I.O.S.

MOORINGS : A REVIEW OF THE PROBLEMS AND SOME SOLUTIONS

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ABSTRACT

A large increase in the number and types of mooring deployments during the past ten years has led to an increased awareness of the inherent problems associated with gathering data from the sea using this technique. The principal parameters, some solutions and comparisons with field results are described. Comparisons of observed subsurface buoy depressions from I.O.S. - Bidston Continental Shelf rig deployments with theoretical calculations obtained using the modified Bidston Computer Programme, MOOR, yielded exceptionally favourable results. It still appears that a substantial amount of research, both engineering and theoretical, has to be undertaken in order to more accurately describe the force field, motion and configuration of all types of moorings. One of the major difficulties is to resolve the complex motion and time-varying forces into a form from which a three-dimensional analysis can be undertaken.

INTRODUCTION

A number of mathematical formulations and computer programs have developed to define the force field, motion and configuration of moorings. One problem is the lack of precise quantitative and qualitative data on many of the parameters used. In addition, the dynamic behaviour of in-situ motion and resulting interactions are not perfectly understood. In order to cope with the complex environmental situation a two rather than three-dimensional reference system is used and the in-situ dynamic motion is regarded as static or quasi-static.

The aim of this paper is to draw attention to those areas in need of more intensive investigation; describe some of the available mathematical formulae and computer programs; comparisons between theory and in-situ results and lastly, suggestions for future investigations.

THE FIELD OF MOTION

I Currents

One of the major difficulties in physical oceanography has been to define the field of motion. The development of high sampling rate recording current meters demonstrated the existence of low and high frequency current fluctuations, (WEBSTER 1964). These current fluctuations can cause cable excitation leading to sensor motion, higher drag coefficients and high loading forces. Unless a current meter is secured to a fixed object or the sea floor, there is no method yet devised for avoiding sensor motion. Cable "strumming" is believed to cause an "effective" increase in cable diameter by as much as 50%. As the drag force is proportional to the cable diameter any such increase will result in large calculation errors. The limitations of using the "vibrating string" equation for determining the amplitude of this type of cable motion is described by POFFENBERGER, CAPADONA and SITER (1966). He criticises the lack of consideration for flexural rigidity, tension due to loop formation and internal or external damping of the cable.

II Waves

The effect of wave energy and motion on a mooring is one of the most complex factors to resolve. In a developed sea, the waves may consist of a spectrum of frequencies each differing in direction of travel, i.e. cross seas. Where in-situ wave measurements are not available and wind data is to be used for wave forecasting or hindsight calculations, several theories are available (Sverdrup-Munk-Bretschneider, Pierson-Neuman-James and Darbyshire). These appear to be reliable for specific situations (KING 1972). The results from two long series of wave observations obtained during the recent "Joint North Sea Wave Project" do not coincide with the currently assumed relationships between wind stress and bottom friction on wave development and attenuation respectively (CARLSON 1973).

The formula used to derive the wave drag is similar to the expression used for currents and wind stress but differs in being proportional to the Froude number. Variations in the values of drag and inertial coefficients reported by various investigators are described by MUGA and WILSON (1970) to :

(a) different wave theories used in the calculations resulting in non-uniform particle velocity and acceleration estimates.

(b) the tests covered a limited range of Reynolds numbers, and

(c) dissimilar flow conditions and experimental set-ups were devised.

Other complications arise, partially due to the simplification of physical factors. A body having 6 degrees of freedom has 21 inertial coefficients. These coefficients have been evaluated under ideal fluid considerations and neglect the effects of viscosity (MUGA and WILSON 1970).

What is generally not considered in mooring calculations is the effect of wave motion at the surface and at depth, causing drag, lift, depression and excitation on the subsurface elements of a mooring.

COEFFICIENTS AND SOLUTIONS

I Currents

The drag force due to currents is generally defined as :

$$F = \tfrac{1}{2} \rho C_D A V^2$$

where : F = drag force
ρ = mass density
C_D = drag coefficient
A = exposed area
V = relative current speed

Some typical, empirically derived drag coefficients for various shaped objects are shown in Figure I (MARKS 1958).

