PACK: A Pragmatic Snowmelt Model for Real-time Use

Report to the Flood Protection Commission
Ministry of Agriculture, Fisheries and Food

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1. INTRODUCTION

An analysis of currently available snowmelt models reveals that they are made up of essentially four components:

(i) an input transformation, correcting for the representativeness of climatic inputs, especially precipitation;

(ii) a surface melt component, often using a simple excess temperature mechanism as a substitute for a full energy budget controlled melt formulation;

(iii) a snowpack storage mechanism, controlling how surface melt is retained within the pack; and

(iv) a drainage term, defining the release of water from the pack, and often formulated as an integral part of (iii).

The model formulated here, and referred to as the PACK model, is based on these four components and employs representations particularly suited to UK conditions and for real-time application. For further information on the background to the model formulation and to an initial form of the model the reader is referred to the research report by Harding and Moore (1988).

The basic structural form of the PACK model is described in the next section. Here, the concept of a subdivision of a snowpack into "dry" and "wet" stores is introduced and an areal depletion curve is invoked to allow shallow packs to only cover a fraction of the basin area. Consideration is then given to how the model may be corrected in real-time using snow survey data on the depth and water equivalent of the pack at prescribed locations. This leads to a formulation based on identifying an empirical state adjustment to the model pack at a survey point and the subsequent transfer of the adjustment to a model pack of the basin for which flow forecasts are required. Estimation of the parameters of the final model with state updating is not straightforward due to the absence of snowfall measurements. A screened objective function is formulated to circumvent this problem.

The model is available for calibration to snow survey data within the RFFS Model Calibration Facilities (Institute of Hydrology, 1991) and point and basin forms of the model are incoporated as Model Algorithms within the operational RFFS (River Flow Forecasting System).

2. MODEL FORMULATION

Input Transformation

Any precipation measurements are first corrected for representativeness using a factor, c, for example to correct for gauge loss or altitude and aspect effects. A temperature threshold, T_s, is used to discriminate precipitation, p (after correction for representativeness) into rainfall, R, and snowfall, P. Thus we have R=p and P=0 when the standard air temperature $T \geq T_s$ and R=0 and P=p otherwise. It will be assumed at this stage that measurements of precipation in the form of snow are available. A typical value for T_s under UK conditions is 1°C.

Melt Equation

Whilst melt can be computed from a full energy balance calculation this demands extensive climate data that are often not available in real-time. Such an approach may also only give an impression of greater accuracy which in practice is spurious because of the complex effects of spatial variability at the basin scale. A simple temperature excess representation of the rate of melt, M, is adopted here of the form:

$$M = \begin{cases} f(T-T_m) & T > T_m \\ 0 & \text{otherwise} \end{cases} \qquad (2.1)$$

where T_m is a critical temperature above which melt occurs and f is a melt factor in units of mm/day/°C. The critical temperature T_m is usually taken to be 0°C. An extension to incorporate wind speed may be important for UK conditions (Harding and Moore, 1988) but the added level of complexity has not been incorporated in the current implementation of the model.

Snowpack Storage and Drainage

Water accounting within the snowpack is accomplished by introducing the concept of "dry" and "wet" stores. New snow falling on the pack contributes to the dry store and water continuity is defined by the equation

$$\frac{dW}{dt} = P - M \qquad (2.2)$$

where W is the water equivalent of the snow in the dry store which is added to by snowfall, P, and is depleted by melt, M.

A second wet store receives water as rainfall R, as melt from the dry store, M, and loses water via losses to drainage, Q, which can subsequently form the input to a catchment scale rainfall-runoff model. Continuity in this case gives

$$\frac{dS}{dt} = R + M - Q \tag{2.3}$$

where S is the water equivalent of the snow in the wet pack store.

Release of water from the wet pack as drainage occurs at a slow rate proportional to the wet pack storage. When the total water equivalent of the pack exceeds a critical limit, referred to as the Critical Water Capacity S_c, then release of water begins to occur at a faster rate. This mechanism can be conceptualised by viewing the wet store as a tank with two orifices, one located at the base of the store and releasing water slowly and a second one at a height which varies dynamically as a function of the total water equivalent of the pack (Figure 2.1).

