I.O.S.

A NEW METHOD FOR THE ANALYSIS OF EXTREMES APPLICABLE TO ONE YEARS' DATA

by

P G CHALLENOR

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1. INTRODUCTION

When return values of wave height are needed for engineering purposes, often a data set covering only one year is available from which to estimate these values. In this case, the classical method of extreme value analysis, in which a distribution is fitted to annual maxima, cannot be used; at least five years' data are needed. Instead various empirical approaches have to be employed, such as fitting the data to a Weibull distribution - for details of this and other methods see for example Carter and Challenor (1981). Estimates of 50-year and 100-year return values from one year's data will be very uncertain, and with these empirical methods of doubtful validity, it is impossible to calculate confidence limits.

This report describes, and slightly extends, the method of Weissman (1978), in which the k largest data values are fitted to a distribution - where k is small compared with the total number of data values, but greater than one. This method is theoretically justified, and provided that certain specific assumptions are met it can be applied to only one year's data, and confidence limits can be derived for return values.

The application of this method to estimating return values assumes - as indeed do all methods in use at present - that there is no variation in wave climate from year to year. (Even if the wave climate remains constant, any large between-year variability will reduce the accuracy of the estimated return value.) The validity of this assumption and others made in the method are tested by applying it to the several years of data recorded by the Seven Stones Light Vessel during the past fifteen years.

2. WEISSMAN'S METHOD

2.1 Derivation

The method of Weissman (1978) is similar to that of Pickands (1975), described briefly by Carter and Challenor (1981), in that it uses the largest values of the sample to obtain estimates of return values. Pickands' method does this by fitting a 'tail distribution' to these upper points and extrapolating to obtain the results. There are serious problems in estimating the parameters of this 'tail distribution', in particular deciding where in the ordered data the 'tail' starts. Weissman's method, on the other hand, uses the asymptotic joint distribution of the largest k data points. As will be seen the estimators can be expressed in a closed analytic form and hence no numerical methods are necessary. However the value of k has to be chosen and no satisfactory objective method has yet been proposed for this.

It can be shown, e.g. Galambos(1978), that any distribution of extreme values of samples from a population must tend asymptotically to one of three distributions, the

Fisher-Tippett distributions. The Fisher-Tippett Type 1 (FT-1) distribution is unbounded above and below, and is given by

$$P(X<x) = \exp[-\exp\{-(x-A)/B\}]$$

Types 2 and 3 are bounded from below and above respectively.

Weissman(1978) derives estimators for all three Fisher-Tippett distributions, assuming that the bounds are known for Types 2 and 3. Here only the FT-1 is considered. The method of derivation is not as rigorous as Weissman's but is considerably easier to understand.

Consider a sample, size n, from a distribution in the domain of attraction of the FT-1. Let $X_1>X_2...>X_n$ be the order statistics. Then, if the asymptotic distribution of the largest value (X_1) is FT-1, the asymptotic joint probability density function of the k largest order statistics (Galambos 1978) is

$$B^{-k}\exp\ [-\exp\{-(x_k-A)/B\}-\sum_{i=1}^{k}(x_i-A)/B]$$

Therefore the likelihood of the largest k values in our sample is also

$$L(A,B) = B^{-k}\exp[-\exp\{-(x_k-A)/B\}-\sum_{i=1}^{k}(x_i-A)/B]$$

Using this likelihood it is easy to derive the maximum likelihood estimators (MLE), these estimators are in a closed analytic form so there are no non-linear equations that have to be solved numerically. However they are biased and therefore it may be preferable to use the minimum variance unbiased estimators (MVUE). Because of the analytic nature of the MLE's these are also simply derived and are in a closed form. These estimators (using ^ for MLE's and * for MVUE's) are given by

$$\hat{B} = \overline{X}_k - X_k \quad ; \quad \hat{A} = \hat{B}\ln(k) - X_k$$

$$B^* = \overline{X}_{(k-1)} - X_k \quad ; \quad A^* = B^*(S_k - \gamma) + X_k$$

where

$$\overline{X}_k = \sum_{i=1}^{k} X_i / k$$

$$S_k = \sum_{j=1}^{k-1} j^{-1}$$

and $\gamma = 0.5772 \ldots$ is Euler's constant.

