I.O.S.

THE ESTIMATION OF WAVE PARAMETERS FOR THE DESIGN OF OFFSHORE STRUCTURES

by

H.M. TANN

A DESCRIPTION OF THE METHOD PRESENTLY USED BY IOS

REPORT NO 23

1976

NATURAL ENVIRONMENT RESEARCH COUNCIL

INSTITUTE OF OCEANOGRAPHIC SCIENCES

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1. ABSTRACT

This report contains a description of the method of wave data analysis used by IOS. This covers: (i) the manual method for calculating height and period parameters from an analogue record of the variation of surface elevation at a point and (ii) the techniques used to extrapolate results to rare events in order to estimate design heights for offshore structures. The scientific theory on which the method is based is given, including a summary of the assumptions made and some error estimates. The procedure is illustrated by a worked example.

2. INTRODUCTION

The Institute of Oceanographic Sciences and its forerunner. the National Institute of Oceanography. have been measuring waves for both research and practical purposes for nearly thirty years. During that time standard methods for the interpretation of wave recordings have been developed. These methods have been widely used not only by IOS but by other organisations and individuals working in many areas of marine science and engineering both in the United Kingdom and overseas.

Their success has been due largely to their simplicity of application and to the fact that well defined statistical parameters of general applicability result. Both of these advantages were made possible by incorporating the major contributions to the statistical theory of sea waves which were made at NIO in the 1950's.

This theory is available in works by Cartwright (1958) and Longuet-Higgins (1952) and the application of the method in practical situations has been described in papers by Tucker (1961) and Draper (1963). However it has been apparent for some time that there existed a lack of appreciation of how the practical and theoretical aspects of wave study are combined at IOS to give a consistent interpretation method.

The present report aims to answer this need; it takes the form of a critical review and can thus provide the basis for future refinements and improvements of the methods it describes.

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3. THE DEFINITION OF SOME WAVE PARAMETERS

3.1 A wave may be defined by considering every maximum on a record as a separate wave, its amplitude ξ being measured from the 'mean line' (i.e. the line that would be produced on the record if the sea were perfectly calm). Note that negative values of ξ are possible. It is this definition which is most accessible to a theoretical approach to sea waves.

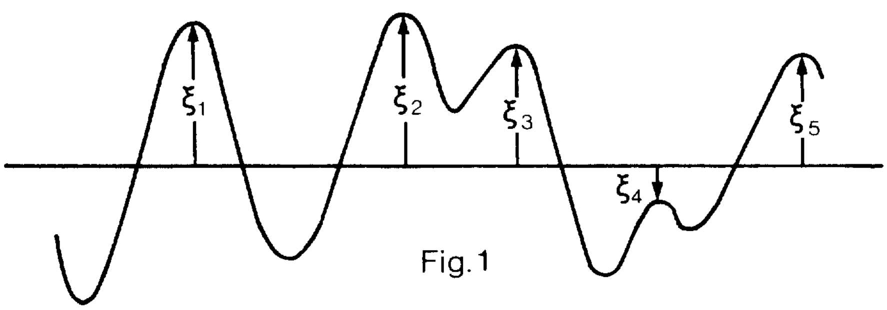


Fig. 1

3.2 The 'crest to trough' definition probably agrees best with one's intuitive idea of a wave. Each crest is a wave whose height H is the vertical distance between the crest and the preceding trough.

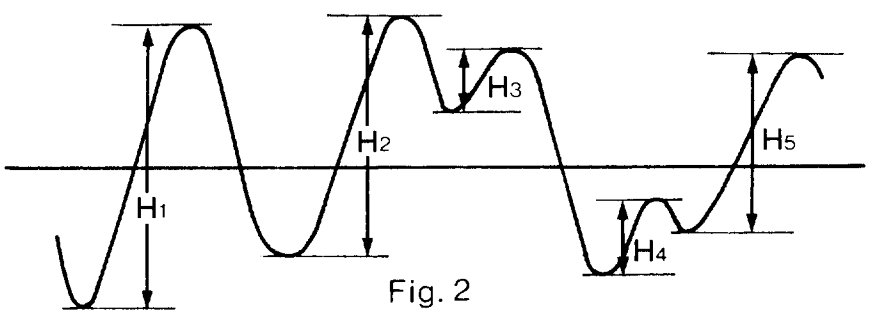


Fig. 2

3.3 In practice a useful definition is the 'zero up-cross' wave. A zero up-crossing is considered to occur when the surface passes through the mean line in an upward direction. A zero up-cross wave is the portion of the record between adjacent zero up-crossings. Its height H is equal to the vertical distance between the highest and lowest points of the wave.

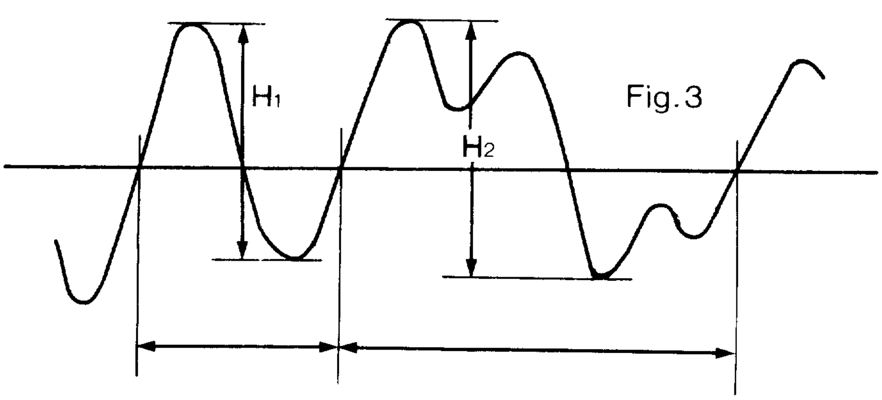


Fig. 3

3.4 The root mean square surface elevation, $E^{\frac{1}{2}}$, (sometimes referred to as the r. m. s. value of the ordinate) is defined by

$$E = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t)^2 dt$$

where f(t) is the vertical distance from the mean line to the surface at time t.

3.5 For **n** given zero up-cross waves H_1, H_2 ..... H_n, the root mean-square zero up-cross wave height, H_{rms}, is given by

$$H^2_{rms} = \frac{1}{n}\left(H_1^2 + H_2^2 + \ldots + H_n^2 \right)$$

3.6 A much-used indicator of the sea-state is the 'significant wave height', H_S, defined as the mean height of the highest third of the waves. Given 3N zero up-cross waves the significant height is

$$H_S = \frac{1}{N}\left(H_{2N+1} + \ldots + H_{3N} \right)$$

where the heights H_1 , ... H_{3N} are arranged in increasing order. Note that earlier workers sometimes used crest-to-trough heights in this definition.

3.7 The period of a zero up-cross wave is defined as the time interval between the two zero up-crossings which bound it. Given a record of duration t minutes, the mean zero up-crossing period is then

$$T_Z = \frac{t \times 60}{\text{no. of zero up-cross waves on the record}} \text{ sec.}$$

3.8 Similarly we may define the mean crest period

$$T_c = \frac{t \times 60}{\text{no. of crest-to-trough waves on the record}} \text{ sec.}$$

4. INTERMITTENT RECORDING

Wave data at IOS has almost invariably been collected in the convenient chart-roll form. It is a reliable method and has the advantage that one can see at a glance whether one has recorded realistic wave profiles. To leave the recorder running continuously would result in unmanageably large volumes of paper. A practical alternative is to record for short periods of time at regular intervals and assume 'short-term stationarity', i. e. that the record is typical of the sea-state for periods either side when the recorder was inactive. The record must, however, be sufficiently long that it is an adequate statistical sample of the wave pattern. IOS analyses a twelve-minute length of record every three hours, and in practice this gives a reasonable balance between sampling errors and changes in the underlying wave characteristics. Both errors are random and they tend to average out in the long run.

