}{L_\Omega} \tag{6}$$

$$t_p = \frac{0.44\ L_\Omega\ (R_B/R_A)^{0.55}}{R_L^{0.38}\ v} \tag{7}$$

If the GIUH is approximated by a triangle, these expressions are then sufficient to its specification.

(2) A simpler and more general description of the Giuh

Gupta et al (1980) have given a simpler and more general formulation of Rodriguez-Iturbe and Valdes's model of drop travel times in the basin. The states of the process are defined to be a_i, for a hillslope region draining to a stream of order i, and r_i, for a stream of order i. A drop falling randomly on the catchment, can follow a finite number of paths to the outlet. The number of possible paths is $2^{\Omega-1}$. For instance, for a third order basin, there are 4 possible paths given by:

s_1: $a_1 \rightarrow r_1 \rightarrow r_2 \rightarrow r_3 \rightarrow$ outlet
s_2: $a_1 \rightarrow r_1 \rightarrow r_3 \rightarrow$ outlet
s_3: $a_2 \rightarrow r_2 \rightarrow r_3 \rightarrow$ outlet
s_4: $a_3 \rightarrow r_3 \rightarrow$ outlet

It should be noted that the paths as defined here or in the paper by Rodriguez-Iturbe and Valdes, are not synonymous with trajectories: each path may represent a number of different routes, all characterised by the same succession of states.

The cumulative density function of the travel time to the outlet, for a basin of order Ω, is then:
where $S=\{s_1,s_2,\ldots\}$,

$$P(T_B<t) = \sum_{i=1}^{N} P(T_{si}<t)\ P(S_i) \tag{8}$$

Ts_i is the travel time in path s_i,
$P(s_i)$ is the probability of following this path,
$N=\Omega+1$ is the state of the outlet.

The travel time Ts_i is the sum of travel times in the different states which constitute path s_i. Its probability density function is then the convolution of the probability density functions of the individual states. For example, in the case of the third order catchment, Ts_1 is:

$$Ts_i=Ta_1+Tr_1+Tr_2+Tr_3 \tag{9}$$

where Ta_1, Tr_1, Tr_2 and Tr_3 are the times spent in states a_1, r_1, r_2 and r_3, respectively. The corresponding probability density functions are then related by:

$$f(ts_1)=f(ta_1)*f(tr_1)*(tr_2)*f(tr_3) \tag{10}$$

where * represents the convolution operation and $f(ts_1)$ represents the probability density function of Ts_1.

The probability of following a given path, $P(s_i)$, is given by

$$P(s_i)=\Theta_i(0).p_{ij}.p_{jk}..p_{l\Omega} \tag{11}$$

where the subscripts $i,j,\ldots,\Omega$ refer to the states constituting the path. For instance, for s_1 in the third order basin,

$$P(s_1)\ =\Theta_1(0).p_{12}.p_{23}. \tag{12}$$

The GIUH can now be given as:

$$h(t) = \sum_{i=1}^{N} f(T_{si})\ P(s_i) \tag{13}$$

The model of Gupta et al is more flexible than the one proposed by Rodriguez-Iturbe and Valdes. It can handle any distribution for the holding times of the streams (in principle), whereas the model of Rodriguez-Iturbe and Valdes, has to assume exponential distributions to make the mathematics tractable. Gupta's model, unlike Rodriguez-Iturbe and Valdes', also allows the time spent in the hillslope to be considered. However, if this time is neglected and if exponential distributions are chosen for the waiting times, Gupta et al obtain the same results as Rodriguez-Iturbe and Valdes.

(3) Extensions and variations

Since the GIUH was first presented, a few extensions to the original model have been proposed.

Wang et al (1981) have introduced a new feature in the model developed by Gupta et al (1980). The mean holding time in the channel, $1/m_i$, is allowed to vary over time as a power function of rainfall intensity: $1/m_i \sim i^{-\alpha}$, $\alpha > 0$. Values of α have then been computed for a number of catchments, and it was shown that α decreases as the size of the catchment increases. The authors suggest that α can be considered as a measure of the nonlinearity of catchment response to rainfall.

Rodriguez-Iturbe and Gonzalez-Sanabria (1982) have given a similar extension to the model of Rodriguez-Iturbe and Valdes, by allowing the velocity to vary with rainfall intensity: $v \sim i^{\alpha}$. By doing so, they have derived new expressions for the time-to-peak and flow peak of the GIUH, as a function of rainfall intensity instead of the velocity in the channels.

Van der Tak and Bras (1990) have replaced the exponential distributions used to model holding times in the streams by gamma distributions and have also incorporated hillslope effects in the GIUH.

Other variations are given by Rodriguez-Iturbe et al (1979), Agnese et al (1988) and Georgakakos and Kabouris (1989).

(4) Discussion

The GIUH is widely used and referred to in the literature, but surprisingly, little attention has been brought to check its underlying hypotheses. Yet, strong criticisms can be made on these assumptions, as shown in the list below:

1. The GIUH assumes that hydrological properties of channels, such as holding times, are related to the order of the stream. Yet, as Scheidegger (1965) points out, this assumption presents three flaws: (i) the order of a link varies with the map scale; (ii) the order of a stream does not always increase when two streams join together, yet the discharge and the width do vary at junctions; (iii) the rule for combining streams is not associative (or additive) and as a consequence, the Strahler ordering system is incompatible with the principle of mass conservation; symbolically $2+(1+1)=3$ but $(2+1)+1=2$.

2. Equations (6) and (7) are derived by means of a regression analysis in which the variables are extracted from both synthetic and real networks. The validity of this analysis might be questionned by the following observations: (i) some authors, including Strahler (1952) and Broscoe (1959) found that the law of stream lengths, when applied to real catchments, does not always fit the data very well. (ii) the parameters R_A and R_L, which are used as independent variables in the regression, are probably highly correlated.

3. The assumption of an exponentially distributed holding time is investigated by Kirshen and Bras (1983). They derive the hydrograph of an individual channel by solving the continuity and momentum equations for the boundary conditions defined by the GIUH and then deduce the distribution of holding times. This distribution is compared to the exponential distribution used by Rodriguez-Iturbe and Valdes. The results indicate that the two models do not agree and that the overall catchment response is sensitive to the choice of channel response (i.e. the distribution of holding times).

4. The choice of a probabilistic model for the representation of flow paths amounts to a waste of information, as shown in the following example.