It can be shown that if F(.) is in the domain of attraction of the FT-1 and η_p is the p-quantile of F(.) then

$$\eta_{1-c/m} \simeq A - B\ln(c)$$

where c<<n and n is the sample size. (This is approximately equal to calculating the percentile directly from the FT-1 with parameters A and B). In the case of the m-year return value C = 1/m. Therefore the estimators (MLE and MVUE) are

$$\hat{\eta}_{1-c/m} = \hat{A} - \hat{B}\ln(c) \qquad = \hat{A} + \hat{B}\ln(m)$$

$$\eta^*_{1-c/m} = A^* - B^*\ln(c) \qquad = A^* + B^*\ln(m)$$

Because of the analytic nature of both the MLE's and MVUE's it is relatively easy to find confidence limits on the parameters. Weissman gives confidence limits for both MLE's and MVUE's, because the MVUE's are more likely to be used in practice only their confidence limits will be given here. $2(k-1)B^*/B$ is distributed $\chi^2_{2(k-1)}$ and therefore a 100(1-)% confidence limit is given by

$$\frac{2(k-1)B^*}{\chi^2_{2(k-1),\alpha/2}} < B < \frac{2(k-1)B^*}{\chi^2_{2(k-1),(1-\alpha/2)}}$$

The distribution of $U_k = (x_k-A)/B^*$ is tabulated, for various k, in Weissman (and an extended table is given in appendix II), so that the confidence limits are given by

$$x_k - B^*U_{k,\alpha/2} < A < x_k - B^*U_{k,(1-\alpha/2)}$$

Using a similar method to that used by Weissman to obtain confidence intervals on A, confidence intervals for return values have been derived. Details of the derivation are given in appendix I. The distribution of $W_k = (x_k-\eta)/B^*$ has been obtained and is tabulated in appendix III. The confidence limits are therefore

$$x_k - W_{k,\alpha/2}B^* < \eta < x_k - W_{k,(1-\alpha/2)}B^*$$

2.2 Assumptions

The assumptions made in the above derivation are:

a) Fisher and Tippett's extreme value theory is applicable to the data set, in particular the data are identically distributed.

b) The FT-1 is the appropriate asymptotic extreme value distribution.

c) The largest k values are independent - otherwise the joint probabilty distribution given above is incorrect. (Strictly Fisher and Tippett's theory also requires independence of the data, but it has been shown that some small dependence is not important.)

d) There is no between year variability in the k largest values, otherwise the derivation of 50-year return value from one year's data is invalid.

Assumption (a) is not generally justified for a year's environmental data, which usually has a large within-year variation, with for example a markedly different distribution of wave height in the winter than in the summer. The effect of this variation can be considerably reduced by analysing data from each month separately.

There is no proof that assumption (b) is valid for environmental data; but for waves the rather small amount of data available for analysis gives no evidence that the assumption of FT-1 is incorrect. Physical constraints might suggest some upper limit to wave height, even in deep water, but the existence of an upper bound on the data is insufficient to ensure that the asymptotic extreme distribution is bounded. The distributions usually fitted to wave data, such as the Weibull, have distributions of maxima that are asymptotic to the FT-1.

Assumptions (c) and (d) are more difficult to justify, and are discussed below in Section 4.1.

3. APPLICATION

3.1 Data

The longest series of significant wave height data available is from a Shipborne Wave Recorder fitted in the Seven Stones Light Vessel (near 50°N 6°W, between Cornwall and the Isles of Scilly). This set consists of data from 1968 to 1977, unfortunately with gaps of approximately two years so that there are in all seven or eight years' data available depending upon the month. Estimates of significant wave height from traces recorded for 15-20 minutes every 3 hours are analysed in this report.