The purpose of the IOS analysis is to enable well-defined, statistically meaningful data to be abstracted from the analogue records as simply as possible. To this end estimates of the following three parameters are made from each twelve-minute record:

1. T_z - the mean zero up-crossing period
2. H_s - the significant wave height
3. $H_{max}(3hr)$ - the most probable height of the largest zero up-cross wave in three hours.

A statistical analysis of waves may be considered in two parts: the analysis of short-term statistics (i. e. statistics which characterize an individual record) and that of long-term statistics. The short-term analysis as implemented by IOS computes estimates of H_s, T_z and $H_{max}(3hr)$ for a given three-hour period, while problems concerning the distribution of these parameters over longer periods of time are dealt with in the long-term analysis.

5. THE ANALYSIS OF SHORT-TERM STATISTICS

5.1 Summary of the method of analysis of a twelve-minute record

The zero up-crossing period T_z is found by dividing 12 x 60 sec by the number of zero up-cross waves on the record. Height parameters are calculated from measurements of the two highest crests and two lowest troughs in the record as follows. It is assumed that wave heights defined as maxima of surface elevation relative to mean level have the probability distribution derived by Cartwright and Longuet-Higgins (equation (5.4-2) below). In order to specify this distribution one needs the values of a bandwidth parameter, ε , and the root-mean-square surface elevation, $E^{\frac{1}{2}}$. The expected height of the largest wave in the twelve-minute record (i. e. the mean of its probability distribution) could then be found. Reversing the argument we see that $E^{\frac{1}{2}}$ may be calculated from a knowledge of ε and the expected height of the highest maximum, measured from the mean, in the record. This expected height is approximated as the average size of the lowest trough and the highest crest on the twelve-minute record, and ε may be evaluated as a function of the number of crests and zero-crossings on the record.

Similarly, $E^{\frac{1}{2}}$ is also estimated using the sizes of the second largest trough and crest, and the two values of $E^{\frac{1}{2}}$ are averaged.

H_s is estimated by assuming that the heights of zero up-cross waves follow a Rayleigh probability distribution. No exact theory is available for the finite bandwidth case, but assuming that the relationships are the same as for the narrow bandwidth case, then $H_s = 4E^{\frac{1}{2}}$.

Finally the probability distribution function, F(x) say, of the largest of the $(3 \times 60 \times 60)/T_z$ zero up-cross waves in the three-hour period centred on the twelve -minute record is derived from the Rayleigh distribution of individual wave heights and $H_{max}(3hr)$, defined as the mode of this distribution, is evaluated as the solution of

$$F''(x) = 0$$

The remainder of this section restates the short-term analysis in greater detail.

5.2 The measurements made from each record.

Each twelve-minute record is manually analysed as follows:

The mean line is drawn in by eye (see below) and the values A, B, C D, N_c, N_z are noted, where

A = height of largest crest, measured from the mean line

B = height of second largest crest

C = depth of largest trough) By convention, these are taken as positive values

D = depth of second largest trough)

N_c = number of crests (i.e. maxima)

N_z = number of zero up-crossings

5.3 Estimating T_z from the measurements

The mean zero up-crossing period for the twelve-minute record is immediately obtained as

$$\frac{12 \times 60}{N_z} \quad \text{sec.}$$

On the basis of the stationarity assumption this value is taken as T_z, the mean zero up-crossing period for the complete three-hour interval.

5.4 Estimating E from the Measurements

The theory of Cartwright and Longuet-Higgins (1956) and Cartwright (1958) is used to estimate E.

The wave trace is considered as part of a random function, f, given by the sum of an infinite number of sinusoids.

$$f(t) = \sum c_n \cos(\omega_n t + \delta_n) \tag{5.4-1}$$

where the frequencies ω_n are densely distributed in the interval $(0, \infty)$ and the phases δ_n are random and uniformly distributed in $[0, 2\pi)$, and the amplitudes c_n are such that in any small interval of frequency $d\omega$

$$\sum_{\omega_n=\omega}^{\omega+d\omega} \frac{1}{2} c_n^2 = S(\omega)d\omega$$

where $S(\omega)$ is a continuous function known as the energy spectrum of f(t).

The nth moment of the energy spectrum is defined by

$$m_n = \int_0^{\infty} \omega^n S(\omega) d\omega$$

and the mean square surface elevation is

$$E = m_o$$

The functions f(t). f' (t). f"(t) may be considered as random variables. Each consists of an infinite sum of components with zero mean and random phase so by the central limit thereom they have a joint normal distribution which is determined by their matrix of correlations.

At a maximum of f. f has some value ξ , f' is zero and f" is negative. We may integrate the joint probability distribution over regions of negative f" and zero f' to get the probability function of ξ .

Writing η for $\xi / E^{\frac{1}{2}}$ we get the probability density function

$$p(\eta) = \frac{1}{\sqrt{2\pi}} \left[\varepsilon e^{-\frac{1}{2}\left(\eta/\varepsilon\right)^2} + \sqrt{1-\varepsilon^2}\, \eta e^{-\eta^2/2} \int_{-\infty}^{\frac{\eta}{\varepsilon}\sqrt{1-\varepsilon^2}} e^{-\frac{1}{2}x^2} dx \right] \qquad (5.4\text{-}2)$$

where ε (known as the bandwidth parameter) is a measure of the range of frequencies present. given by

$$\varepsilon = \frac{m_0 m_4 - m_2^2}{m_0 m_4}$$

Graphs of this density function for different values of ε are shown in Fig. 4. Note that equation (5. 4-2) reduces to the Rayleigh distribution as ε tends to zero.

The probability that a given maximum shall exceed $xE^{\frac{1}{2}}$ is then

$$q(x) = \int_x^{\infty} p(\xi) d\xi$$

The largest of N_c crests is less than x if and only if each of the N_c crests is less than x.