Consider the catchment of Figure 2 where a network of order 4 is represented. The streams of different orders are indicated by lines of different thickness. Using this map, the matrix of transition probabilities, P, can be computed. There are, in total, 72 streams whose frequency distribution is given by the matrix:

$$N = \begin{pmatrix} 0 & 45 & 9 & 3 \\ 0 & 0 & 10 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

where n_{ij} is the number of streams of order i draining into a stream of order j. The number of streams of order i is obtained by summing the elements of row i: $m_1=57$, $m_2=11$, $m_3=3$. The transition probabilities are computed as $p_{ij}=n_{ij}/m_i$. We have:

$$P = \begin{pmatrix} 0 & 0.79 & 0.16 & 0.05 \\ 0 & 0 & 0.9 & 10.09 \\ 0 & 0 & 0 & 1.00 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure 2 An axample of Strahler classification for a river network of order 4

where p_{ij} is the probability that a stream of order i connects to a stream of order j. The GIUH then uses this matrix to derive a list of all possible paths. For example, one possible path is given by the following sequence of streams: (1,2,4). So, if we know that a drop of rain lands in a stream of order 1, the probability of following this sequence, rather than the sequence (1,3,4) or (1,2,3,4), is computed as $p_{124} = p_{12}.p_{24} = (0.79).(0.09) = 0.07$. This expression, is statistically correct, but in our particular case it does not provide the right proportion of paths (1,2,4): there are in total 57 paths starting with a stream of order 1 and among these, only two follow the sequence (1,2,4), as shown by the map. Hence, the observed proportion is 2/57 = 0.03. The discrepancy comes from the fact that we compare an observed proportion with an estimated one.

If we used a sample of catchments for which $p_{12}=0.79$ and $p_{24}=0.09$, statistically, the proportion of paths 1,2,4 would be $p_{124} = p_{12}.p_{24}$ (provided that the transitions between states are independent). For an individual network, however, different values might be observed, and this is in fact what happens in our example.

The problem described above is discussed in detail by Gupta and Waymire (1983, pp. 105-108). Through a simple (but lengthy) worked example, they show that there exist discrepancies between the true network response and the response obtained using the GIUH probabilistic representation. In their view, these discrepancies show that the Strahler ordering system is inadequate for characterising catchment response. Although we share the view that the Strahler ordering scheme is hydrologically of little use, we believe that the observed discrepancies come from the use of a Markov chain for representing flow paths, rather than the inadequacy of the ordering system. Discrepancies would still be observed, even with a good, or better numbering scheme, like for instance the numbering by magnitude.

In conclusion, the GIUH appears as a complicated model whose basic assumptions are physically not realistic. In addition, we observe that its probabilistic representation of flow paths amounts to a waste of information. This last criticism does not apply to the models presented in the next section which represent the movement of water along real trajectories, rather than through hypothetical states. These models are also simpler to understand and hence less space is required for their description.

1.3 MODELS BASED ON TRAVEL TIMES ALONG FLOW PATHS

The models described in this section are all based on the same hypothesis: they assume that travel times throughout the catchment are determined by the distance to the outlet, this distance being calculated along the trajectory of the runoff. The differences encountered concern the modelling of travel times which can be either probabilistic or deterministic and the representation of hillslope effects. So, two drops landing at different locations, but at an equal distance from the outfall will take the same time to leave the catchment, or will have equal probability distributions of travel times.

The model described in part 1 is based on a probabilistic representation of travel times along the paths. It includes a representation of waiting times in the hillsope and as such is more complete than the two following models.

The model described in part 2 mixes a probabilistic and a deterministic approach: first, it assumes that the rainfall is kept out of the channel for a duration specified by a probability distribution. The water is then released into the channel where it travels at a constant velocity. Both the storage time distribution and the velocity are spatially uniform, so that the time spent in the basin can be directly related to the distance. However, the input to the channel can vary spatially, so that two points which are equidistant from the outlet will not necessarily contribute the same amount of runoff.

The model described in part 3 is entirely deterministic and assumes that the velocity in the channel is spatially uniform and constant in time. Its formulation is such that it can only be used in the case of a spatially uniform input. Hence, two different points which are equidistant from the outlet will contribute equal amounts of runoff and this contribution will be registered simultaneously at the outlet.

(1) Example of a model with a probabilistic representation of travel times

Mesa and Mifflin (1986) propose a model which represents the time spent in the catchment as a random variable. This variable is the sum of two independent variates: the time spent in the overland phase and the time spent in the channel. The probability distribution of the hillslope holding time is assumed to be spatially uniform. With these hypotheses, the probability density function of the basin holding time is simply

$$f_b(t) = \int_0^{\infty} f_n(t-\tau)\, f_h(\tau)\; d\tau \tag{14}$$

where τ and t represent time, and where $f_n(\tau)$, $f_h(\tau)$ and $f_b(\tau)$ represent the probability density function of the time spent in the network, the hillslope and the catchment, respectively.

$f_h(z)$ is decomposed in two terms: a slow and a fast component which are both taken to have a triangular distribution

$$f_h(t) = \pi_f f_{hf}(t) + \pi_s f_{hs}(t) \tag{15}$$

where π_f and π_s are the probabilities of following the fast and the slow way and where f_{hf} and f_{hs} represent the holding time density functions of the fast and slow response, respectively.

The movement of a particle in the channel is modelled as a one dimensional Brownian motion with diffusion σ^2 and drift velocity v. σ^2 and v are assumed to be constant over time and spatially uniform. Hence, if a particle starts its journey at a distance x from the outlet, the probability density function of its holding time in the network is represented by an inverse Gaussian density

$$h(x,t) = \frac{x}{\sqrt{2\pi\sigma^2 t^3}} \exp\left[-\frac{(x-vt)^2}{2\sigma^2 t}\right] \qquad (16)$$

If independence can be assumed, the waiting time of a particle chosen at random in the river is then

$$f_n(t) = \int_0^\infty h(x,t)\ N(x)/L_T\ dx \qquad (17)$$

where L_T is the total length of the network and N(x) is the width function (see Appendix).

The model proposed by Mesa and Mifflin is quite simple in its concepts, assuming that hillslope response is spatially uniform and modelling travel times in the channels as a function of distance alone. However, despite its conceptual simplicity, the equations and its application turn out to be quite complicated. The solution to equation (17) and its application in discrete form are given by Naden (1992).

(2) Example of a model with a semi-probabilistic representation of travel times

Surkan (1969) assumes that a network with a uniform drainage density can represent the response of the catchment if a storage delay is introduced to represent the retention of water in the soil and the channels. The rainfall which may vary in time and space is input at the nodes of the network where it is first stored and then progressively released at an exponential rate. It then travels towards the outlet at a constant velocity. If no storage is accounted, i.e. if the whole of the input is immediately released, the hydrograph of the catchment is given as

$$U(t) = \sum_{m=1}^{N} Q(n,t-x_n/v) \qquad (18)$$

where N is the total number of nodes,
x_n is the distance of the n^{th} node from the outlet (m),
v is the velocity in the channel (m/s),
U(t) is the outflow (m3/s) and
Q(n,t-x_n/v) is the amount of water released at node n at time t-x_n/v (m3/s). This quantity of water travels for a duration x_n/v and reaches the outlet at time t (s).