3.2 Method of Analysis

Although up to now consideration has been given to analysing one year's data, in practice, because of considerable seasonal variation in the Seven Stones data (as in all wave data around the UK), it is necessary to analyse the data by month. (For details of the effects of seasonal variation on the analysis of extremes see Carter and Challenor,1981.) The return value of wave height throughout the year may be obtained, assuming the monthly maxima are independent, by

$$P(X<x) = \prod_{i=1}^{12} \exp(-\exp(-(x-A_i)/B_i))$$

However, confidence limits are not available for these estimates.

3.3 Choosing a value for k

One problem that is immediately apparent with the above method is that k has to be chosen subjectively. Obviously we would like to have k as large as possible to produce the narrowest confidence limits, but if we make k too large the asymptotic theory on which the method is based will no longer hold. At present there has been no work published on what a suitable choice for k would be and so some care must be exercised in choosing a value.

In analysing the Seven Stones data an attempt was made to find an optimum k, that is the largest value of k for which the asymptotic theory would hold. By examining a few months with various values of k it was decided initially to use k=40 and the analysis was performed with this value of k. However it was then noticed that the estimates of A (the most likely highest value of significant wave height in a month) were all much too high implying that 40 was too large a value for k. Therefore k=30 was tried, but during the analysis it became plain that the same error was present. Since a value of k that is smaller than the optimal would be satisfactory it was decided to use k=10; which appears to give reasonable estimates of A. All the results quoted below have been obtained with this value of k. However no claim is made that this value of k is in any way the 'best'. Obviously further work is necessary on methods of choosing k.

4. RESULTS

4.1 Estimates of distribution parameters

The estimates obtained by month and year are shown in figures 1 and 2 together with their 90% confidence limits. (The estimates of A are shown by +'s and B by x's.) Also included in these diagrams are the estimates of A and B from 7 years data as given in Carter and Challenor (1978). If we were to assume that the estimate from 7 years' data to be approximately the correct value then we would expect nine out of ten of the confidence bands to intersect this value. Unfortunately the estimates from the analysis of extremes are likely to be far from the correct value (the confidence limits given in Carter and Challenor (1978) are very broad indeed). However from the definition of confidence intervals one would expect at least nine out of ten of the 90%

confidence bands to overlap. This is simply not the case here, which implies that the estimates are inconsistent with each other and therefore not surprisingly differ significantly from the results of the analysis of extremes. In most cases given in the figures it would be impossible to draw any line that intersected nine out of ten of the confidence bands!

Why should this be? One answer would be that the new method simply does not work, possibly because 240 values is not enough to produce convergence to their asymptotic distributions of the top ten values. However the estimates obtained, despite being significantly different from one another, are all reasonable, particularly the return values (given in Table 1), and some fuller explanation seems to be needed.

Both methods require the data to be independent and identically distributed within the months. However since the analysis of extremes uses only one value per month any dependence in the data would not affect the results unduly and theoretical work, described in Galambos(1978), shows that in general lack of independence is not a serious problem. The new method, on the other hand, uses the joint distribution of the highest values and so may be seriously affected if the data were not independent. No theoretical work dealing with this problem has yet been published and so no estimate can be made of its effect on the estimators.

This could therefore be the cause of the difference in estimates derived by the new method. It could be a particular problem with small k, when all k values fitted might be from one large storm. This might explain the high values estimated for A and B for March 1976 (giving a 100-year return value of significant wave height of 20m). During this month there was a severe storm with wave heights greater than 6m recorded for 8 consecutive 3-hourly periods with a maximum of 11m. This value is the largest ever recorded at Seven Stones, so the analysis might simply be reflecting the effect of an outlier or the results of fitting wave heights during a storm and not during a month.