The mass density is the weight of a foot^3 or meter^3 of sea water depending on the system of units used. A small, but significant, error can occur when the vertical variations of temperature, salinity and pressure are not considered. For deep water moorings the error can be large. For example, the change in density due to the compressibility of sea water having a temperature of 0°C and a salinity of 35 $^{o}/oo$ over a 10,000 db range is 4.3% (RILEY and CHESTER 1971). These variations in density are shown in Table I. Another small increase in density can occur due to suspended sediments, especially near the sea floor in areas of high currents.

II Waves

A number of equations are available for predicting the wave forces on a rigid structure. These formulas have been included in this discussion with reservation about the degree of applicability to a non-rigid mooring array.

As mentioned earlier, wave drag is in the category of pressure (form) drag. The magnitude of the wave drag is proportional to the Froude number :

$$F = \frac{V}{\sqrt{gL}}$$

where : F = Froude number

V = relative velocity between the object and current

g = local gravitational acceleration

The expression for determining the wave drag on a rigid structure is:

$$F = \frac{1}{2} \rho C_J V^2 DL$$

where : F = force in pounds

ρ = mass density in slugs/ft^3

C_J = force coefficient (includes direct current and particle acceleration effects (WIEGAL 1964))

V = current speed in ft/sec

D = diameter of object in feet

L = length of object in feet

The various formulas for calculating the phase velocity, particle velocity and energy of deep and shallow water waves are :

(1) Deep water waves :

$$C = \left(\frac{gL}{2\pi}\right)^{1/2} ; \quad u = v = \pi \frac{H}{T} e^{2\pi z/L}$$

(2) Shallow water waves :

$$C^2 = \frac{gL}{2\pi} \tanh \frac{2\pi d}{L} ; \quad u = \frac{\pi H}{T} \frac{\cosh [2\pi (d - z)/L]}{\sinh 2\pi d/L}$$

where : C = wave phase velocity

g = local gravitational acceleration

L = wave length

d = water depth

u = horizontal particle speed/amplitude

v = vertical " " "

H = wave height (crest-to-trough)

T = wave period

z = vertical distance below mean surface

Shallow water waves are defined as :

$$\frac{d}{L} =< 0 \cdot 5$$

When the depth of water is less than 0.05 $^{d}/_{L}$, Tan h $\frac{2\pi d}{L}$ becomes almost $\frac{2\pi d}{L}$. The equation now becomes (KING 1972) :

$$C^2 = gd$$

(3) The wave energy is defined as :

$$E = 41\ H^2 T^2$$

where E = wave energy in foot-pounds/foot of wave crest
H = wave height in feet
T = wave period

The relationship between wave length (L) and wave period for deep water waves is :

$$L = 5\cdot 12\ T^2$$

where L is in feet, and
T is in seconds

High drag forces can develop throughout the upper regions of the water column. A breaking wave will have water particles moving at approximately the same speed as the wave, which could be on the order of 20 knots. The attenuation of orbital velocity with depth is related to the wave length. There is a 50% attenuation at a depth approximately 1/9 of the wavelength and 1/535 of the surface velocity at a depth equal to one wavelength (KING 1972).

Measurements of wave induced currents during a force 10 storm in the Celtic Sea showed that the surface generated currents had a speed of 8.7 knots at 10 fathoms which was attenuated to 1 knot at a depth of 100 fathoms (BERTREAUX and WALDEN 1964).

The results from long period recording tensiometers inserted in taut Woods Hole Oceanographic Institution deep water moorings using surface toroidal buoys, in one case, had mean tensions ranging from 1250-2500 pounds. The spread of values around the mean was about 600-700 pounds (BERTREAUX and WALDEN 1964). The conclusions reached by these two investigators demonstrate the difficulty in trying to analyse the forcing functions acting on a mooring.

"The amplitude of the dynamic tension does not seem to be related to the amplitude of the quasi static tension. High tension fluctuations are observed around low and high means.

The dynamic response to the same surface excitation is larger in compound moorings than in all nylon moorings.