With a spatially uniform storage delay, the hydrograph is obtained as the convolution of U(t) and the storage function
where

$$U'(t) = \sum_{s=1}^{s} [e^{-(s-1)/K} - e^{-s/K}] \; U(t-s) \tag{19}$$

$$\sum_{s=1}^{s} [e^{-(s-1)/K} - e^{-s/K}] \tag{20}$$

represents the fraction of water released between the instants s-1 and s and where the time step is equal to one.

The effects of network geometry on the hydrograph are studied by simulating synthetic networks with various shapes, sizes and storage decays. Whilst the shape does not seem to influence the hydrograph, a linear relationship is found between the duration of the impulse response and the total number of links or nodes in the network. The effect of the exponential storage is to delay the time to peak, to reduce its magnitude and also to smooth the impulse response obtained by the network alone. The greater the delay constant, the stronger the effect.

Surkan's model is conceptually very simple and also very simple to apply. It is formulated in a flexible way, allowing a heterogeneous rainfall input and making it easy to consider spatially variable velocities in the channel.

(3) Example of a model with a deterministic representation of travel times

Calver et al (1972) assume that the input to the network is provided from the hillslope as a spatially uniform flow per unit length of channel. The velocity of water in the river is assumed constant over time and spatially uniform. With these hypotheses, the hydrograph of the catchment can be expressed in the following way:

$$Q(t) = \int_{0}^{D} q(t-x/v) \; N(x) \; dx \tag{21}$$

where x is the distance along the river (m),
D is the diameter of the network (m),
N(x) is the width function,
v is the velocity in the channel (m/s),
q(t-x/v) is the hillslope input from both banks of the channel at time t-x/v (m2/s) and
Q(t) is the output flow (m3/s).

Q(t) can also be expressed as an integral over time by substituting $x = v\tau$ into the last equation:

$$Q(t) = \int_{0}^{D/V} q(t-\tau) \; N(v\tau) \; v \; d\tau \tag{22}$$

where τ represents time.

The model proposed by Calver et al does not have the flexibility of Surkan's model, but its simplicity and parsimonious use of parameters are quite appealing. However, to be complete, a representation of q, the input to the channel, has to be specified. In fact the authors do propose a model for the hillslope but it is not reviewd here as it involves the use of finite elements.

(4) Discussion

The models described in this section are much simpler than usual routing models, even if some of the equations or concepts are quite complicated. A major simplification comes from the fact that factors like the slope, the width and the depth of the channels are not incorporated in the models. Another important feature is that the flow, or the velocity, at a given point is derived independently of upstream or downstream conditions. The idea of mapping distances into times of travel is however interesting, in that it is simple and reasonably realistic from a physical point of view.

In the models of section 1, the description of hillslope processes is either neglected or represented in a very simplistic way. This is because a map of the river network does not convey enough information for this purpose. The models described in the next section provide a remedy to this problem. They use a Digital Terrain Model to represent the hillslope and describe the generation of runoff which is known to be largely controlled by the topography.

2 Models using a digital terrain model and the concept of contributing area

This section describes and compares two models focusing on hillslope processes. They use the concept of contributing area which is briefly reviewed in section 2.1. Section 2.2 then describes an Australian model called Saturation Zones Model, whereas section 2.3 describes TOPMODEL. The two models are then compared in section 2.4.

2.1 A BRIEF REVIEW OF THE ORIGINS AND DEVELOPMENT OF THE CONCEPT OF CONTRIBUTING AREA

A review by Hewlett (1974) shows that contributing areas have long been recognised as an important factor in controlling catchment response.
However, the concept was introduced in rainfall-runoff modelling relatively late, by the United States Forest Service (1961) and by Betson (1964). Their work suggested the need for field investigations of the location, extent and variation of saturated areas in a catchment, and also of their role in the production of runoff.

Experiments were later set up, for instance by Ragan (1968), Betson and Marius (1969) and Dunne and Black (1970). Groundwater level data, as well as rainfall and runoff measurements were recorded and used to map the locations of saturated areas in the catchment. These locations were related to the area drained, slope and soil properties. It was also observed that the extent of wet areas was linked to antecedent soil moisture and could increase or decrease with rainfall intensity during the course of a storm.

The experiments were not confined to mapping wet zones. They also gave an account of infiltration and runoff production in relation to the state of the soil. In humid regions, where infiltration capacities exceed rainfall intensities, stormflow was observed to be originated mainly from the saturated zone, either as saturation overland flow, or as throughflow. Rainfall intercepted by the unsaturated zone was reported to move vertically, rather than laterally, to feed the saturated zone responsible for baseflow generation.

Several models have used the results of these experiments to represent the efficency of rainfall-runoff transformation, by assuming that percentage runoff is given by the percentage of catchment area which is saturated. Examples include Artega and Rantz (1973), NERC IEM4 (1975), Lee and Delleur (1976) and Hughes (1984). These models, however, do not derive the contributing area from the topography of the catchment, but as a function of antecedent conditions and rainfall history. So in fact, they represent the concept of contributing area only implicitly.

Two models using the topography to locate wet zones inside a catchment are reported in the literature. Both models represent runoff as a function of the wetness of the catchment which is directly related to the percentage of saturated areas. A mass balance equation is used to update, at each time step, the extent of the contributing area. These models are described in the next sections.

(1) Description of the model

2.2 SATURATION ZONES MODEL

O'Loughlin (1981, 1986) has proposed a simple model, the Saturation Zones Model or SZM, based on Darcy's law, to predict saturated areas in a catchment. At any point in the catchment, if the soil is saturated to the surface, the drainage flux of the profile is given by (see Figure 3):

$$q = \int_0^l K(z)\ \partial h/\partial s\ dz \qquad (23)$$

where q is a flow per unit contour length (m2/s),
z is the vertical distance from the surface (m),
K is the hydraulic conductivity of the soil (m/s),
h is the water head (m),
s is the distance along the tangent to the hillslope profile (m),
l is the depth of the profile (m).

$\partial h/\partial s$ is assumed to be independent of z. Hence,

$$q = \partial h/\partial s\ T \qquad (24)$$

where $T = \int_0^l K(z)\,dz$ is the transmissivity of the profile (m^2/s).

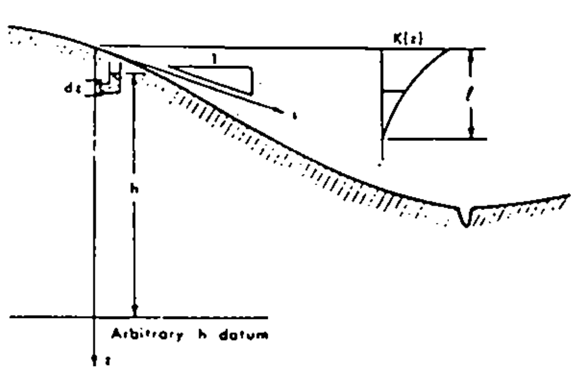

Figure 3 Definition sketch for flow in a real hillslope (from O'Loughlin, 1981)

The maximum value of $\partial h/\partial s$ is given by the local slope. Hence, when the profile is saturated, the maximum subsurface flow, i.e. the hillslope capcity, becomes:

$$C = \max(q) = T \tan\beta, \qquad (25)$$

where $\tan\beta$ is the local slope. The model assumes that when the profile is saturated, q=C.