There is, however, yet another possibility and this is that the distribution of extremes does actually vary from year to year. If this were true it would invalidate the analysis of extremes (and indeed any other analysis using more than one year's data and make the results of any analysis of only one year's data difficult to interpret and of limited application). There is some evidence for this in the results given here, for instance both the winter and spring of 1972 give higher than average estimates of A. This correlation between adjacent estimates implies that there could be some non-random variation within the data which is picked up by the new method but is impossible to discern when using the analysis of extremes.

4.2 Estimates of return values

Estimates of 50-year and 100-year return values from various 12-months, using the distribution given in section 3.2 are given in Table 2. This table also gives estimates of the 50-year return value from Fortnum and Tann(1977), and for the 100-year return value from Tucker and Fortnum(1981). Both these papers derive estimates of the most likely highest wave in 3 hours by fitting an FT-1 distribution to the 12 months data, Fortnum and Tann by least squares and Tucker and Fortnum by eye. Significant wave heights in Table 2 have been obtained by dividing by 1.9.

The values for the year April 75 - March 76 estimated by Weissman's method seem very high; they result from the high parameter values for March 1976 , discussed above. The other values estmated by this method are generally higher than those of Fortnum and Tann - except for the year July 73 - June 74 - but do not seem unreasonable, with a range from year to year of about 2.5m (omitting 75-76), which is similar to the range obtained by Fortnum and Tann and by Tucker and Fortnum. The mean of the seven yearly values for the 50-year return value is 14.8m in agreement with the estimate obtained by Carter and Challenor(1978) by analysing monthly extremes.

5. CONCLUSIONS

A new method of estimating return values from only one year's data has been discussed, it is theoretically sound, but does assume that the data are independent and identically distributed. Confidence limits are given for the parameters of the distribution and also for estimates of 50 and 100-year return values.

However the estimates derived by the new method, applied to Seven Stones L.V. wave data, while not being wildly different, do differ significantly from those obtained by an analysis of extremes and are not consistent with each other. Four reasons have been proposed for this. The first, that the method does not work because the sample size is too small, has been rejected because the results are not obviously wrong. The possibility that the rather small value of k (k=10) is still too large is a worrying one. Intuitively one would expect the top 5% of the data to be near its asymptotic distribution, but there is no guarantee that this is so. There is an urgent need for research into an objective method of estimating k so that the real cause of these discrepancies can be ascertained. The third explanation is that dependence in the data is seriously affecting the new method. However it might be expected that this should affect each year's data equally, which is not so in these results. Patterns in the discrepancies between the new method and the analysis of extremes point to the fourth possible explanation: that there are differences in the distribution of extreme wave heights from year to year which are greater than would arise by chance, i.e. there is some non-random year to year variation. However there is insufficient evidence at present to either prove or disprove this. The final conclusion must be that while the new method is an interesting addition to extreme value analysis its practical aspects are too little understood to make it viable for use. More work is needed to show whether the difficulies encountered in its application are due to short comings in the method or to some non-random between year variation in the distribution of significant wave height.

7. ACKNOWLEDGEMENTS

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APPENDIX I

THE DERIVATION OF CONFIDENCE LIMITS ON RETURN VALUES

As stated in section 2 the confidence limits on return values, η (= A + Blnc), are obtained from the distribution of $W_k = (x - \eta)/B^*$. This distribution will now be derived.

$$(x_k - \eta)/B = (x_k - A + B\ln c)/B$$

$$= (x_k - A)/B + \ln c$$

but the distribution of $(x_k - A)/B$ is given in Weissman, (2.4), as

$$f\left(\frac{x_k - A}{B}\right) = \frac{1}{(k-1)!} \exp\left(-e^{-x} - kx\right)$$

the distribution of $y = (x_k - A)/B + \ln c$ is

$$f(y) = \frac{1}{(k-1)!} \exp\left\{-e^{-y+\ln c} - k(y - \ln c)\right\}$$

The distribution of $(x_k - \eta)/B^*$, W_k say, is then obtained by finding the joint density of y and $(k-1)B^*/B$ (which is gamma) and transforming the variables. This gives us the joint density of W_k and $(k-1)B^*/B$, t say. Integrating out t from $(0, \infty)$ and integrating W_k from $-\infty$ to z gives the distribution of W_k as

$$P(W_k < (k-1)z) = \int_0^\infty \sum_{j=0}^{k-1} \left\{ e^{-ce^{-tz}} \frac{c^j e^{-tzj}}{j!} \right\} \frac{e^{-t} t^{k-2}}{\Gamma(k-1)} \, dt$$