The instantaneous dynamic response is a complex function of wind strength and wind history. Graphs of tension and wind versus time (Fig.12) can show trends but cannot permit to establish an explicit expression uniquely relating wind to dynamic tension. Maxima of dynamic tension can however be approximated by assuming that the dynamic tension varies as the wave height and that the wave height varies as the square of the wind. The dynamic tension can then be expressed as

$$T = K\,V^2$$

where

T = dynamic tension (lbs.)
K = wind response coefficient (lbs/knot^2)
V = wind speed (knots)

The wind response coefficients for the single point taut moored systems W.H.O.I. station number 298 (compound) and 299 (all synthetic fiber) were determined from maximum dynamic tension values and found to be 0.62 and 0.25 respectively.

The surface dynamic tension is attenuated down the mooring line. The experimental attenuation coefficient or ratio between two values of the dynamic range seems to be a function of the mooring material and of the excitation frequency. Under severe conditions, the wave action is felt all the way down where the dynamic bottom tension can be 1/4 of the surface value. An average value of wire rope attenuation coefficient in taut compound moorings is 0.3×10^{-3} lbs/meter of wire rope.

One formula for determining the dynamic loading on a mooring for a specific wave frequency is :

$$\tau = \frac{1}{f} = 2\pi \left(\frac{\frac{TL}{AE}}{g} \right)^{1/2}$$

where τ = natural period in seconds
f = frequency
T = cable tension in pounds
L = cable length in feet
A = cable cross sectional area in inches squared
E = modulus of elasticity in pounds per square inch

The static deflection of the mooring $Ys = \frac{TL}{AE}$

The final item is the inertial effect of "added mass" to wave forces. The inertial force is defined as :

$$\text{Force inertial} = (m + m')2$$

where :
m = mass of body
m' = added mass
a = acceleration

The "added mass" coefficient C_i is given by the expression :

$$C_i = \frac{\frac{\Delta F_T}{\Delta z}}{1/4\,\rho\,\pi D^2 \left(\frac{dv}{dt}\right)}$$

where :
C_i = added mass coefficient
F_T = elemental inertial fluid force in pounds
Δz = cartesian coordinate
ρ = mass density of fluid in slugs/ft^3
D = diameter of object in feet
$\frac{dv}{dt}$ = local acceleration

or

$$F_I = C_i \frac{\rho}{g} \frac{\pi}{4} D^2 u$$

where :
F_I = inertial force
D = diameter of object
C_i = added mass coefficient
u = time-varying particle acceleration

The inertial coefficient appears to be a function of body shape and the nature of fluid motion and properties (MUGA and WILSON 1970). The values of C_i for various shaped fixed structural objects are shown in Figure 2 (McCORMICK 1973).

FORMULAE AND COMPUTER PROGRAMS USED IN CALCULATING MOORING FORCES

The methods used to describe the forces, motion and configuration of mooring arrays range from simple to solutions requiring computers. The most simple method considers the sum of all the forces to be acting on the surface or subsurface buoy, steady state flow conditions and the mooring cable always normal to the current flow. Improvements on this model consider vertical variations in current speed (FOFONOFF 1965) derived a single level model. He assumed that the mooring cable is always normal to the current flow, the net buoyancy of the mooring acts at the subsurface buoy, only the top $\frac{1}{3}$ of the wire length is used for wire drag calculations, the resulting horizontal force acts at the buoy and the cable tension below the buoy is equal to the net buoyancy. Fofonoff's expression for lateral displacement along the axis is :

$$X = \frac{\rho C_D A L V^2}{2 F_B}$$

where :

X	=	horizontal axis
ρ	=	mass density of sea water
C_D	=	drag coefficient
A	=	diameter of object
L	=	length of object
V	=	current speed
F_B	=	net buoyancy

Another approach was devised by LAMPIETTI and SNYDER (1965). The L-S method considers the current and drag forces to be in the horizontal plane, the cable is near vertical and the drag and weight of the cable is constant throughout the mooring. The displacement in the X plane is given by :

$$X = \left(\frac{H}{W} + \frac{gT_F}{W^2}\right) \ell_n \frac{T_F}{T_S} - \frac{gL}{W}$$

where :

g	=	drag
W	=	weight of cable
H	=	drag on buoy
T_F	=	vertical component of tension on top of cable
T_S	=	vertical component of tension on bottom of cable.