The topography of the catchment is represented by an ensemble of elementary hillslopes characterized by their drainage area, a, and their width at the bottom, w. Now, let Q be the discharge per unit area at a given point. If steady-state conditions are observed, the outflow from the hillslope can be derived, either by integrating q along the bottom line of the hillslope, or by integrating Q over the area of the elementary hillslope

$$\int_w q \, dw = \int_a Q \, da. \qquad (26)$$

Saturation occurs if q exceeds T $\tan\beta$, i.e. if

$$\left(\int_a Q \, da\right)/w > T \tan\beta, \qquad (27)$$

assuming that q is homogeneous along s.

Provided the values and the spatial distribution of T and Q are known, this equation can be used to derive saturated zones across the catchment. These can then be summed to give the total contributing area.

Figure 4 gives an example of the spatial variation of wetness derived from these equations.

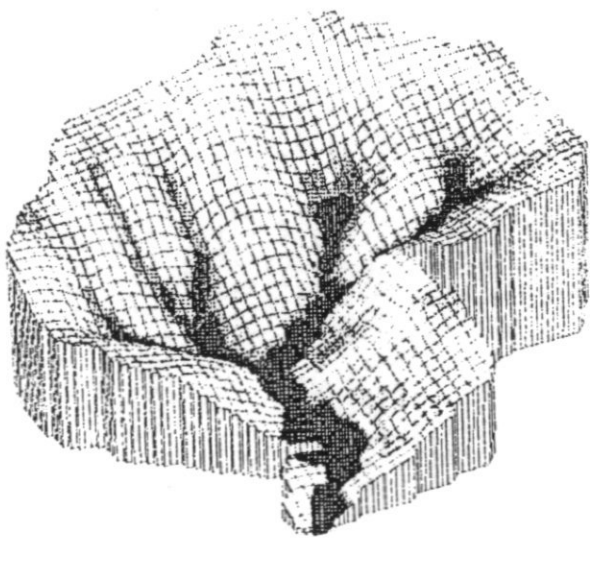

Figure 4 Location of wet zones in the Geebung Creek catchment. Wet zones are identified by dark shades (From O'Loughlin, 1986)

Although the model is simple in its principle, it is not straightforward to apply, as values of a, w and tanβ are required for a large number of points in the catchment. Moore et al (1988) describe a methodology and a computer program designed to extract these variables automatically from a Digital Terrain Model. However, the major difficulty is still the determination of T and Q across the catchment.

The use of the model is illustrated by O'Loughlin (1986), under the assumption of spatial uniformity of T and Q. For a given value of T, which can be estimated from field measurements, the extent of the contributing area is derived for a range of values of Q. The percentage of saturated area (contributing area/total area) is then plotted against $1/Q_0$, where $Q_0 = Q.A_t$ represents the baseflow of the catchment and where A_t represents its total area. The curve obtained in this way is compared to the predictions given by the analysis of event data. For each event, the value of percentage runoff (quickflow/net rainfall) is plotted against $1/Q_0$, where Q_0 is the baseflow at the beginning of the event, or alternatively, the average baseflow for the whole event. If percentage runoff is assumed equal to the percentage of saturated area, these data can be superimposed on the curve obtained from topographic analysis, as shown in Figure 5.

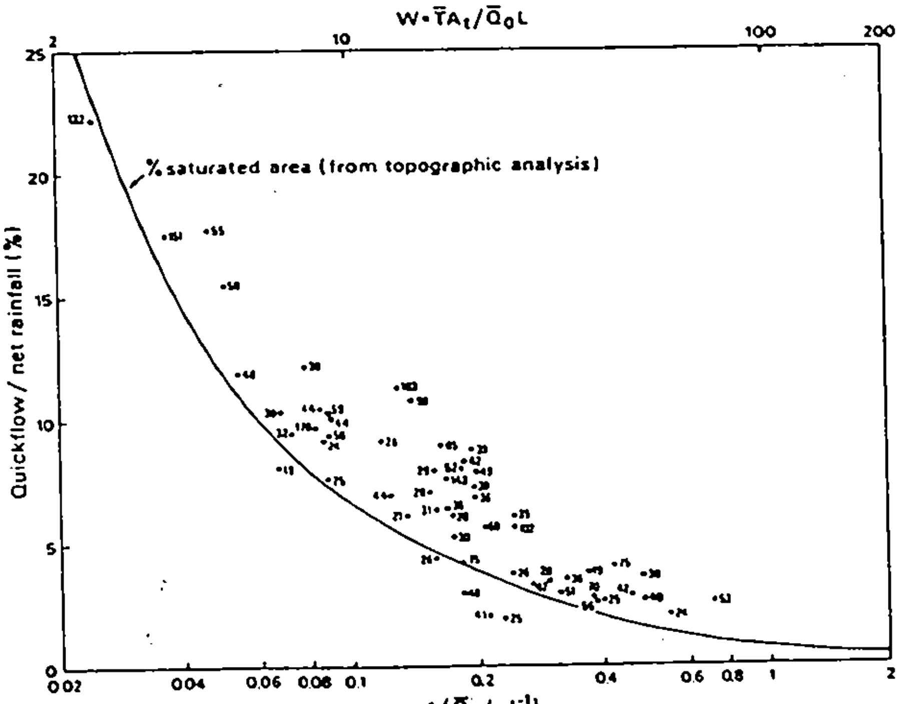

Figure 5 Relationship between percentage runoff (or contributing area) and 1/baseflow (From O'Loughlin, 1986)

Given a value of the baseflow, this curve can be used to determine the spatial extent of the contributing area. Inversely, for a given value of percentage runoff or contributing area, it allows the calculation of the baseflow.

(2) Example of a model using saturation zones model

Moore et al (1986) have coupled O'Loughlin's Saturation Zones Model with a conceptual rainfall-runoff model which expresses baseflow and overland flow as direct functions of the contributing area. Baseflow is determined by the state of a soil-moisture store which is updated at each time step, presumably by means of a mass balance equation. The relationship between percentage of saturated area and baseflow, as derived in SZM (Figure 6), is an input to their model and is used to derive percentage runoff. For a given time step, the quickflow, R, is then calculated as

$$R = P/100\ A_t\ r, \tag{28}$$

where P is the percentage of saturated area (or equivalently, the percentage of runoff and r is the net rainfall intensity obtained by subtracting interception and evaporation rates from the gross rainfall rate. The model possesses other features, like a flow routing procedure, a soil-moisture store, but these are not directly relevant to the subject and are not detailed here.