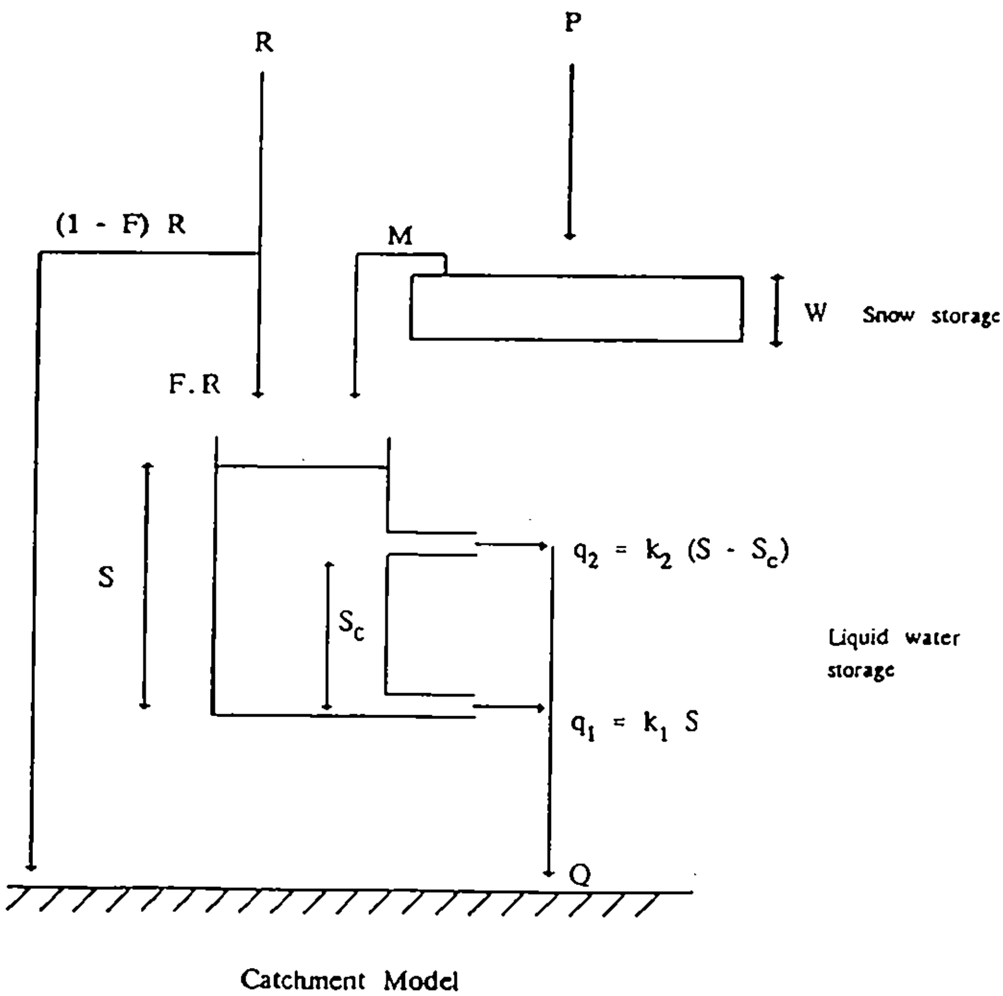

Figure 2.1 Schematic of the pragmatic snowmelt model PACK

The total drainage from the store can be expressed mathematically as

$$Q = q_1 + q_2 = \begin{cases} k_1 \, S & S \le S_c \\ k_1 \, S + k_2 (S - S_c) & S > S_c \end{cases} \tag{2.4}$$

where k_1 and k_2 are storage time constants with units of inverse time. The dynamically varying level of the upper orifice, S_c, is defined through the relation

$$S_c = S_c^* \, (S + W) \tag{2.5}$$

where S^*_c is the maximum liquid water content (expressed as a proportion of the pack water content), a fixed parameter of the model. This formulation is similar to that used in the Japanese Tank Model (Sugawara et al, 1984). As a special, simplified case the lower orifice may be removed and the upper orifice replaced by an open tank by setting k_1 to 0 and k_2 to 1.

Solution of the equations of continuity in conjunction with the dynamics as represented above is achieved using a simple discrete time formulation. Representing the dry and wet store contents at the discrete time point t by W_t and S_t then the set of water accounting equations for use between the discrete time points t-1 and t of duration Δt are:

$$W_t = W_{t-1} + (P_t - M_t)\Delta t$$

$$S_t' = S_{t-1} + (R_t + M_t)\Delta t$$

$$S_c = S_c^* \, . (S_t' + W_t)$$

$$Q_t = \begin{cases} k_1 S_t' & S_t' \le S_c \\ k_1 S_t' + k_2 (S_t' - S_c) & S_t' > S_c \end{cases} \tag{2.6}$$

$$S_t = S_t' - Q_t \Delta t \; .$$

A further refinement is introduced to inhibit drainage during cold periods. A cold period is defined to be below a temperature of T_c, taken to be 0°C in the present implementation.