This has been tabulated for $c=0.02$ in appendix III, and $c=0.01$ in appendix IV. N.B. The distribution of U_k given in Weissman is incorrect. A factor of $(\Gamma(k-1))^{-1}$ has been omitted.

APPENDIX II

Percentage Points for $U_k(p)$

K \ P	0.01	0.025	0.05	0.10	0.50	0.90	0.95	0.975	0.99
10	-6.02	-5.10	-4.46	-3.83	-2.33	-1.48	-1.31	-1.17	-1.03
12	-5.83	-5.04	-4.47	-3.91	-2.51	-1.67	-1.50	-1.36	-1.22
14	-5.71	-5.02	-4.50	-3.98	-2.67	-1.85	-1.66	-1.53	-1.39
16	-5.64	-5.01	-4.53	-4.05	-2.79	-1.99	-1.82	-1.68	-1.53
18	-5.59	-5.02	-4.57	-4.11	-2.91	-2.13	-1.94	-1.80	-1.66
20	-5.57	-5.02	-4.60	-4.18	-3.02	-2.24	-2.05	-1.92	-1.77
25	-5.54	-5.06	-4.70	-4.32	-3.24	-2.47	-2.31	-2.17	-2.02
30	-5.54	-5.10	-4.79	-4.44	-3.42	-2.69	-2.52	-2.38	-2.23
35	-5.56	-5.17	-4.86	-4.53	-3.57	-2.86	-2.69	-2.56	-2.41
40	-5.58	-5.22	-4.93	-4.62	-3.71	-3.01	-2.85	-2.71	-2.57
45	-5.61	-5.27	-4.99	-4.70	-3.82	-3.15	-2.98	-2.85	-2.70
50	-5.64	-5.31	-5.05	-4.77	-3.93	-3.27	-3.10	-2.97	-2.83

APPENDIX III

Percentage Points for $W_k(p)$ (c=50)

K \ p	0.01	0.025	0.05	0.10	0.50	0.90	0.95	0.975	0.99
10	-15.91	-13.55	-11.87	-10.25	-6.35	-4.24	-3.81	-3.48	-3.14
12	-14.73	-12.79	-11.38	-9.99	-6.55	-4.51	-4.09	-3.77	-3.43
14	-13.96	-12.29	-11.06	-9.79	-6.69	-4.74	-4.33	-4.01	-3.68
16	-13.41	-11.93	-10.83	-9.72	-6.80	-4.94	-4.53	-4.22	-3.89
18	-12.77	-11.67	-10.67	-9.64	-6.91	-5.11	-4.71	-4.40	-4.08
20	-12.69	-11.47	-10.54	-9.59	-7.00	-5.26	-4.88	-4.57	-4.25
25	-12.16	-11.13	-10.34	-9.51	-7.21	-5.59	-5.22	-4.92	-4.61
30	-11.82	-10.92	-10.22	-9.49	-7.38	-5.85	-5.49	-5.21	-4.90
35	-11.59	-10.79	-10.15	-9.48	-7.53	-6.07	-5.73	-5.45	-5.15
40	-11.43	-10.70	-10.11	-9.49	-7.65	-6.26	-5.93	-5.66	-5.36
45	-11.31	-10.63	-10.09	-9.50	-7.77	-6.43	-6.10	-5.84	-5.55
50	-11.22	-10.58	-10.07	-9.52	-7.87	-6.57	-6.26	-6.00	-5.72