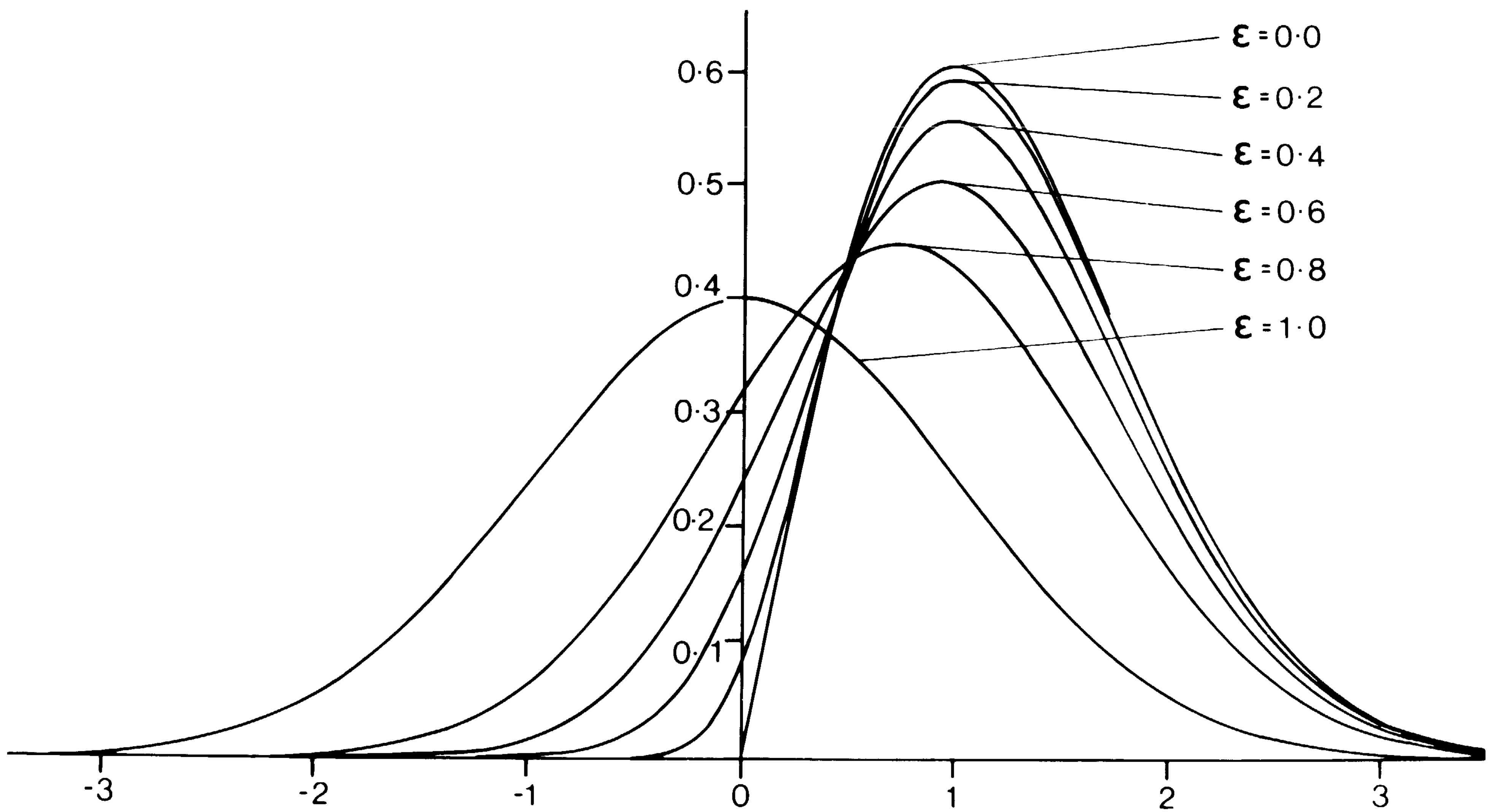


Fig. 4

The distribution of maxima of surface elevation for various values of the band width parameter

Therefore, provided the crests may be considered independent, the cumulative distribution function of the height, ξ_1 , of the largest of N_c crests is

$$p_{N_c}(x) = \left(1 - q(x)\right)^{N_c}$$

Cartwright (1958) found that the errors incurred in assuming the waves to be independent were small.

The mean of the distribution of ξ_1 may be shown to be

$$\bar{\xi}_1 = E^{\frac{1}{2}}\sqrt{2\theta}\left(1 + \tfrac{1}{2}A_1\theta^{-1} - \tfrac{1}{8}A_2\theta^{-2} + \tfrac{1}{16}A_3\theta^{-3} - \dots\right) \qquad (5.4\text{-}3)$$

where

$$\begin{aligned} \theta &= \ln\left(N_c\sqrt{1-\varepsilon^2}\right) \\ A_1 &= 0{\cdot}5772 \\ A_2 &= 1{\cdot}9781 \\ A_3 &= 5{\cdot}4449 \end{aligned} \qquad (5.4\text{-}4)$$

Similarly, if ξ_2 is the height of the second highest of N_c crests, we may derive its distribution function and obtain its mean value as

$$\bar{\xi}_2 = E^{\frac{1}{2}}\sqrt{2\theta}\left(1 - \tfrac{1}{2}(1-A_1)\theta^{-1} + \tfrac{1}{8}(2A_1 - A_2)\theta^{-2} - \tfrac{1}{16}(3A_2 - A_3)\theta^{-3} + \dots\right) \qquad (5.4\text{-}5)$$

Equations (5.4-3) and (5.4-5) give two methods of estimating $E^{\frac{1}{2}}$, provided ε, $\bar{\xi}_1$ and $\bar{\xi}_2$ are known.

To find $\bar{\xi}_1$ and $\bar{\xi}_2$ we first note that since the trace is assumed to be statistically symmetrical about its mean line, we may consider the upper and lower halves as two realisations of the same stochastic process. Therefore A and C are both sample values of the random variable ξ_1 , and so the sample mean $\frac{1}{2}(A + C)$ is the best estimator of $\bar{\xi}_1$. Similarly $\frac{1}{2}(B + D)$ is the best estimator of $\bar{\xi}_2$. Note that these sums are not dependent on the position of the zero-line drawn in by the operator, so long as it is parallel to the true mean line.

To find ε we turn again to the theory of Cartwright and Longuet-Higgins (1956). Here it is shown that

$$\varepsilon^2 = 1 - \left(1 - 2r\right)^2$$

where r is the proportion of negative maxima in the record. Any difference between