2.3 TOPMODEL

TOPMODEL is a conceptual model which represents the catchment by a sequence of storage elements. Although its parameters are lumped, it takes the heterogeneity of the catchment into account as it relates the content of the soil-moisture store to the extent of the contributing area and routes the runoff on the hillslope and in the channels. The volumes of runoff and baseflow are directly related to the extent of the contributing area which can expand and contract as a result of varying inputs and outputs.

A detailed description of the model is given by Beven and Kirkby (1979). The use of a Digital Terrain Model for the derivation of saturated zones is described by Quinn et al (1989) who also give an updated version of the model, taking into account the transmissivity of the soil and expressing soil moisture in terms of deficit rather than relative storage. Applications of the model can be found in Beven et al (1984) and Beven and Wood (1983).

The derivation of saturated areas in a catchment is based on the relationship between soil-moisture content and subsurface flow. At any point in the hillsope,

$$q = T.\exp(-S/m).\tan\beta \tag{29}$$

where q is the subsurface flow per unit contour length (m2/s),

T is the transmissivity of the profile (m2/s),
S is the storage deficit of the profile (m),
m is a constant (m),
$\tan\beta$ is the local slope.

When S=0, i.e. when the soil is saturated,

$$q = T.\tan\beta \tag{30}$$

as in O'Loughlin's SZM. However, when S>0, i.e. when the profile is unsaturated, TOPMODEL gives an expression of q, whereas this is unspecified in SZM.

An important hypothesis is made at this stage, which allows the model to be developed further: it is assumed that the subsurface flow at a given point, q, is a direct function of the area drained:

$$q = a.\rho \tag{31}$$

where a is the drainage area per unit contour length of the elementary hillslope (m) and ρ (m/s) is spatially uniform but varies in time with the wetness of the catchment. The variation of ρ with time is assumed to be slow, so that within a time step steady-state can be assumed. Equation (31) implies that two points situated at different locations but draining the same area, must have the same value of q. If these two points have different values of $T.\tan\beta$, the storage deficits will then be different and will take values which allow q to be the same at both locations. The storage deficit at one point can be obtained by combining (29) and (31):

$$S = -m\ln\rho - m\ln\left(\frac{a}{T\tan\beta}\right) \tag{32}$$

This relationship can be used to determine the average storage deficit for the whole catchment:

$$\begin{aligned} \bar{S} &= \frac{1}{A_t}\int_{A_t} S\,dA \\ &= \frac{1}{A_t}\int_{A_t}\left[-m\ln\rho - m\ln\left(\frac{a}{T\tan\beta}\right)\right]dA \\ &= -m\ln\rho - m\gamma \end{aligned} \tag{33}$$

where A_t is the total area of the cathment, dA is an increment of area (m2) and

$$\gamma = \int_A \ln\left(\frac{a}{T\tan\beta}\right) dA \tag{34}$$

is a constant for the catchment which represents the areal average of the soil-topographic index ln $(a/T\tan\beta)$. The value of γ can be evaluated by analysis of a Digital Terrain Model, provided that T is known. (The determination of T will be considered later.)

The relationship between S and ρ (equation (32)) can be used to provide the catchment baseflow as a function of S:

$$Q_0 = A_t \rho = A_t e^{-\gamma} e^{-\bar{S}/m} \tag{35}$$

where Q_0 is expressed in cumecs.

The storage deficit can also be evaluated as a function of S, by substituting for ρ:

$$S = \bar{S} + m\,\gamma - m\, ln\,(a/\,T\tan\beta \tag{36}$$

where a, T and $\tan\beta$ are local values. Hence, saturation occurs wherever

$$ln\,(a/\,T\tan\beta) > \gamma + \bar{S}/m \tag{37}$$

Equations (35), (36) and (37) are sufficient to determine the outflow, provided that a number of parameters, as well as initial values of the variables are known.

The parameters to be estimated are T, γ, A_t and m. The knowledge of the transmissivity, T, is required for the derivation of γ. This parameter, which for simplification is assumed constant across the catchment can be estimated by field measurements, or alternatively, by setting it as a calibration parameter in the model. The value of A_t and γ are then derived from the Digital Terrain Model, whilst m is estimated by recession analysis.

The value of Q_0 prior to the first storm is then used to derive the initial value of S, which can provide the initial distribution of S across the catchment, and in the process, the initial value of the contributing area.

At each time step, a value of the baseflow and of the overland flow (over the contributing area) is computed. The baseflow is computed from the equation above, whereas the overland flow, P, is simply obtained through equation (28).

The value of S is then updated by means of a mass balance equation.

Although TOPMODEL is conceived as a storage model, procedures are included for the routing of overland flow and channel flow (see Beven and Kirkby, 1979).

2.4 COMPARISON BETWEEN SATURATION ZONES MODEL AND TOPMODEL

Saturation Zones Model and TOPMODEL share several hypotheses:

1. Darcy's law is used to model subsurface flow.
2. Subsurface flow per unit area is assumed to be spatially uniform. In principle, Saturation Zone Model allows it to vary, but in practice it is kept constant in space.
3. The transmissivity of the soil is assumed to be spatially constant implying that

the generation of flow is entirely controlled by the topography.

The major difference between the two models is that TOPMODEL can predict the subsurface flow for an unsaturated profile, by means of equation (30), whereas Saturation Zones Model cannot. In spite of this difference, both models obtain a direct relationship between the baseflow and the percentage of saturated area.

3 Suggestions for the use of ih digital maps in rainfall-runoff modelling

From this review, it appears quite clearly that the features characterising the various models depend on the data used. When a map of the river network is used, the models focus on the routing of flows and do not give too much attention to the storage function of the catchment. This is so, because the river network alone cannot be used to represent water content in the overland phase. This type of model is therefore usually applied to large catchments where the river network is thought to play a dominant role over hillsope processes in determining catchment response to rainfall.

When a DTM is used, the reversed situation is observed. These models focus on the spatial variation of soil moisture content, but give less attention to modelling the transport of water to the outlet. This type of model is therefore usually applied to small catchments in which the response to rainfall is determined by the behaviour of the hillslope.

As both DTM and river network are available at IH, it is possible to combine features of both types of model. In addition, as a map of soil-type is also available, information relating to the depth and permeability of soil could be used with the topography for the prediction of wet zones.

3.1 DIGITAL MAPS AVAILABLE AT IH

1 DTM

The DTM is being developed for the whole of the UK. At present, it is only available for part of Northern Ireland and parts Great Britain. The DTM is represented on a grid with a horizontal resolution of 50 m. For each point on the grid, the following features are available:

- the altitude to the nearest 10 cm
- the area drained to this point
- the direction of flow at this point

Using these data, it is possible to derive, for each point, the slope and the pathline to the outlet. A description of the DTM and its generation is given by Morris and Flavin (1990).

2 River network

The river network has been digitised from OS maps at the scale of 1:50000 and is available for England and wales (Moore 1983). In its present form, it provides only planar coordinates of the rivers, but the DTM is currently being used to add elevations to it.