A third method using a computer program called SHAPE is based on the original work undertaken by MIHOFF (1966) which was later modified by EAMES (1968). The parameters taken into consideration are basically the same as the L-S method except that the drag coefficient formulation is completely different. The drag coefficient for a perfectly faired cable is 1.0 whereas a rough cable has values ranging from 0.02 - 0.05. In addition, the normal and tangential drag forces are resolved into their component parts. The equations governing the equilibrium shape of the cable are :

$$P\,dS + dT = 0$$
$$Q\,dS + T\,d\alpha = 0$$

where : P = normal and tangential drag forces per unit length of cable.

Q = cable weight

S = arc length

α = horizontal angle

T = tension

Furthermore :

$$\frac{dT}{dS} = \mu R \cos\alpha + WT \sin\alpha$$

and

$$\frac{d\alpha}{dS} = \frac{\mu R \sin\alpha + (1-\mu) R \sin^2\alpha + WT \cos\alpha}{T}$$

The drag normal to per unit length of cable is :

$$R = {}^1/2\ \rho \cdot C_D \cdot D\ V\,|V|$$

where : R = normal drag force

C_D = drag coefficient

D = cable diameter

V = current speed

A computer evaluation of these three methods was undertaken by BARBER (1971). The models chosen consisted of a taut wire mooring in 2000 m of water using a 6 mm diameter cable, a 4 ft diameter spherical subsurface buoy and 4 current meters inserted in the moor. Attached to this subsurface buoy, for some of the calculations, was a length of buoyant rope and a small Dan buoy. Various weak to strong current profiles were assumed over the length of cable.

Barber concluded from this exercise that the L-S and SHAPE equations yielded similar mooring excursion results, whilst the results from Fofonoff's method ranges from approximately half to twice that of either. An expanded W.H.O.I. computer program (BERTREAUX and CHHABRA 1968) based on the earlier work of MARTIN (1968) has been developed for a single point mooring system. This program takes into account buoy shape, mooring line elastic properties, reduction of cable diameter due to tension, mooring line scope, insertion of instruments or buoyancy elements into the line, loads incurred during launching and vertical variations of current including reversals in direction. The computer program provides for the numerical integration of the forces on finite segments of cable. This is followed by an iteration process which compares the results with known physical constraints. If the results are outside the bounds of selected constraints, corrections are inserted and the program re-run. This evaluation technique developed by Bertreaux and Chhabra is two-dimensional and does not take into consideration non-planar variations in flow conditions, dynamic response of the mooring to waves and fluctuating currents.

An I.O.S. Bidston computer program (Moor) based on a W.H.O.I. program (MARTIN 1968), was designed by M. J. Howarth. Instead of the W.H.O.I. surface buoy and composite steel and nylon mooring line a subsurface buoy and all steel cable line was substituted.

COMPARISON OF THEORY AND OBSERVED RESULTS

Comparisons of observed and calculated tensions using the W.H.O.I. program were undertaken for 2 mooring arrays by BERTREAUX (1970). At Woods Hole station 298 tensiometers were deployed 10m and 2500m beneath the surface buoy. The near surface comparisons differed by 1% whilst at 2500m depth the observed value was 19% lower than the computed tension. At 50m depth (station 299) the computed tension was 13% lower than the in-situ value.

At I.O.S. Bidston, 13 comparisons were undertaken. The moorings were all deployed on the Continental Shelf of the U.K. in currents ranging from 80-220 cm/sec. The mooring arrays consisted of a subsurface buoy, eight millimeter diameter steel cable, several Bergen current meters and a 1200 cwt chain anchor. Different types of subsurface buoys were deployed at depths varying from 5-15m below chart datum. The top current meter located within

10 m below the subsurface buoy contained a strain gauge pressure sensor capable of resolving changes in water level in steps of 15 cm. Four types of subsurface buoys were used in this study: a bullet shaped Cosalt buoy, a spherically shaped solid Slingsby, a slightly elliptical free flooding Slingsby and a Marconi hydrodynamic designed discus. Observations in shallow water indicated that the Cosalt buoy tended to lie broadside, rather than face into the current. Therefore, two cases using different drag coefficients and buoy dimensions were used in the theoretical calculations for the Cosalt buoy. The subsequent analysis showed that the Cosalt buoy was indeed lying broadside to the current stream.