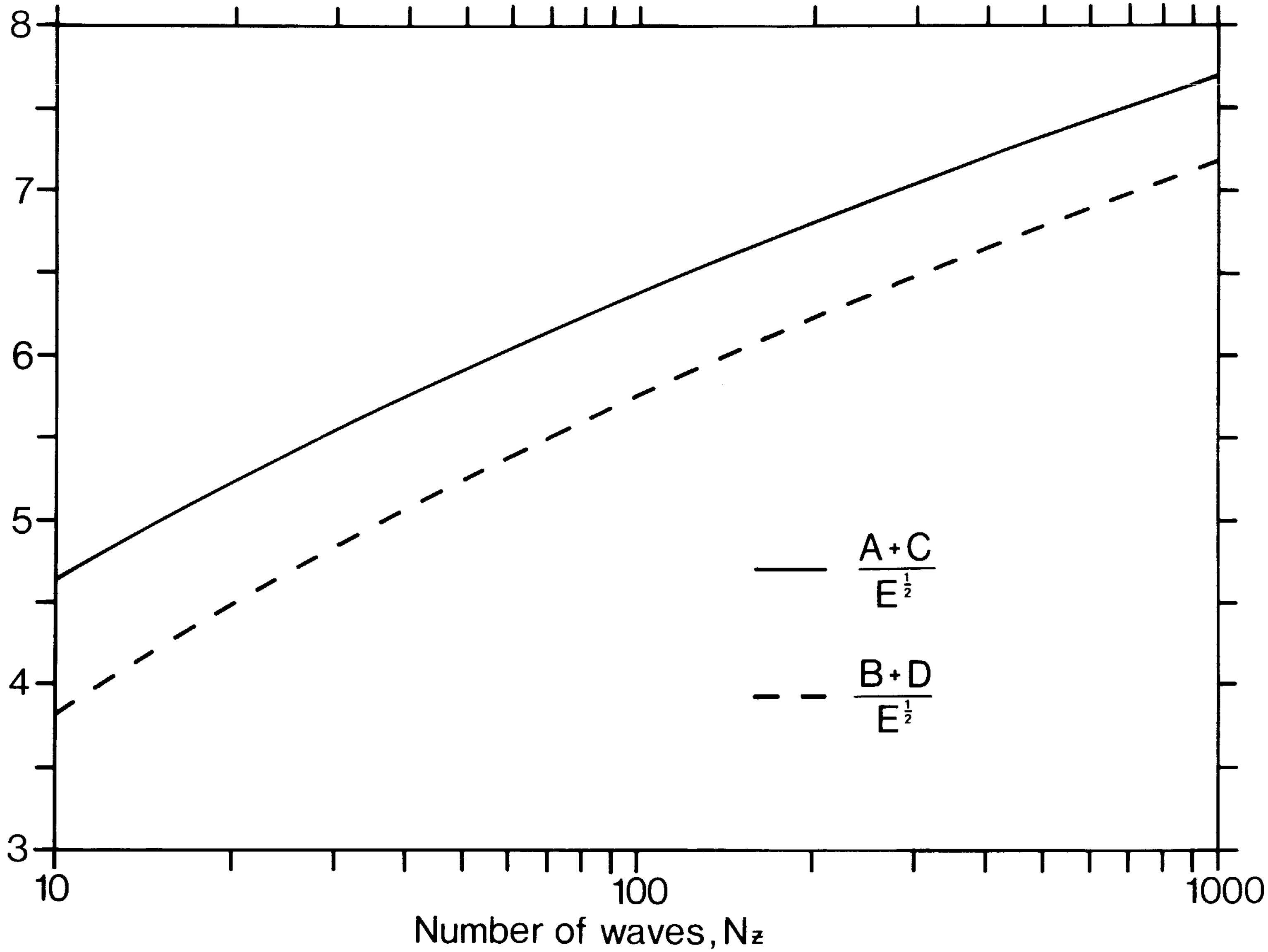


Fig.5

Graphs for the estimation of root-mean-square surface elevation.

N_c and N_z must be due to either positive maxima or negative minima. so

$$N_c - N_z = \text{no. of positive minima} + \text{no. of negative maxima}$$
$$= 2 \text{ (no. of negative maxima)}$$

since the surface is assumed statistically symmetrical.

$$\therefore \text{ proportion of negative maxima} = \tfrac{1}{2}\left(1 - \frac{N_z}{N_c}\right)$$
$$\therefore \varepsilon^2 = 1 - \left(\frac{N_z}{N_c}\right)^2$$

and from the equation (5. 4-4)

$$\theta = \ln N_z$$

Hence, truncating the series in equations (5. 4-3) and (5. 4-5) we get two values of E. namely

$$E_1^{\frac{1}{2}} = \frac{A+C}{2\sqrt{2\theta}}\left(1 + \tfrac{1}{2}A_1\theta^{-1} - \tfrac{1}{8}A_2\theta^{-2}\right)^{-1} \qquad (5.\ 4\text{-}6)$$

and

$$E_2^{\frac{1}{2}} = \frac{B+D}{2\sqrt{2\theta}}\left(1 - \tfrac{1}{2}(1-A_1)\theta^{-1} + \tfrac{1}{8}(2A_1 - A_2)\theta^{-2}\right)^{-1} \qquad (5.\ 4\text{-}7)$$

Typical values of the first neglected term in the series are . 003 and . 0003 respectively.

The equations may conveniently be used in graphical form (see Fig. 5).

Our final estimate of $E^{\frac{1}{2}}$ is taken as the average of $E_1^{\frac{1}{2}}$ and $E_2^{\frac{1}{2}}$.

The above process is the heart of the analysis method proposed by Tucker (1961).

5. 5 Estimating H_s from E

Zero up-cross waves are assumed to follow a Rayleigh distribution (Fig. 6). There is good empirical evidence in support of this assumption (see, for example, Goda 1970), though theorists have failed to explain the goodness of fit.

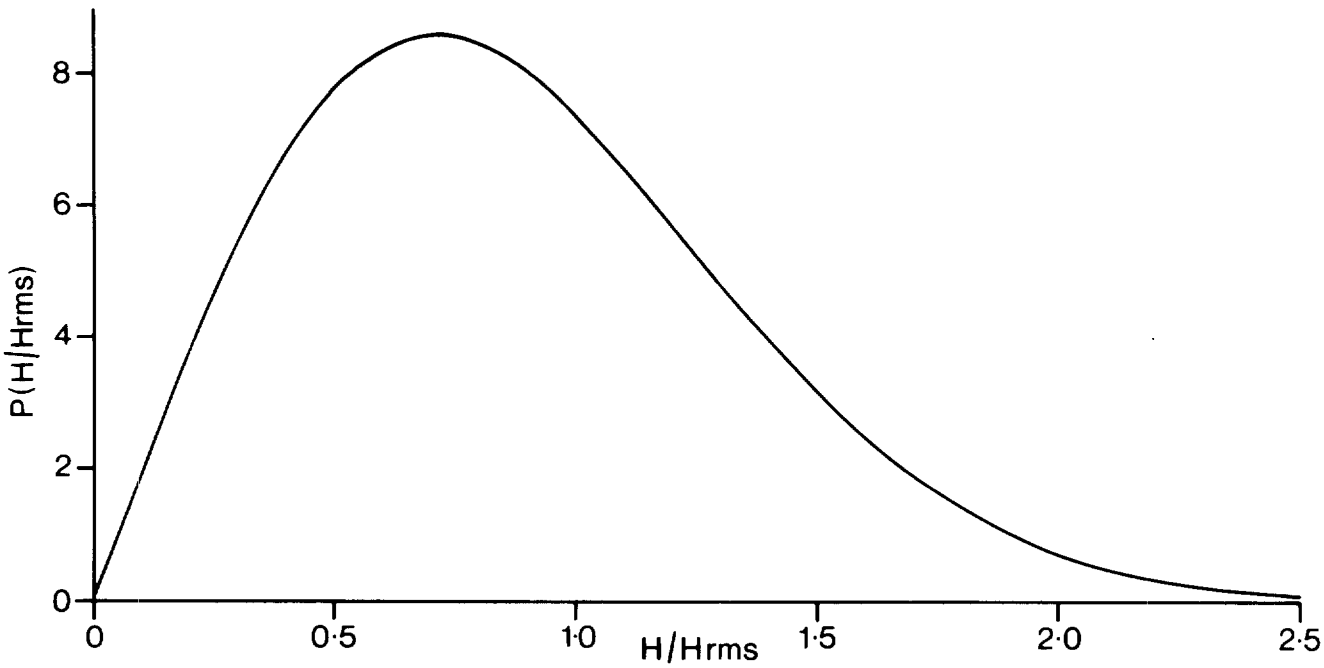


Fig. 6

The Rayleigh distribution for zero up-cross waves.

The probability that the height, H, of a given zero up-cross wave is less than x is thus

$$Q(x) = 1 - \exp\left(-\left(\frac{x}{Hrms}\right)^2\right) \qquad (5.5\text{-}1)$$

For large numbers of waves, the highest one-third will be those of height greater than h, where

$$Q(h) = \frac{2}{3}$$

The mean height of these will be

$$H_s = \frac{1}{1-Q(h)} \int_h^{\infty} tQ(t)\,dt$$

$$= 1{\cdot}416\ Hrms \qquad (5.5\text{-}2)$$

(see Longuet-Higgins, 1952) $\simeq \sqrt{2}\ Hrms$

It is now necessary to relate H_{rms} to E. Here we need to assume a narrow bandwidth.