3 Soil types

The maps of soil-types produced by the Soil-Survey and Land Research Centre for England and Wales, have been used to produce a map which displays the hydrological properties soils. The HOST map (Hydrology Of Soil Type) distinguishes 29 soil-hydrological classes based on parameters like the depth of the aquifer and the soil water storage capacity. Details about this classification scheme are given by Boorman and Hollis (1990).

3.2 HYDROLOGICAL DATA

1) IH has collected flood event data for a set of approximately 200 catchments in the UK. The number of events per catchment corresponds to an average of 10. For each event, the data available are:

- a sequence of hourly flows at the outlet of the catchment
- a sequence of hourly rainfall which represents the areal average for the catchment
- measurements of soil moisture deficits at one or several points in or near the catchment

These data are used to derive

- the percentage runoff of the event
- the unit hydrograph of the event

An average value of percentage runoff, as well as the average unit hydrograph are also derived and are used to represent the catchment average response.

2) A few continuous records of hourly flows and rainfalls are also available.

3) Long term daily records of flow and rainfall are available for a large number of UK catchments.

3.3 SUGGESTIONS

1) The average percentage runoff and the average unit hydrograph can be modelled by assuming a fixed contributing area and by computing a frequency distribution of travel times which, if scaled properly, is comparable to the time-area diagram.

Three zones will be distinguished inside the catchment: the unsaturated zone in which rainfall infiltrates and moves vertically, the saturated zone in which water moves as overland flow or throughflow at slow speeds and the channels in which the water moves faster. From every point on the DTM grid, a pathline to the outlet can be defined, as well as the velocities along this path. The velocity at a given point will depend on its location (unsaturated zone/ saturated zone/ river). Travel times can then be derived by integrating the inverse of the velocity along the path line. The derivation of the contributing area can be done using the approach of SZM or TOPMODEL. Soil might be taken into account in this model by applying it to catchments with homogeneous soil-types.

2) The scheme described in 1) constitutes a frame from which further developments can take place. For example, the modelling of velocities can be made more or less sophisticated and the estimation of the contributing are can be made in various ways. At a later stage, the modelling could be extended to continuous records by allowing the contributing area to vary with time, as in SZM and TOPMODEL. Attention will be given to modelling, in a very simple way, the transfer of subsurface flow. The assumption of a uniform subsurface flow might be revisited, as Moore et al (1988, p.314) suggest that it is not an inevitable hypothesis. It might be the only way of obtaining a tractable model, but the problem is worth investigating. The map of soil-types might be useful for the modelling of varying contributing area, by providing information, like soil-depth and conductivity, which influence the rate of moisture variation in soils.

3) Prior to any modelling, some data analysis is required. Using the flood event data base, it is possible to produce, for a given catchment, a plot of $1/Q_0$ against percentage runoff, each point representing a flood event. It is worth producing such plots for a number of catchments, and to analyse the results in order to check if SZM is applicable to UK catchments and to the flow separation method used in IH.

Another preliminary study can be made using the DTM and HOST, in order to investigate the possibility of deriving the river network from the topography, soil-type and perhaps climate variables. The digitised river network can be used as a base of comparison. Drainage area, slope, soil-type and pluviometry are expected to have an influence and the problem will be to quantify the importance of each variable. This study is seen as a useful step for the understanding of the formation of wet zones in a catchment. If it is possible to predict the location of rivers using this type of information, then it might be possible to predict the location of saturated areas. The mapping of soil-moisture data is still difficult and rare, but it is an area of growing interest. Brun et al (1990), for instance, describe experiments designed to map the contributing area by using a helicopter-borne Scatterometer.

Conclusion

This review has shown that, although there are only a few conceptual rainfall-runoff models using distributed data of catchment geomorphology, they are varied in their concepts.

The GIUH is a well known model and has been reviewed in detail. It is found to be very complicated and unrealistic in its assumptions. The three other models based on the river network appear to be conceptually simpler and have a better physical basis.

Two of these models represent the transport of water in the river network as a simple translation and assume that the velocity is spatially uniform. One of these, the model proposed by Surkan, is formulated in a very flexible way, allowing the input to the river to vary spatially. The second one, proposed by Calver et al does not have this capability but as the advantage of being very simple. The third model, proposed by Mesa and Mifflin, although conceptually simple is complicated mathematically and difficult to apply. In this model, the transport of water in the river network is represented as a convection-diffusion process. The response of the network is then convolved with the hillslope response which is assumed to be spatially uniform, to provide the unit hydrograph of the catchment. This unit hydrograph depends on the calibration of eight parameters.

River network models can be considered as improved variations of the time-area diagram and should be useful for the derivation of unit hydrographs. They have however, a shortcoming which is their inability to describe the generation of flow and the distribution of wetness in the catchment. They appear therefore as incomplete and would require, in order to be applicable, an extension devoted to the modelling of subsurface flow.

The shortcoming described above is remedied in two topography-based models, the Saturation Zones Model and TOPMODEL which represent the wetness at a point as a function of the draining area, the local slope and the average soil-moisture deficit in the catchment. These two models are quite similar in their underlying assumptions, but have different areas of application. On the one hand, TOPMODEL models a continuous sequence of flows, and describes a continuous variation of the contributing area. On the other hand, the Saturation Zone Model, or rather the application described here, is an event-based model in which the contributing area varies between events but not within an event.

To the author's knowledge, no model uses information provided by soil-maps. This might be due to the fact that these maps are rarely available in digital form. It might also reflect the difficulty of including this type of information in a model.

A map of soil-types (HOST) and a map of the river network are now available at IH in a digital form and a Digital Terrain Model is being produced. These maps are available, or will be available for the whole of Great Britain. It will therefore be possible to apply any of the models reviewed in this document. The Digital Terrain

Model will be useful for the representation of flow generation, whilst the river network will be useful, especially in large catchments, to model its transport to the outlet. Problems might, however, arise from the overlay of a vector map (the river network) on a gridded map (the Digital Terrain Model) and it might be useful to use a gridded approximation of the network in the modelling. The soil map might be useful if its classes can be related quantitatively to parameters like transmissivity which is used in TOPMODEL and the Saturated Zone Model, or the depth of the water table.

This review shows that distributed data can provide useful information in rainfall-runoff modelling. It is however not sufficient. Attention will also need to be given to the modelling of losses due to evaporation and interception. In addition, a calibration procedure is needed for the model application. Ideas are however obtained on simple ways of formulating the generation and the transport of flow in a catchment.

Appendix - Characterisation of the river network

This Appendix gives the terminology used to describe river networks and their components. Parameters or functions commonly used in characterising river networks are then defined. These include Strahler ordering system and the laws of drainage composition, the magnitude of a network and the width function.