APPENDIX IV

Percentage Points for $W_k(p)$ (c=100)

K	0.01	0.025	0.05	0.10	0.50	0.90	0.95	0.975	0.99
10	-17.68	-15.06	-13.19	-11.39	-7.07	-4.72	-4.24	-3.88	-3.51
12	-16.32	-14.17	-12.61	-11.07	-7.26	-5.01	-4.54	-4.18	-3.81
14	-15.43	-13.59	-12.22	-10.83	-7.39	-5.25	-4.79	-4.44	-4.07
16	-14.80	-13.17	-11.95	-10.72	-7.51	-5.46	-5.01	-4.66	-4.30
18	-14.32	-12.86	-11.75	-10.62	-7.61	-5.64	-5.20	-4.86	-4.50
20	-13.96	-12.62	-11.60	-10.55	-7.71	-5.80	-5.37	-5.04	-4.68
25	-13.33	-12.21	-11.34	-10.44	-7.91	-6.14	-5.73	-5.41	-5.06
30	-12.94	-11.96	-11.19	-10.38	-8.08	-6.41	-6.02	-5.71	-5.37
35	-12.67	-11.79	-11.09	-9.75	-8.23	-6.64	-6.26	-5.96	-5.63
40	-12.47	-11.67	-11.03	-10.35	-8.35	-6.83	-6.47	-6.18	-5.86
45	-12.33	-11.58	-10.99	-10.35	-8.46	-7.01	-6.65	-6.37	-6.05
50	-12.21	-11.52	-10.96	-10.38	-8.51	-7.16	-6.82	-6.54	-6.23

Table 1

	1968	1969	1971	1972	1973	1974	1975	1976	1977
J	10.9	12.2	-	9.5	11.6	13.1	-	10.7	10.6
F	5.8	8.0	-	10.6	13.6	11.3	-	8.7	8.3
M	8.4	5.5	-	14.4	9.1	8.0	-	18.6	10.1
A	7.8	9.1	-	10.1	8.4	7.2	9.3	8.9	-
M	6.5	4.4	-	9.1	8.4	8.3	5.3	5.6	-
J	4.8	6.7	-	6.2	5.0	9.4	4.5	5.3	-
J	4.9	7.6	3.9	4.5	6.7	-	5.7	4.3	-
A	8.3	4.7	5.8	6.5	6.9	-	4.9	3.3	-
S	11.5	5.1	9.5	4.8	9.0	-	12.2	8.0	-
O	7.7	6.8	8.5	8.7	6.7	-	7.0	13.8	-
N	8.5	15.0	9.6	11.9	7.4	-	9.7	8.7	-
D	14.3	11.0	13.4	9.6	11.0	-	8.9	12.7	-

Estimates of 50-year return value of significant wave height (m) by month

Table 2

Year of data	This report 50-year	This report 100-year	Fortnum and Tann(1977) 50-year	Tucker and Fortnum(1981) 100-year
1968	14.4	15.3	12.8	13.2
1969	15.1	16.0	12.8	13.5
July 71-June 72	14.9	15.7	14.5	14.0
July 72-June 73	13.9	14.7	13.0	12.7
July 73-June 74	12.6	13.2	15.2	15.2
April 75-March 76	18.6	20.0	-	15.0
April 76-March 77	14.1	15.0	-	13.8

Estimates of return value of significant wave height (m) at Seven Stones from 12 months wave data

(Fortnum & Tann and Tucker & Fortnum estimate $H_{max,3hrs}$, their results have been divided by 1.9 to obtain the values given here)