Letting $\varepsilon \to 0$ in (5.4-2), we have that the probability density function of $\frac{\xi}{E^{\frac{1}{2}}}$, the normalised height of maxima of surface elevation, is

$$xe^{-\frac{1}{2}x^2}$$

Therefore $$\xi^2_{rms} = E\int_0^\infty x^2 xe^{-\frac{1}{2}x^2}dx$$
$$= 2E$$

When the frequency spectrum is narrow the trace appears as a carrier wave whose amplitude is controlled by a slowly changing envelope. So the height H of a zero up-cross wave is approximately twice the height ξ of the crest measured from the mean line.

$$H = 2\xi$$

Therefore $$H_{rms} = 2\xi_{rms}$$

so $$H_{rms} = 2\sqrt{2E} \qquad (5.5\text{-}3)$$

Finally, from (5.5-2)

$$H_S \simeq 4E^{\frac{1}{2}} \qquad (5.5\text{-}4)$$

6. THE ESTIMATION OF DESIGN HEIGHTS

This process may be considered in two parts. Firstly, for each twelve-minute record the short-term analysis is extended to estimate $H_{max}(3hr)$, the most probable height of the highest wave in the three-hour recording interval. Secondly, the statistics of a long series of values of $H_{max}(3hr)$ are examined and extrapolated to estimate design heights.

6.1 Estimating $H_{max}(3hr)$ from E and T_z

We consider a three hour period centred on the twelve-minute record in question, and examine the probability density function of the highest wave to occur in three hours. $H_{max}(3hr)$ is the mode (peak) of this distribution. Typically the mean of the distribution is 3% higher, and the probability of $H_{max}(3hr)$ being exceeded is about 0.6.

The number of zero up-cross waves in the three hour recording interval is

$$N = \frac{3 \times 60 \times 60}{T_z}$$

The largest wave to occur in three hours, H', is less than x if and only if each of the N waves is less than x. Assuming the heights of successive waves to be independent random variables distributed according to the Rayleigh distribution with variance H^2_{rms} gives

$$\text{Prob}\,(H' \leqslant x) = \left(1 - e^{-\left(\frac{x}{H_{rms}}\right)^2}\right)^N$$

The probability density function is the derivative

$$\frac{2N}{H^2_{rms}}\, x e^{-\left(\frac{x}{H_{rms}}\right)^2} \left(1 - e^{-\left(\frac{x}{H_{rms}}\right)^2}\right)^{N-1}$$

$H_{max}(3hr)$ corresponds to the turning point of this function, which occurs when

$$\Psi = \ln N - \ln\left(1 - \frac{1}{2\Psi}\left(1 - e^{-\Psi}\right)\right) \qquad (6.1\text{-}1)$$

where $$\Psi = \frac{x^2}{H^2_{rms}}$$

(see Longuet-Higgins 1952)

Equation (6.1-1) may be solved using Newton's method of successive approximations, though a good polynomial approximation to the root is

$$\Psi = \left(0{\cdot}566405 + 0{\cdot}316548Y + 0{\cdot}330573Y^2 - 0{\cdot}073968Y^3 + 0{\cdot}006361Y^4\right)^2 \qquad (6.1\text{-}2)$$

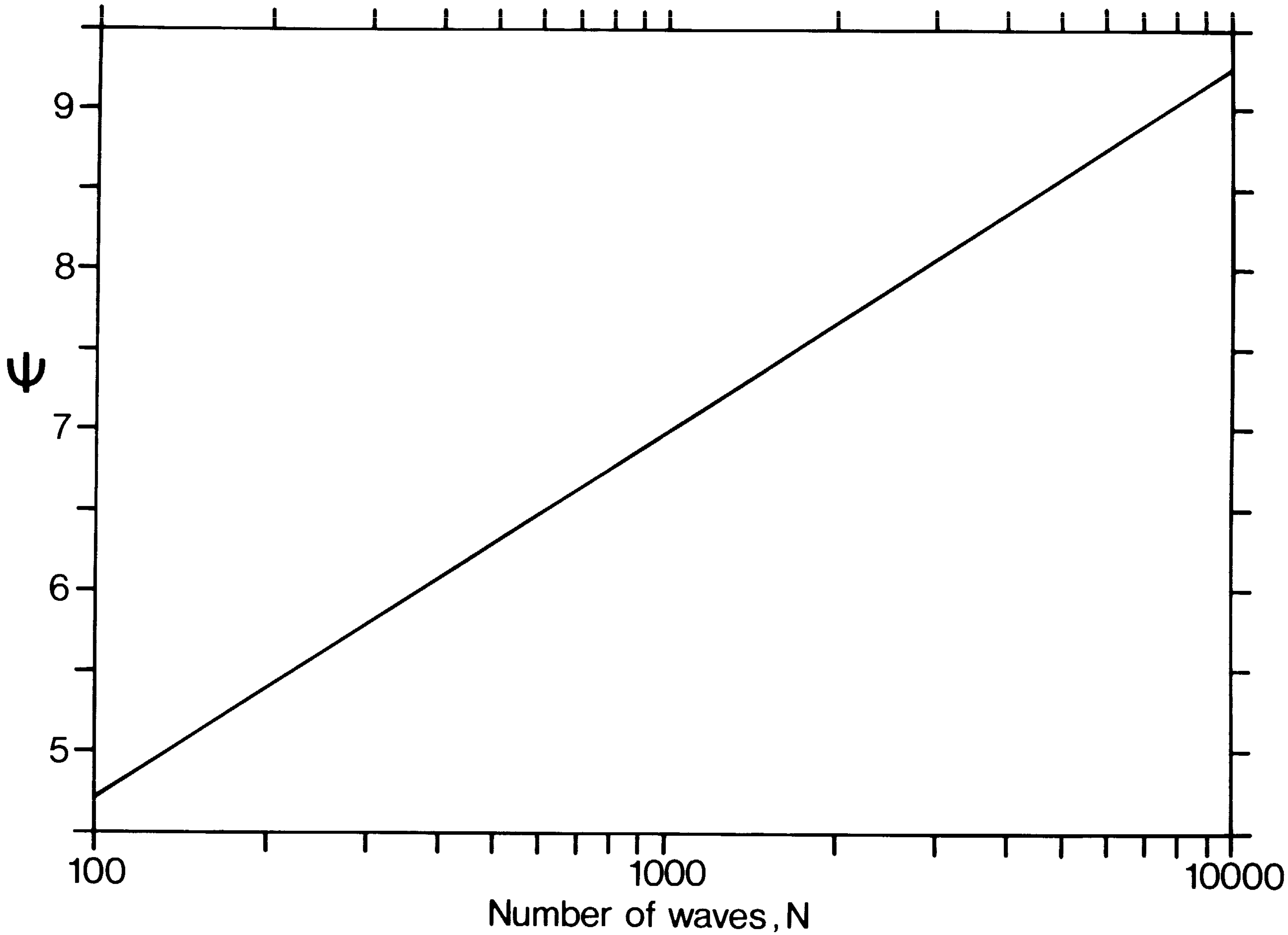


Fig.7

Graph of Ψ for estimation of $H_{max}(3hr)$.

where $Y^2 = \ln N$

Alternatively, Ψ may be obtained from the graph shown in Fig. 7.

Finally, equation (5. 5-3) gives

$$H_{max}\,(3hr) = 2\sqrt{2E\Psi} \qquad (6.\,1\text{-}3)$$

6. 2 Extrapolation from a long series of values of $H_{max}(3hr)$

The derived parameters H_s, T_z, and $H_{max}(3hr)$ may be used to determine the long-term distribution of wave heights. A designer of an offshore structure needs to know the height of the highest wave that the structure will encounter during its lifetime. We estimate $H_{max}(T\ yr)$, i. e. the most probable height of the highest wave in T years. Usually values of $H_{max}(T\ yr)$ with T = 10, 50, 100 are quoted.