DEFINITIONS

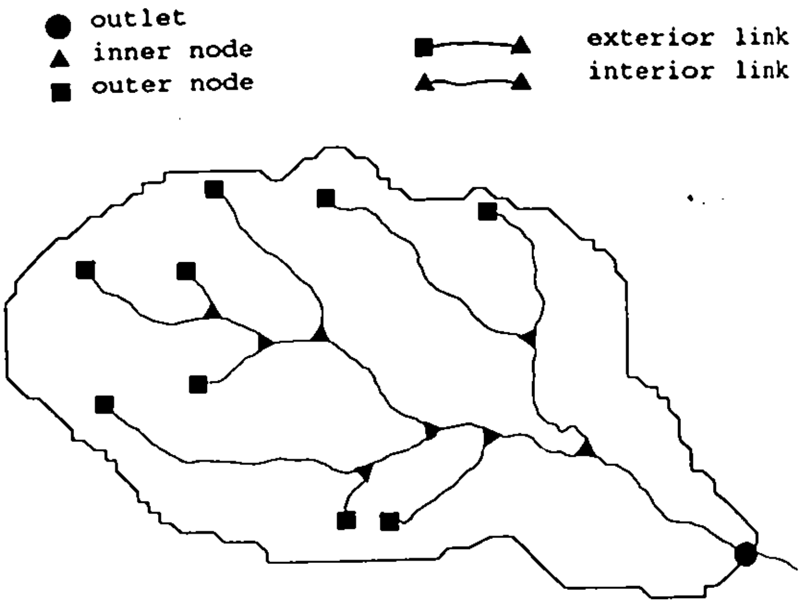

Figure 6 River network terminology

The terminology used to describe a river network is illustrated in Figure 6. The network is usually assumed to possess a tree-like structure, without islands or bifurcations, and multiple junctions (more than 2) are not supposed to occur. It is composed of a number of segments called links which are connected at points called nodes. Outer nodes, called sources, are nodes which originate a path to the outlet. There are no links connected upstream of an outer node, whereas there are always 2 links directly upstream of an inner node. The outlet is not considered to be a node. A distinction can also be made between exterior links and interior links. The former always originate from a source whereas the latter always connect two inner nodes or an inner node to the outlet.

STRAHLER ORDERING SYSTEM AND THE LAWS OF DRAINAGE COMPOSITION.

Horton (1945) has proposed a method of classifying streams and suggested 2 empirical laws: the law of stream numbers and the law of stream lengths. A similar law, the law of stream areas was later proposed by Schumm (1956). Strahler (1957) revised Horton's classification and proposed a simpler and less subjective system, to which Horton's laws are still applicable. This system has since always been favoured by both hydrologists and geomorphologists and for this reason it is the only one reffered to in this text.

The Strahler ordering scheme is represented by 3 rules:

1. Exterior links are defined to be first-order streams.
2. When 2 streams of order m merge, a stream of order (m+1) is created. 3. When 2 streams of different orders, say m and n, join, the order of the link downstream is max(m,n).
   The order of the basin is the highest order of the network.
   An illustration of this ordering system is given in Figure 5.

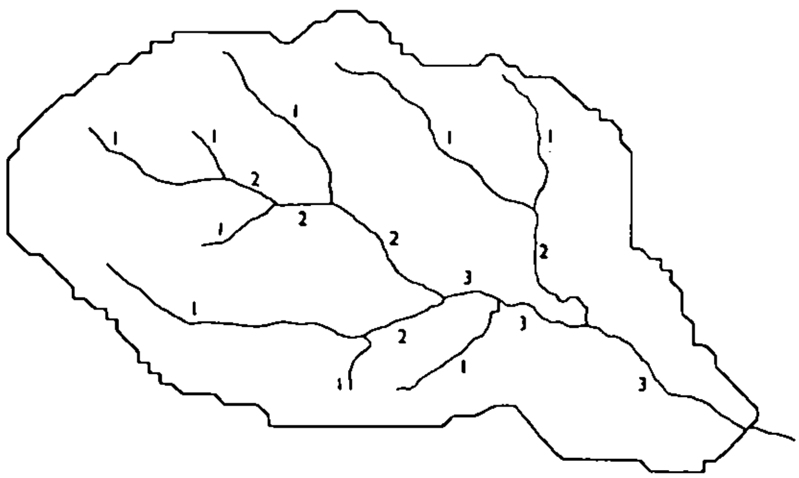

Figure 7 Strahler ordering system

It should be noted that streams and links are not equivalent and that a stream of order greater than 1 can be composed of several links (see Figure 7).

THE LAWS OF DRAINAGE COMPOSITION

The laws of drainage composition are empirical laws observed for natural basins. Let i be a given order, let N_i be the number of streams of order i, let L_i be the average length of streams of order i and let A_i be the average area draining to a stream of order i. Then, according to the laws of drainage composition, the ratios R_B, R_L and R_A defined below are constant.

Law of stream numbers	$R_B = N_{i-1}/N_i$,	$i = 2,3,\ldots,\Omega$
Law of stream lengths	$R_L = L_i/L_{i-1}$,	$i = 2,3,\ldots,\Omega$
Law of stream areas	$R_A = A_i/A_{i-1}$,	$i = 2,3,\ldots,\Omega$

If these laws are observed, a plot of $\log(N_i)$, $\log(L_i)$ or $\log(A_i)$ against i should appear as a straight line. The law of stream numbers and the law of stream lengths have been applied to the River Tamar at Gunnislake which has been digitised at a scale of 1:50000. The law of stream areas has not been tested, because the definition of draining areas from a map of the river network alone is thought to be arbitrary. The results are shown in Figure 6 and it can be seen that for this particular case, the ratios of stream lengths do not plot as a straight line, i.e. do not agree very well with the law of stream lengths.