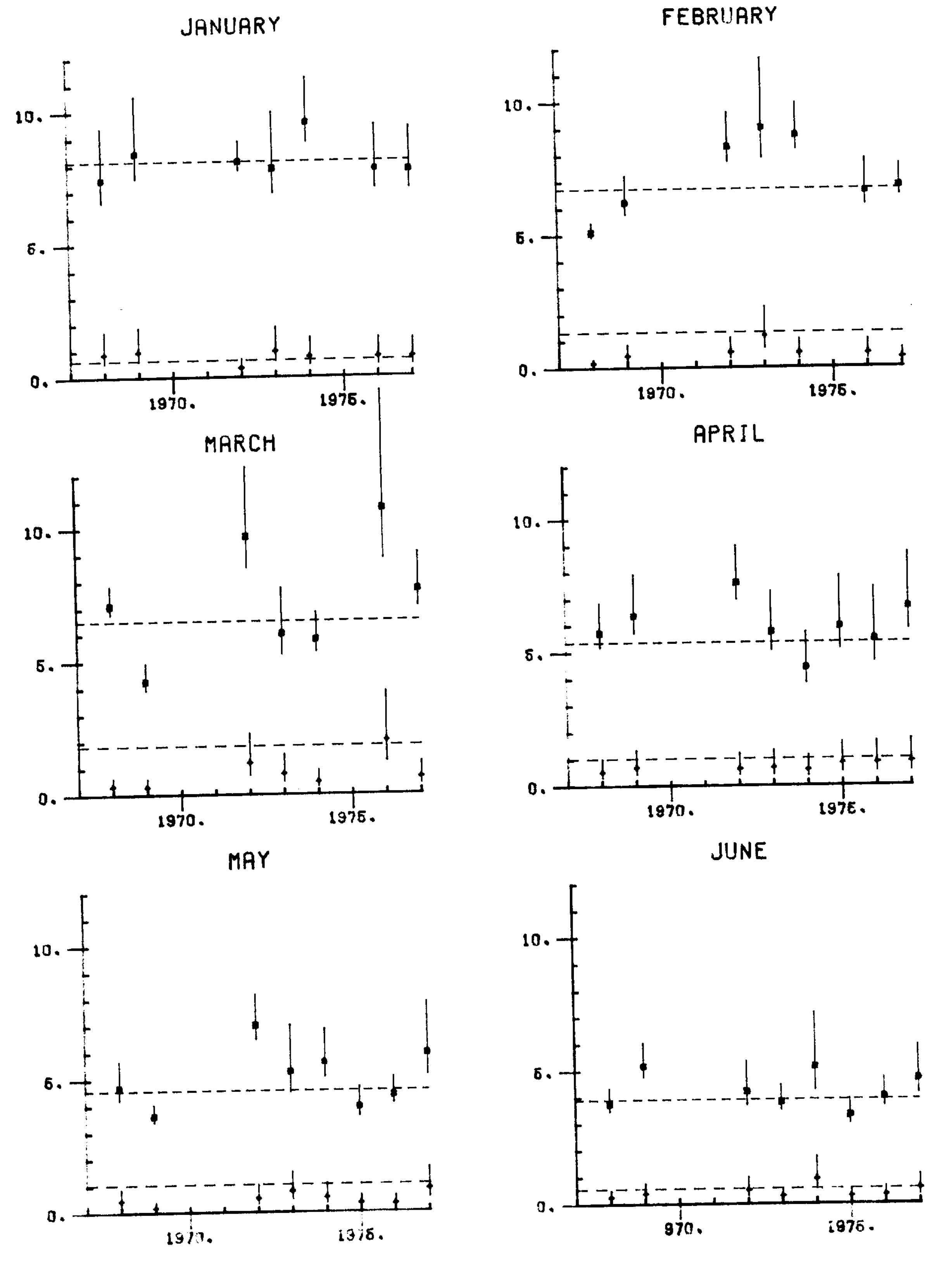

Figure 1: Estimate of location (*) and scale (+) parameters of H_s for Seven Stones 1968-1977 January-June. (The dashed lines show the estimates from monthly maxima).