Of course $H_{max}(T\ yr)$ is not an upper bound on the height of the highest wave in T years, though it does represent a parameter of its probability distribution on which risk estimates may be based. For example, one can be 90% sure that the largest wave in 50 years will not exceed $H_{max}(50yr)$ by more than 15%.

6. 2. 1 The data required

The estimate is based on values of $H_{max}(3hr)$ collected over a period of time. The data need not be continuous in time, though it is important that it gives an accurate picture of the distribution of stormy and calm months, since it is assumed that the data used is representative of the following T years. Often it is necessary to estimate $H_{max}(50yr)$ from only one year's data, when even ten years' data falls short of the ideal. One may check that the data was not collected during an unusually stormy or calm period by referring to the meteorological reports for the time, though at present it is not possible to make accurate allowance for such conditions.

6.2.2 Plotting the data

The height range is divided into a number, n, of small intervals (say 0.0 - 0.2m, 0.2 - 0.4m, ... 24.8 - 25.0m), and the number of occurrences of $H_{max}(3hr)$ in each interval counted. The next step is to plot these frequencies on graph paper and then fit an extreme value distribution. Following Gumbel (1958), the mid-point of the mth interval is plotted against the probability $\frac{S_m}{N+1}$ where N is the total number of observations in the time period under consideration, and S_m is the number of occurrences of $H_{max}(3hr)$ in the first m intervals (so $N = S_n$). Jasper (1956) found that certain wave parameters appear to follow a log-normal distribution and this is the distribution that was fitted to data by IOS up to 1970.

Fitting is facilitated by the use of graph paper whose scales are such that a log-normal distribution function appears as a straight line. The line is extrapolated to find heights which have the required low probability of being exceeded. Confidence in the extrapolation is based upon how well the straight line fits the data.

It has been found (Battjes, 1970) that a Weibull distribution generally gives a better fit than the log-normal. Since 1970 IOS have used the two methods side-by-side, comparing the goodness-of-fit to decide upon the distribution to be taken.

The Weibull distribution is a three-parameter distribution given by

$$\text{Prob}\,(X \leqslant x) = \begin{cases} 1 - \exp\left(-\left(\frac{x-A}{B}\right)^C\right) & \text{for } x \geqslant A \\ 0\ldots\ldots\ldots\ldots\ldots & \text{for } x < A \end{cases}$$

where B and C are positive (see Weibull, 1951).

We have thus estimated the distribution function, P(x), of $H_{max}(3hr)$ for very low probabilities of exceedance.

6.2.3 Derivation of a time scale for the probability axis

The distribution function P(x), gives the probability that $H_{max}(3hr)$ is less than x metres for a randomly chosen three hour period. In order to deduce return periods for given heights we need to know the extent of correlation between successive values of $H_{max}(3hr)$. However, it appears that very little error is incurred if we assume successive values of

$H_{max}(3hr)$ to be independent (see 7. 8). Then a height H which $H_{max}(3hr)$ has probability q of exceeding will be exceeded on average once every $\frac{1}{q}$ trials. The return period of H is therefore

$$T = \frac{1}{q} \times 3hr$$
$$= \frac{1}{1-P(H)} \times \frac{3}{365 \times 24} \quad yr$$

So the height, H_T, which $H_{max}(3hr)$ exceeds on average once every T years is given by

$$P(H_T) = 1 - \frac{3}{365 \times 24} T^{-1}$$
$$= 1 - 0{\cdot}00034 \; T^{-1} \qquad (6.\ 2.\ 3\text{-}1)$$

Thus H_T may be read off the extrapolated graph of P(x) as the height corresponding to probability $P(H_T)$. It is this value which is taken as our estimate of $H_{max}(T\ yr)$.

7. A SUMMARY OF ASSUMPTIONS

In this section the major assumptions are summarized and sources of random error indicated. An estimate of the resulting error is made where possible.

7.1 The method is based on a linear model. The sea surface is considered as the resultant of a large number of summable sinusoidal components of random phase (equation (5.4-1)) leading to a Gaussian distribution of surface elevation and statistical symmetry about the mean line. Yet these are features only approximately exhibited by actual waves. Moreover, it seems likely that non-linear phenomena, such as wave-breaking will occur most frequently in the very high waves in which we are particularly interested.

7.2 When E is calculated sampling errors occur in the estimation of $\bar{\xi}_1$ and $\bar{\xi}_2$ (see Section 5.4). Using the formulae for the moments of the distribution of the highest wave in a record derived by Cartwright (1958), the estimate of $E^{\frac{1}{2}}$ turns out to have a standard error of about 6%. Correlation between crests in the twelve-minute record is ignored, but as stated in 5.4, this should not significantly reduce the accuracy of the method.

7.3 One of the more serious assumptions is made when we require a narrow spectrum to derive equation (5.5.-3) which gives the link between the distribution of zero up-cross waves and the mean square surface elevation, E. When using data recorded by instruments which exhibit the 'hydrodynamic filter' effect (e.g. the shipborne wave recorder) this assumption is probably more valid than with surface measuring instruments (such as the Waverider buoy) which have a more extended high frequency response. It is worth noting that the root-mean-square surface elevation, $E^{\frac{1}{2}}$, has been obtained without making any bandwidth assumptions.

7.4 Having obtained H_s and T_z for a twelve-minute record we then make the fundamental assumption that these values persist for the rest of the three-hour interval. This is the stationarity assumption.

7.5 Any error in the estimation of H_s and T_z will be passed on to H_{max}(3hr). In addition the theoretical distribution of the highest wave in the three hour interval is based on the assumption that heights of adjacent zero up-cross waves are not correlated.

7.6 When extrapolating to find the design wave (see Section 6.2.2) it is not clear which extreme value distribution should be used, for it is possible for two distribution functions to be almost identical in the range for which data is available, yet differ by as much as 15% at the probability corresponding to a 50-year return period.

Furthermore, there is no complete guarantee that the wave population will continue to follow its previous probability curve right into the tail of the distribution.

The extrapolation itself is necessarily subjective to a considerable degree, since it is a matter of judgement as to how much weight should be given to the higher observed points.

7.7 It will be noticed (see Section 6.2.3) that H_T is the most probable height of the highest wave in the most stormy three-hour period in T-years, whereas we are in fact seeking H_{max}(T yr), the most probable height of the highest wave in the complete T years. H_T and H_{max}(T yr) will differ due to probability contributed to H_{max}(T yr) by the probability distribution of the highest wave in the 2nd, 3rd, most stormy three-hour periods. However, these probability contributions should be small, since the probability density function of the highest wave in a three-hour period tails off rapidly after reaching a peak. A rough calculation reveals that an estimate of H_{max}(50yr) which takes account of these contributions is typically 6% higher.

7.8 In the conversion of probabilities to return periods (see Section 6.2.3) successive values of H_{max}(3hr) are assumed uncorrelated. However, in practice storms may extend over several three-hour periods, producing a sequence of high values of H_{max}(3hr) displaying a high degree of dependence. It is possible to make a rough estimate of the errors involved as follows.

We suppose that a storm with a given H_{rms} will have a duration D hours, and that this H_{rms} is constant for the duration and zero outside it. Let P(H) be the long-term probability distribution function of H_{rms}, found by plotting three-hourly values of H_{rms} throughout a few years.

We attempt to estimate the most probable height of the highest wave to occur in storm S, where S is the storm with a 50-year return period, given that the duration of S is D_{50} hours.