In ten of the thirteen cases cited the current meter pressure records were corrected for water level fluctuations using data obtained from a National Institute of Oceanography type offshore tide gauge deployed within twenty miles distance from the moorings. The tide gauge has a resolution of about 2 cm and an overall accuracy better than 10 cm. An arbitrary water level reference datum was selected at the time of slowest current flow during a tidal cycle. These flow speeds ranged between 2-20 cm/sec and at these times the mooring was considered vertical. A small error estimated to be about 1% will occur due to this current flow. Mooring arrays "J" and "TQ" were located too far from an offshore tide gauge. Therefore, the variations in water levels were calculated using Admiralty Tide Tables and the values adjusted using a cotidal chart.

The results of the comparisons are shown in Table 2. In twelve cases the difference between observations of buoy depressions and theory agreed within 0.4 m, which is approximately the magnitude of the combined instruments error. Considering the factors not taken into consideration, the close agreement between observations and theory is remarkable. The difference of 3.5 m at station "J" may possibly be ascribed to errors associated with using cotidal charts, barometric changes and wind stress (BANKS 1974). Tidal observations from various sites in the Irish Sea have on occasion disagreed by as much as one hour with cotidal chart values (Dr. J. M. Vassie, personal communication).

CONCLUSION AND SUGGESTIONS FOR FUTURE EXPERIMENTAL WORK

The I.O.S. Bidston and W.H.O.I. results indicate that despite the deficiencies in the mooring equations, results better than 20% and in many cases 5% have been obtained.

The major problems are to find satisfactory sites and instrumentation where a more rigorous defining of the mooring parameters and interactions can be undertaken. Ideally, the suite of instruments should consist of high sampling rate recording; pressure transducers, inclinometers, tri-axial accelerometers, current meters, tide gauges, wind and wave meters. Several sites should be selected in both shallow and deep water, all offering various degrees of protection from wind and wave action as well as providing a near uniform current field.

Hopefully, a well executed series of experiments on individual components as well as mooring arrays at various sites and sea conditions will yield valuable information on the physical constants and help to resolve the various forces acting on a mooring into their respective components.

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TABLE 1

Variation in sea water density as a function of depth
(after RILEY and CHESTER 1971)

depth (d_b)	specific gravity	% decrease in volume
0	1.02812	0.000
100	1.0260	0.046
1,000	1.03285	0.458
5,000	1.05071	2.150
10,000	1.07104	4.007

TABLE 2

Results of comparisons between theoretical and observed subsurface buoy depressions

Mooring Array	Date	Water Depth (m)	Buoy Type	Assumed C_D	Depth of Buoy (m)	No. of Bergen Current Meters	Current Speed (cm/sec)	Observed Subsurface Buoy Depression (m)	Theoretical Depression (m)
G E	10/72	83	Cosalt (1)	1.2	6	3	115	3.9	4.1
G E	10/72	83	Cosalt (1)	1.2	6	3	90	1.7	1.6
M	10/72	95	Cosalt (1)	1.2	11	3	135	7.7	8.2
M	10/72	95	Cosalt (1)	1.2	11	3	115	4.9	4.8
J	7/73	91	Free Flooding Slingsby	0.7	15	3	167	13.6 (2)	17.1
G S	10/72	76	"	0.7	5	3	110	3.7	4.0
G	10/72	78	Solid Slingsby	0.6	5	3	90	1.5	1.5
G	10/72	78	"	0.6	5	3	100	2.6	2.3
T Q	9/73	22	"	0.6	8	2	175	0.9 (3)	1.3
T Q	9/73	22	"	0.6	8	2	220	2.8 (3)	2.8
Z O	9/73	24	Marconi	0.2	10	2	70	0.4	0.0
Z O	9/73	24	"	0.2	10	2	80	0.3	0.0
Z O	9/73	24	"	0.2	10	2	90	0.3	0.1

(1) Buoy broadside to current stream

(2) Tidal data from Admiralty Tidal Tables (Milfordhaven) and cotidal chart

(3) " " " " " " (Dover)