Areal Depletion Curve

A further extension of the model to incorporate the phenomenon that shallow snowpacks may occupy only a fraction, F, of the basin may be important in some instances. The fraction of snow cover may be allowed to vary as a function of the total water equivalent of the pack $\theta = S + W$ ranging from zero when $\theta=0$ to unity when θ exceeds a critical value θ_c and complete snow cover occurs. The functional form for $F = F(\theta)$ adopted here derives from

$$\theta = (\theta_c + 1)^{F(\theta)} - 1 \qquad (2.7)$$

suggested by Laramie and Schaake (1972). In terms of $F(\theta)$ we have

$$F(\theta) = \frac{\log(\theta + 1)}{\log(\theta_c + 1)} \ . \qquad (2.8)$$

Given the current water content of the pack θ the fraction of the basin covered by snow is readily calculated from the above. In the event of a fresh snowfall, $\Delta\theta$, it is assumed temporally that the fraction reverts to 1 until a fraction $(1-\alpha)\Delta\theta$ has melted. A linear reversion to the original point on the areal depletion curve (Figure 2.2) occurs in melting the remainder of the new snow, $\alpha\Delta\theta$. Normally the proportion, α, is set to 0.25. Any rain falling on the fraction devoid of snow is available immediately for input to the rainfall-runoff model for the basin.

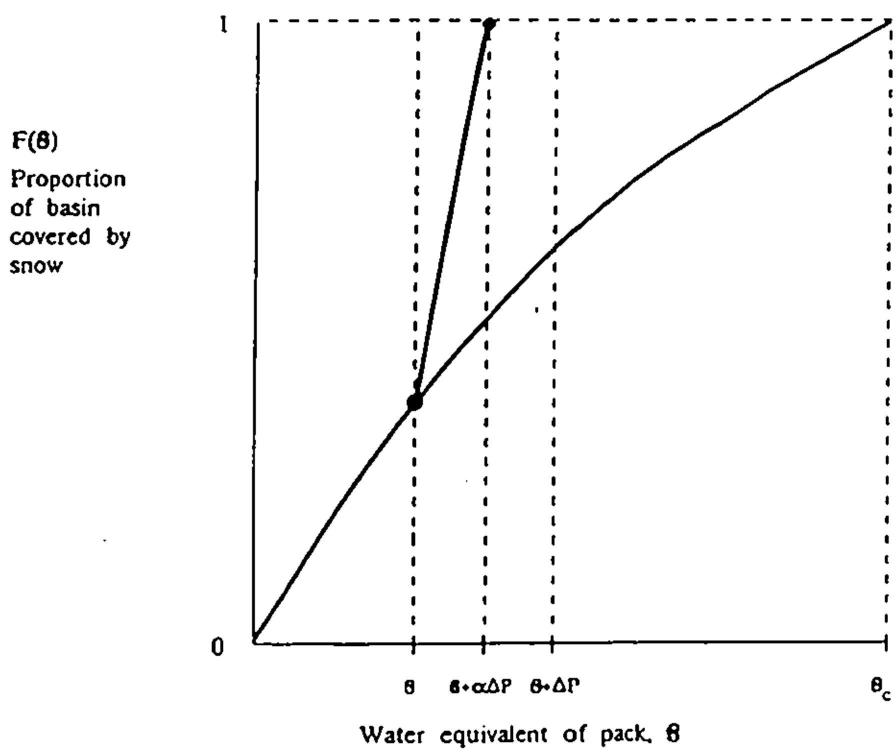

Figure 2.2 Areal depletion curve used in the PACK model

Table 2.1 provides a summary of the model parameters involved in the PACK snowmelt model formulation.

Table 2.1 The PACK snowmelt model parameters

Parameter	Meaning	Typical value	Unit
c	Precipitation representativeness factor	1	dimensionless
T_s	Temperature threshold below which precipitation is snow	1	°C
T_m	Critical temperature above which melt occurs	0	°C
f	Melt factor	4	mm/day/°C
k_1	Storage time constant: lower orifice	0.15	day^{-1}
k_2	Storage time constant: upper orifice	0.85	day^{-1}
S_c^*	Maximum liquid water content, as a proportion of total	0.04	dimensionless
T_c	Critical temperature below which no drainage occurs	0 (fixed)	°C
θ_c	Critical water content below which only a proportion of the basin is snow-covered	100	mm
α	Fraction of new snow remaining below which snow-covered area starts to revert to areal depletion curve	0.25	dimensionless

3. STATE UPDATING AT THE BASIN SCALE

Correction of the snowpack to accord with snow survey measurements can be done at times when measurements of both the pack water equivalent and density are available: typically at 09.00 on days of lying snow. Because these measurements relate to a snow survey measurement point and not to the basin for which snowmelt is to be forecast, the correction process is not straightforward. A correction is first calculated for a "point snowmelt model" and this is subsequently transferred as a correction to the basin scale snowmelt model or models for which the point model is regarded as representative. Thus updating a snowmelt forecasting model involves running two models in parallel, first a point model at the snow survey site using climate data (precipitation, temperature) in the vicinity and second a basin model whose water balance accounting employs climate data representative of the basin and which is updated by transfer of state-correction information from the point model.