If we pick a 3-hour interval at random from the 50 years, the probability of finding the root-mean-square wave height less than H_{50}, the value during the 50 year storm is

$$\frac{50yr - D_{50}hr}{50yr}$$

i. e. $$P(H_{50}) = 1 - \frac{D_{50}}{50 \times 365 \times 24} \qquad (7.8\text{-}1)$$

We can thus find H_{50} from the extrapolated graph of P(H), the distribution function of H_{rms}.

Examination of actual wave data reveals that at a given location the mean zero up-crossing period, T_z, is roughly the same whenever the sea is very high. This value may therefore be obtained from the available data, and we get the number of waves during the 50-year storm as

$$N = \frac{D \times 60 \times 60}{T_z} \qquad (7.8\text{-}2)$$

These waves have root-mean-square wave height H_{50}, so using (6.1-3) and (5.5-3) we get the most probable height of the highest wave in the 50-year storm as

$$H' = H_{50}\Psi^{\frac{1}{2}} \qquad (7.8\text{-}3)$$

where Ψ is given by (6.1-2).

Let us examine how H' varies with D_{50}.

Suppose D_{50} is large. From (7.8-1) this leads to a small value of H_{50}, since P(H) is an increasing function. However, from (7.8-2), N will be larger, since T_z remains approximately constant, and this leads to a larger value of Ψ. Looking at (7.8-3) we see that the two effects will tend to cancel each other out.

A rough calculation confirms this expectation. Using data from the Northern North Sea we find that T_z is about 16 sec. during storms, and H_{rms} has a Weibull distribution given by

$$P(x) = \begin{cases} 1 - \exp\left(-\left(\dfrac{x - 0{\cdot}46}{1{\cdot}22}\right)^{1{\cdot}02}\right) & \text{for } x \geqslant 0{\cdot}46\,\text{m} \\ 0\ldots\ldots\ldots\ldots\ldots\ldots & \text{for } x < 0{\cdot}46\,\text{m} \end{cases}$$

Therefore

$$H_{50} = 0{\cdot}46 + 1{\cdot}22 \exp\left(\frac{\ln\ln(1 - P)^{-1}}{1{\cdot}02}\right)$$

where, from (7. 8-1),

$$P = 1 - \frac{D\,\text{hr}}{50\,\text{yr}}$$

The results for various values of D are tabulated below

D hr	N	Ψ	H_{50}	H'	% Error*
3	675	6. 59	14. 3	36. 7	0. 0
6	1350	7. 28	13. 5	36. 4	0. 8
9	2025	7. 68	13. 0	36. 1	1. 6
12	2700	7. 97	12. 7	35. 8	2. 3
15	3375	8. 19	12. 4	35. 6	3. 0
18	4050	3. 37	12. 2	35. 4	3. 6

*Error incurred by assuming a 3-hour design storm.

A realistic value for the duration of the 50-year storm is 6-12 hours, so we conclude that ignoring correlation between successive values of H_{max}(3hr) produces errors which, while not negligible, are comparatively small.

8. A WORKED EXAMPLE

8.1 The Short-term Analysis

The method of obtaining H_s, T_z and H_{max}(3hr) described in Sections 5 and 6.1 is illustrated by performing the analysis on the twelve-minute trace shown in Fig. 8. The record was taken using a Datawell 'Waverider' buoy in the Southern North Sea (53°N, 3°E).

First the position of the mean line is estimated and drawn in. Then we have

$A = 2.57$m

$B = 2.49$m

$C = 2.71$m

$D = 2.45$m

$N_C = 157$

$N_Z = 108$

The mean zero up-crossing period $T_z = \frac{12 \times 60}{N_z} = 6.67$ sec.

Using (5.4-6) and (5.4-7) we get

$$E_1^{\frac{1}{2}} = \frac{A+C}{2\sqrt{2\theta}}\left(1+0{\cdot}289\,\theta^{-1}-0{\cdot}247\,\theta^{-2}\right)^{-1} \quad \text{where} \quad \theta = \ln 108$$

$$= 0{\cdot}821\text{ m.}$$

and

$$E_2^{\frac{1}{2}} = \frac{B+D}{2\sqrt{2\theta}}\left(1-0{\cdot}211\,\theta^{-1}-0{\cdot}103\,\theta^{-2}\right)^{-1}$$

$$= 0{\cdot}849\text{ m.}$$

Averaging these estimates gives

$$E^{\frac{1}{2}} = 0.835\text{m}$$

By (5.5-4) the significant wave height is

$$H_s = 3.34\text{m}$$

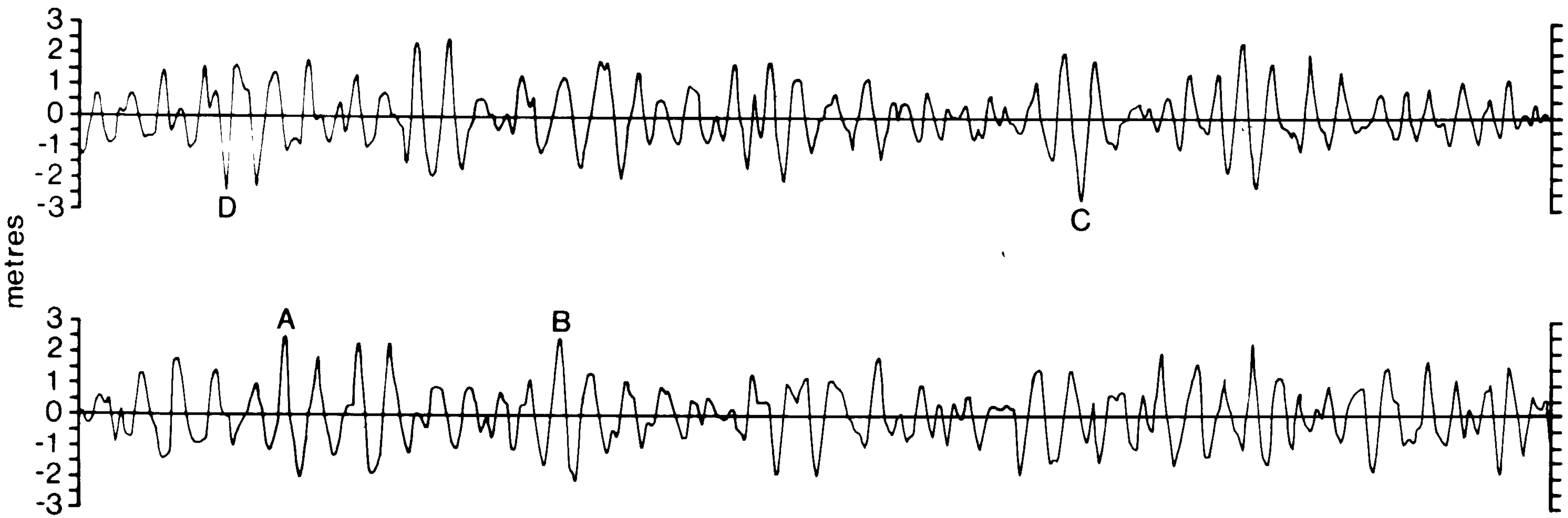


Fig. 8

A twelve minute wave record.