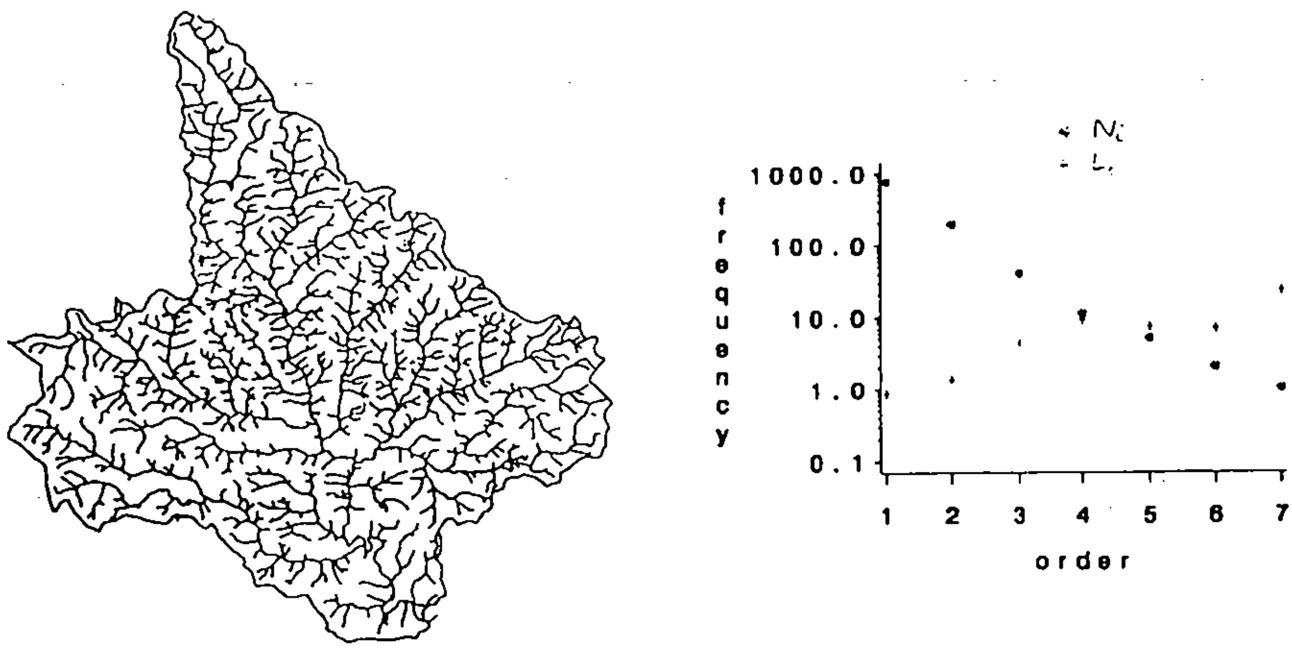

Figure 8 Laws of drainage composition for the Tamar at Gunnislake

MAGNITUDE

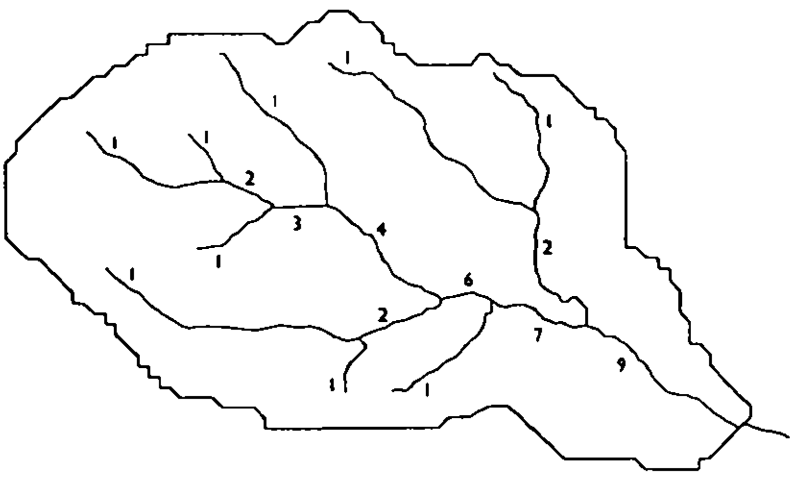

Figure 9 Link numbering by magnitude

The magnitude of a link is the number of sources upstream. A simple procedure can be used to derive magnitudes for the whole network:

1. An exterior link is given magnitude 1.
2. The magnitude of an interior link is the sum of the magnitudes of the links joining its node.
   The magnitude of the network is that of its outlet link.
   These concepts are illustrated in Figure 9.

In a network with a tree-like structure, where multiple junctions do not occur, the total number of links, n, is related to the magnitude of the network, m by $n=2m-1$.

The magnitude of the network reflects quite well the size of a catchment and the magnitude of its mean annual flow, as shown by Naden and Polarski (1990) in a study of 143 UK catchments. However, unlike the network order, the magnitude is not often used in hydrological modelling. It might be interesting to study the relationship between the order of a network and the annual maximum flow or the area of a catchment and compare the results with the study described above.

DIAMETER

From every source in the network, a path to the outlet can be defined. The diameter of the network is the path with the longest topologic length. The topologic length is measured in terms of the number of links from the outlet. The diameter can also be measured as a distance, in which case it is equivalent to the main stream length.

WIDTH FUNCTION

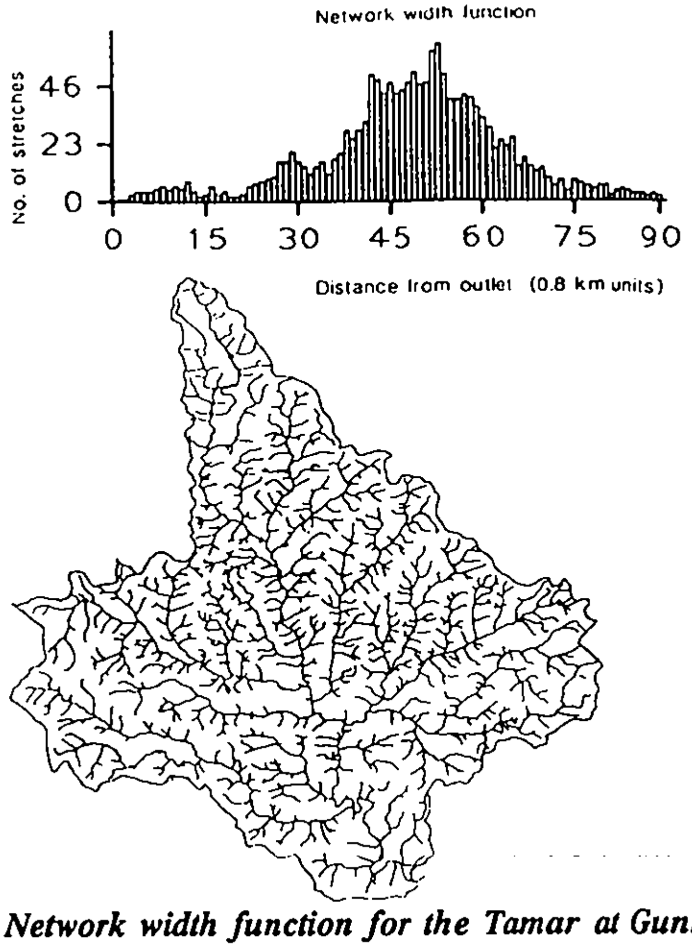

Figure 10 ***Network width function for the Tamar at Gunnislake (From Nadan and Polarski, 1990)***

For every point on the river, it is possible, by following the course of the river, to compute its distance from the outlet. The width function is a frequency distribution of such distances. It is constructed by counting the number of points which are situated on the network, on different reaches, but at the same distance from the outlet. This operation is repeated for a range of distances varying between 0 and the main stream length (or diameter). An example of width function using real lengths, in kilometres, is given in Figure 10.

The width function is useful when the input to the rivers is assumed spatially uniform and when it can be assumed that travel times are the same for points which are equidistant from the outlet. This happens, for instance, if a constant velocity is assumed in the whole network. In this case, it is straightforward to map the width function into a frequency distribution of travel times, by dividing distances by the velocity. The resulting histogram is comparable to the time-area diagram, except that it only applies to the river network, whereas the latter applies to the whole catchment.

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