The state-correction information computed within the point snowmelt model at the survey site comprises two quantities. The first is the snow correction factor, f, computed as the ratio of the measured water equivalent of the pack, θ_m, to the modelled value, θ; that is

$$f = \theta_m / \theta. \tag{3.1}$$

The second is the proportion of dry snow in the pack expressed as

$$\beta = \frac{\rho_m - 1}{\rho_w - 1} \tag{3.2}$$

where ρ_m, ρ_w are the measured snow pack density and the density of dry snow, assumed equal to 0.1. The two state variables of the point snowmelt model, W the water equivalent of the dry pack and S the water equivalent of the wet pack, are then updated as follows:

$$W^{\dagger} = \beta \, \theta_m \tag{3.3a}$$

$$S^{\dagger} = \theta_m - W^{\dagger} = (1 - \beta) \, \theta_m \tag{3.3b}$$

where the superscript dagger is used to denote the updated quantity. Note that this correction ensures that the water equivalent and density of the modelled and measured packs agree, and also establishes the partition between wet and dry pack water storage in the modelled pack.

Transfer of the state correction information, f and β, to the basin scale model used for snowmelt forecasting is straightforward. First the water equivalent of the model pack, $\theta = W + S$ (no change in notation will be introduced as it is clear that here we are referring to the basin scale model), is factored using the point snow correction factor, f, such that

$$\theta^{\dagger} = f \, \theta. \tag{3.4}$$

$$W^{\dagger} = \beta \, \theta^{\dagger} \tag{3.5a}$$

$$S^{\dagger} = \theta^{\dagger} - W^{\dagger} = (1 - \beta) \, \theta^{\dagger} . \tag{3.5b}$$

The correction is thus in proportion to that applied in the point model, relative to the water equivalent of point and basin packs, and also establishes a partition between wet and dry packs by maintaining the same density.

4. SCREENED OPTIMISATION AND PARAMETER ESTIMATION

The availability of snow survey data on the depth and water equivalent of the snowpack as a result of snow coring at selected points provides one means of estimating the parameters of the snowmelt model. A conventional sum of squares objective function can be formulated by comparing observed water equivalent snowpack values at a snow survey site with model estimates using climatic data (precipitation and temperature) for the same site. A practical problem usually arises through the precipitation measurement system being capable of only measuring precipitation in the form of rain: this is the situation prevalent in most regions of the UK where a conventional raingauge is the norm. (A notable exception is the Severn Trent region of the NRA where "super-heated" raingauges capable of melting and measuring the resulting depth of water are deployed). As a result it is impossible to maintain a water budget of the snowpack, independent of the snow survey data, using the PACK model when precipitation falls as snow since no formal measurements of snowfall are available. What can be done is to use the model to discriminate between precipitation in the form of snow and rain using the temperature threshold, T_s, and not to use the raingauge value at times when snowfall is indicated. With the availability of snow survey data recorded once per day the model snowpack can be re-initialised to the snow survey measurement of the pack's water equivalent and density, as discussed in the previous section. Model predictions of water equivalent are only compared to survey measurements on days when no snow has fallen. On days with snow the model water budget is not invoked and the next set of survey observations is used to re-initialise the pack store contents. As a result the objective function includes predictions which may be in part simulation mode (non-updated) predictions for days with no snow and partly updated predictions following days of snow; in the current version of the model state correction is invoked at the time of every snow survey measurement. Since it would be common for the temperature to be conducive to snow (i.e. below 0°C) and for no precipitation to fall (or none recorded by a raingauge) a further criterion for inclusion in the objective function. (and execution of the water budget) is invoked. This requires that the pack water equivalent reduces during the day as indicated by snow survey data on successive days.

5. CONCLUSION

The PACK model provides a very simple approach to snowmelt modelling which encompasses the dominant mechanisms operating in the humid-temperate climate of the UK. It also incorporates an empirical state updating technique which employs a point pack model at a snow survey site to obtain adjustments which are subsequently applied at the basin scale through simple proportioning rules. Problems of parameter estimation arising from the practical difficulty of measuring falling snow are circumvented through the use of a screened objective function and state updating. The PACK model is available within the River Flow Forecasting System in calibration form for parameter estimation using snow survey data and as two model algorithms, for point and basin-scale use, for operational snowmelt flood forecasting.

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