HT RANGE (FT)	MID-PT X (FT)	Number of occurrences	Cumulative totals	P
0 - 1	0.5	1	1	.0003
1 - 2	1.5	2	3	.0010
2 - 3	2.5	7	10	.0034
3 - 4	3.5	60	70	.0238
4 - 5	4.5	98	168	.0573
5 - 6	5.5	139	307	.1048
6 - 7	6.5	136	443	.1512
7 - 8	7.5	153	596	.2034
8 - 9	8.5	156	752	.2567
9 - 10	9.5	176	928	.3168
10 - 11	10.5	174	1102	.3762
11 - 12	11.5	158	1260	.4301
12 - 13	12.5	159	1419	.4844
13 - 14	13.5	174	1593	.5438
14 - 15	14.5	150	1743	.5950
15 - 16	15.5	119	1862	.6357
16 - 17	16.5	128	1990	.6794
17 - 18	17.5	117	2107	.7193
18 - 19	18.5	104	2211	.7548
19 - 20	19.5	111	2322	.7927
20 - 21	20.5	83	2405	.8210
21 - 22	21.5	69	2474	.8446
22 - 23	22.5	71	2545	.8688
23 - 24	23.5	56	2601	.8880
24 - 25	24.5	47	2648	.9040
25 - 26	25.5	31	2679	.9146
26 - 27	26.5	34	2713	.9262
27 - 28	27.5	38	2751	.9392
28 - 29	28.5	26	2777	.9481
29 - 30	29.5	21	2798	.9552
30 - 31	30.5	31	2828	.9655
31 - 32	31.5	17	2845	.9713
32 - 33	32.5	18	2863	.9774
33 - 34	33.5	9	2872	.9805
34 - 35	34.5	9	2881	.9836
35 - 36	35.5	7	2888	.9860
36 - 37	36.5	7	2895	.9883
37 - 38	37.5	6	2901	.9904
38 - 39	38.5	1	2902	.9907
39 - 40	39.5	4	2906	.9921
40 - 41	40.5	10	2916	.9955
41 - 42	41.5	3	2919	.9965
42 - 43	42.5	3	2922	.9976
43 - 44	43.5	1	2923	.9979
44 - 45	44.5	1	2924	.9982
49 - 50	49.5	1	2925	.9986
52 - 53	52.5	2	2927	.9993
53 - 54	53.5	1	2928	.9997

Fig. 9

Tabulation for the long-term analysis.

The estimated number of zero up-cross waves in the three-hour period is then

$$N = \frac{3 \times 60 \times 60}{T_z} = 1620.$$

From (6. 1-2) or using Fig 7

$$\Psi = 7.46$$

and, finally, using (6. 1-3)

$$H_{max}(3hr) = 6.45m$$

8. 2 The Long-term Analysis

For this example we use listings of $H_{max}(3hr)$ derived from data recorded by a shipborne Wave Recorder installed on the Sevenstones Light Vessel (50°N, 6°W) during the period Jan. - Dec. 1968. The cumulative totals of occurrences of $H_{max}(3hr)$ in each region 0-1 ft, 1-2 ft, ..., 54-55 ft. are obtained and divided by the total number of obs ervations plus one to give the probability values, p_i, to be plotted against height, x_i. A tabluation of these quantities is shown in Fig. 9.

The points (x_i, p_i) are plotted using log-normal scales in Fig. 10.

As one would expect, there is greatest scatter at the top end of the graph, where there are few samples. A straight line is fitted to follow the curve of points at the end corresponding to the rougher seas in which we are interested.

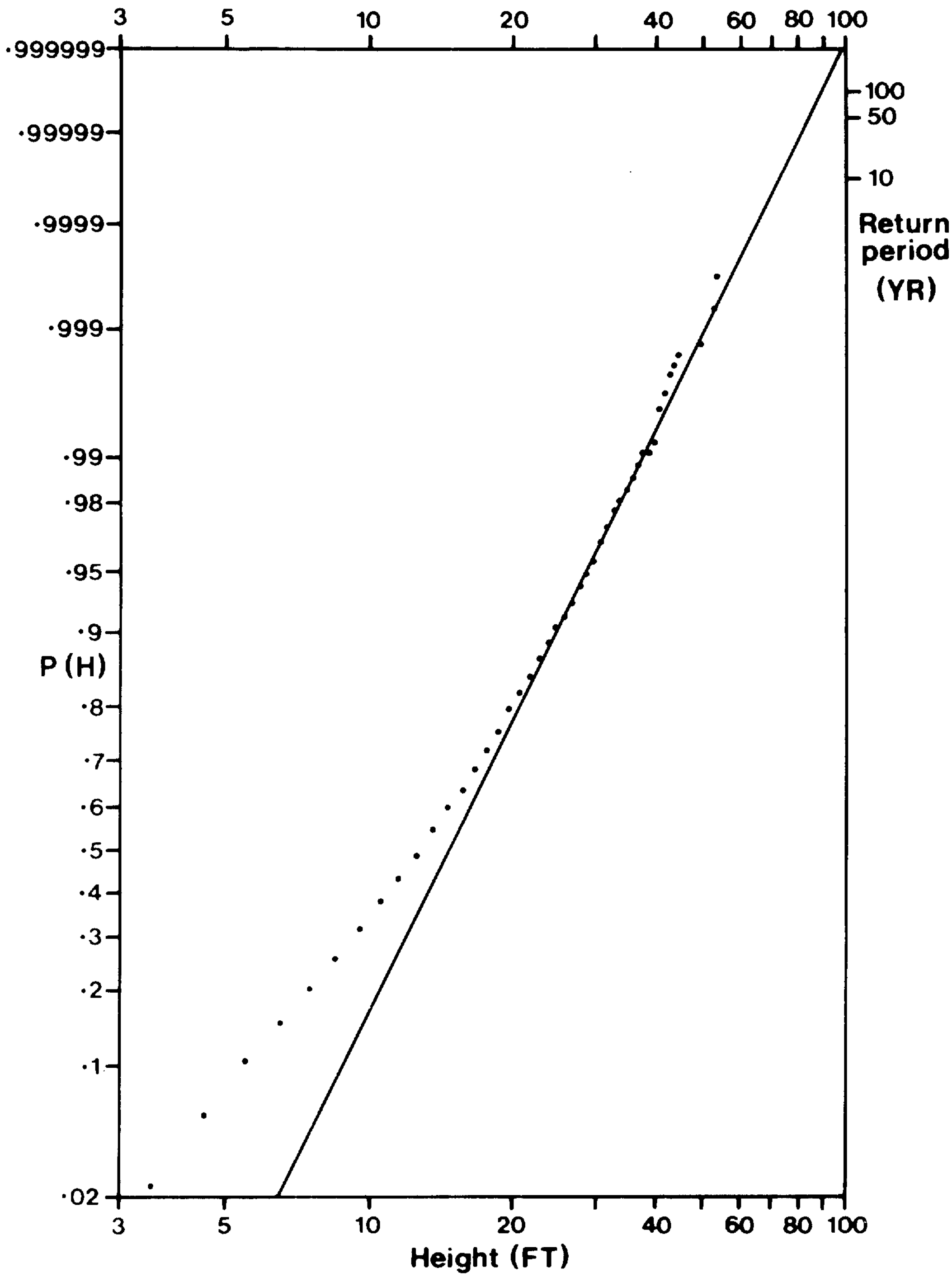
To find $H_{max}(50 yrs)$, the most probable height of the highest wave in fifty years, we need to extrapolate the line to probability

$$1 - \frac{3hr}{50yr} = .9999932 \qquad \text{(from (6. 2. 3 -1))}$$

From the graph, we therefore have

$$H_{max}(50yr) = 80ft.$$

Similarly $$H_{max}(100yr) = 83ft.$$



THE LONG TERM DISTRIBUTION OF H_{max} (3HR)

Fig.10

ACKNOWLEDGEMENTS

The author wishes to thank Dr. D. E. Cartwright, Mr. M. J. Tucker, and Mr. E. G. Pitt of the Institute of Oceanographic Sciences for their valuable suggestions and comments.

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