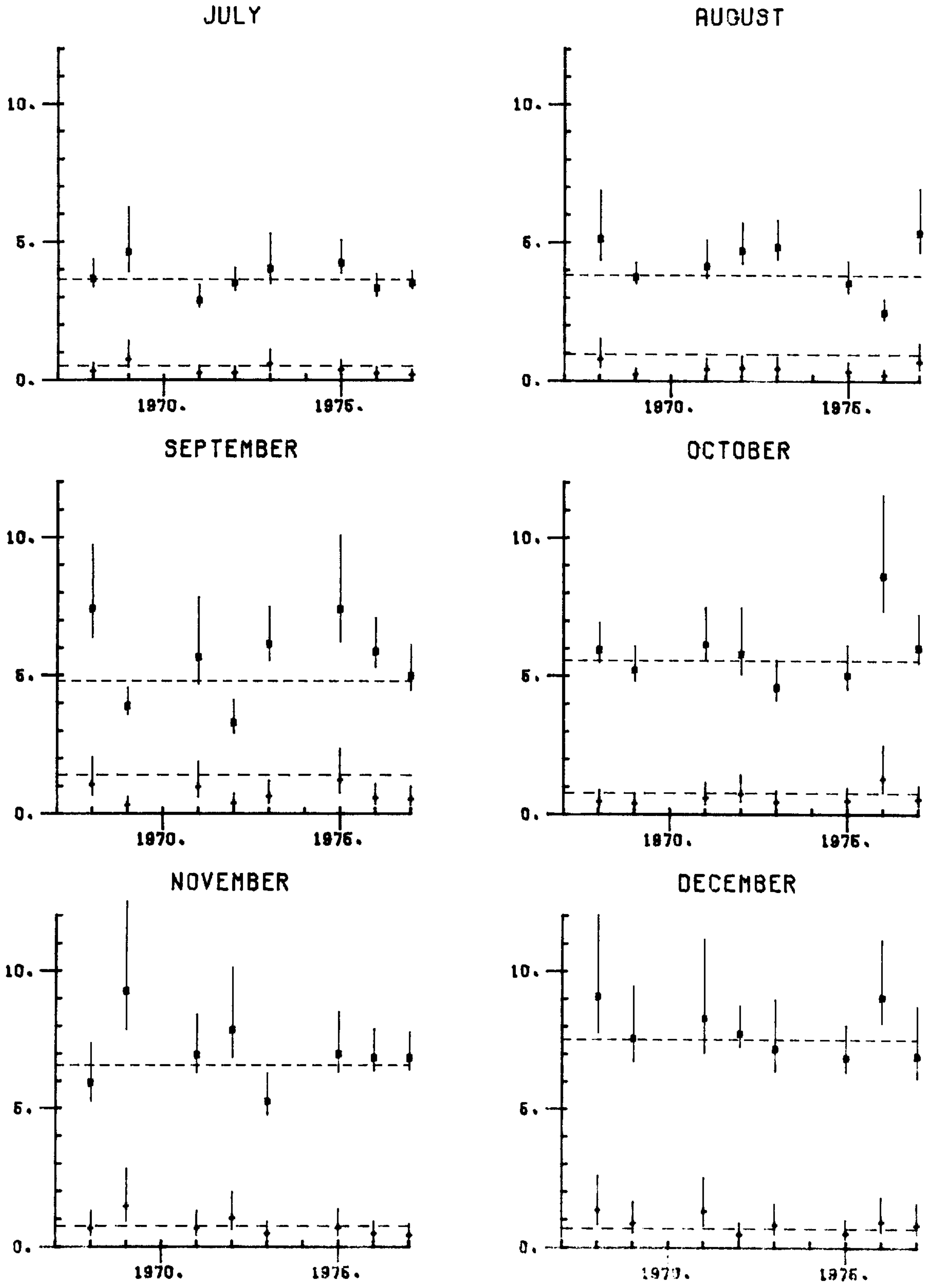

Figure 2: Estimates of location (*) and scale (+) parameters of H_S for Seven Stones 1968-1977 July-December. (The dashed lines show the estimates from monthly maxima).